Survey of theoretical calculations of cross-sections for inclusive reaction
\[ \nu_\mu^{12}C \rightarrow \mu + X \] measured in LSND

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1 Introduction

The \( \nu_\mu \) beam arising from \( \mu^+ \) decay at rest is of low energy: \( \leq 52.8 \) MeV and transitions occur to a few low lying states of \( ^{12}\text{N} \) (60\% to g.s.). The cross-sections can be predicted with an accuracy of \( \sim 2\% \). All 3 experimental results for \( ^{12}C(\nu_\mu, e^-)^{12}N_{g.s.} \) [2] (E225), [3] (KARMEN) and [6] LSND agree with calculations [1].

Also cross-sections calculated by [8] for the transitions to \( ^{12}\text{N} \) excited states \( ^{12}C(\nu_\mu, e^-)^{12}N^* \) using continuum random phase approx. (CRPA) agree with the results reported by the three experiments.

For \( \nu_\mu \) neutrinos produced by pion decays in flight the situation is more complicated. Exclusive cross-sections i.e. transitions to the \( ^{12}\text{N} \) ground state or 15.11 MeV excited state of \( ^{12}\text{C} \) (via NC reaction) measured by Karmen and LSND can be reproduced by different model calculations for neutrino energies up to 300 MeV.

However the above exclusive cross-sections account for a small fraction of the inclusive cross sections at those energies. The calculations of transitions to the unbound continuum states are much more complicated and at variance with experimental results.

2 Experimental Results

LSND have obtained the following results using a \( \pi^+ \) decay-in-flight \( \nu_\mu \) beam at LAMF:

- Analysis of 1993 data [4]
Energy region: $123.7 \leq E_{\mu} \leq 280$ MeV.
The flux weighted average neutrino energy is $\leq E_\nu > = 180$ MeV. Measured cross-section: $8.3 \pm 0.7 \pm 1.6 \times 10^{-40}$ cm$^2$ (out of 210 $\pm$ 17 events only 6 to $^{12}N(g.s)$).
The LSND collaboration uses Coulomb-corrected Fermi Gas Model (FGM) and obtains: $24 \times 10^{-40}$cm$^2$.

The energy range the same as for 1993 data above, but the spectrum is somewhat harder for a part of the data. They have found $56.8 \pm 9.6$ events leading to $^{12}N(g.s)$ which correspond to the exclusive cross-section $6.6 \pm 1.0 \pm) \times 10^{-41}$. Inclusive cross section was measured to be an order of magnitude higher: $(12.4 \pm 0.3 \pm 1.8) \times 10^{-40}$cm$^2$.

3 Models

Almost all the models are based on the random phase approximation (RPA) to describe residual particle-hole interactions. However all the papers referenced below consider only one particle-one hole interactions and two nucleon knock-outs are not taken into account.

- Continuum Random Phase Approximation (CRPA) - Vogel and collab. [7], [8], [9], [10], [11]
  According to authors: “In this approach the usual RPA treatment is combined with a correct description of the particle states in the continuum, i.e. the excited many-body states are coherent superpositions of one-particle-one-hole (1p-1h) excitations”.
  Use different parametrizations to describe (p-h) interactions. They reproduce total $\mu^-$ capture rates in nuclei like $^{12}$C, $^{16}$O and $^{40}$Ca and the inclusive cross section for $^{12}C(\nu_\mu, e^-)^{12}N^*$ for $E_\nu \leq 52.8$MeV. measured in LSND [6].
  For the process $^{12}C(\nu_\mu, \mu^-)^{12}N^*$ they describe well the measured distribution of the muon energy of Ref. [5], but obtain $19.3 - 20.3 \times 10^{-40}$cm$^2$ for total inclusive ($0.63 \times 10^{-40}$cm$^2$ to $^{14}$N ground state).

- “Essentially” RPA, (Oset and collab.) [12], [17], [18]
  $19 \times 10^{-40}$cm$^2$ in [12]
  $16.6 \times 10^{-40}$cm$^2$ in [17]

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1 According to a recent private message obtained by B. Svoboda from R. Inlay the new LSND result is somewhat higher: $(12.4 \pm 0.3 \pm 1.8) \times 10^{-40}$cm$^2$
They conclude that the most recent result agrees with the experiment within errors.

- RPA “with a pairing correction” (Auerbach et al.) [16]
  Announced that they’ve got an agreement with the data: 13.5 - 14.5 × 10^{-40} \text{cm}^2. However their result for muon capture needs an adjustment of a parameter to agree with the data.

  They write that after “a pairing correction is introduced to the RPA results the flux-averaged theoretical \((\nu_\mu, \mu^-)\), \((\nu_\tau, e^-)\) cross-sections and \(\mu^-\)-capture rates in \(^{12}\text{C}\) are in good agreement with experiment”.

- Kim, Piekarewicz and Horowitz [19]
  Claim that relativistic nuclear-structure effects and a strongly reduced value of the nucleon mass in the medium can account for the more than a factor-of-two reduction in the FGM cross section calculations and agree with recently reported by the LSND collaboration. But Kolbe in Ref. [9] criticizes that the same effect would reduce also \(\nu_e\) cross-sections in disagreement with the experiment. Preprint never published.

- Mintz and Pourkaviani, [14]

  Non RPA but “Elementary particle approach” considers nuclei as elementary particles and describes matrix elements in terms of nuclear form factors deduced from muon capture, pion electroproduction and electron scattering data. [13]. Model is criticized by [9],[15] and [17]. They show that Coulomb correction in the final state increases the cross-section by 20% and obtain 13.1 × 10^{-40} \text{cm}^2 with an error in the 25% to 30% range.

4 Conclusions

Both muon capture and \(^{12}\text{C}(\nu_e, e^-)\(^{12}\text{N}^*\) reactions for \(E_\nu \leq 52.8\text{MeV}\) involve very low lying excited states and correspond to low momentum transfers: \(\leq 100\text{MeV}/c\). On the other hand inclusive cross-sections \(^{12}\text{C}(\nu_\mu, \mu^-)\(^{12}\text{N}^*\) involve moment transfers from 100 to 400 \text{MeV}/c. Clearly the theoretical description for this range of momentum transfers is not unique and a variety of approaches and results are possible (see Table).

Although the energies of interest in both atmospheric neutrinos and K2K experiment are higher but low momentum transfers are significant fractions of the total cross sections (especially if forward muon production is selected) and therefore the uncertainties in the LSND region may be also present. In a next note I’ll make a survey of experimental data available at higher energies.
Table 1: Cross-sections for $\nu_\mu^{12}C \rightarrow \mu + X$

<table>
<thead>
<tr>
<th>Year</th>
<th>( \times 10^{-40} \text{cm}^2 )</th>
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<tbody>
<tr>
<td>1997</td>
<td>11.2 ± 0.3 ± 1.8</td>
</tr>
<tr>
<td>1997</td>
<td>19.3 - 20.3</td>
</tr>
<tr>
<td>1996</td>
<td>19</td>
</tr>
<tr>
<td>1996</td>
<td>16.6</td>
</tr>
<tr>
<td>1998</td>
<td>13.5 - 14.5</td>
</tr>
<tr>
<td>1995</td>
<td>0.4 - 0.7 of FGM xsec</td>
</tr>
<tr>
<td>1995</td>
<td>13 ± 4</td>
</tr>
</tbody>
</table>

References


