Physics 142 - November 20, 2008

- No workshops Next week
- Shortish P.S. due Tuesday
- Presentation schedule
  - Tues. Dec 2 grp 7 superconductors + TBA
  - Have preferences from group 1
    Animals + E+M
  - Need prefs from other groups ASAP

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \longrightarrow \int_V (\vec{\nabla}\cdot\vec{E})\,d = \int_V \rho\, dv$$

$$\longrightarrow \boxed{\vec{\nabla}\cdot\vec{E} = \rho}$$

$$\oint_S \vec{B}\cdot d\vec{A} = 0 \longrightarrow \int_V (\vec{\nabla}\cdot\vec{B})\,dv = 0$$

$$\longrightarrow \boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 \int \vec{j}\cdot d\vec{A} + \mu_0\epsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{A}$$

$$\longrightarrow \int_S (\vec{\nabla}\times\vec{B})\cdot dA = \mu_0 \int_S j\cdot dA + \mu_0\epsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{A}$$

$$\longrightarrow \boxed{\vec{\nabla}\times\vec{B} = \mu_0 \vec{j} + \mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}$$

$$\longrightarrow \int_S (\vec{\nabla}\times\vec{E})\cdot dA = -\frac{\partial}{\partial t}\int B\cdot dA \longrightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Differential form of Maxwell's eqns

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \mu_0\epsilon_0 \frac{\partial(\vec{\nabla}\times\vec{E})}{\partial t}$$

from another Maxwell eqn

$$\vec{\nabla}\cdot(\vec{\nabla}\vec{B}) =$$

$$\vec{\nabla}^2\vec{B} = \mu_0\epsilon_0 \frac{\partial(\nabla\times E)}{\partial t} = \mu_0\epsilon_0 \frac{\partial}{\partial t}\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$\nabla^2\vec{B} = -\mu_0\epsilon_0 \frac{\partial^2\vec{B}}{\partial t^2}$$

$$\nabla^2 B_x = -\mu_0\epsilon_0 \frac{\partial^2 B_x}{\partial t}$$

$$\nabla^2 B_y = -\mu_0\epsilon_0 \frac{\partial^2 B_y}{\partial t^2}$$

$$\nabla^2 B_z = -\mu_0\epsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

$$\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} = -\mu_0 \epsilon_0 \frac{\partial B_x}{\partial t^2}$$

Similarly

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$\nabla \times$ $\nabla \times$

$$\nabla^2 \vec{B} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

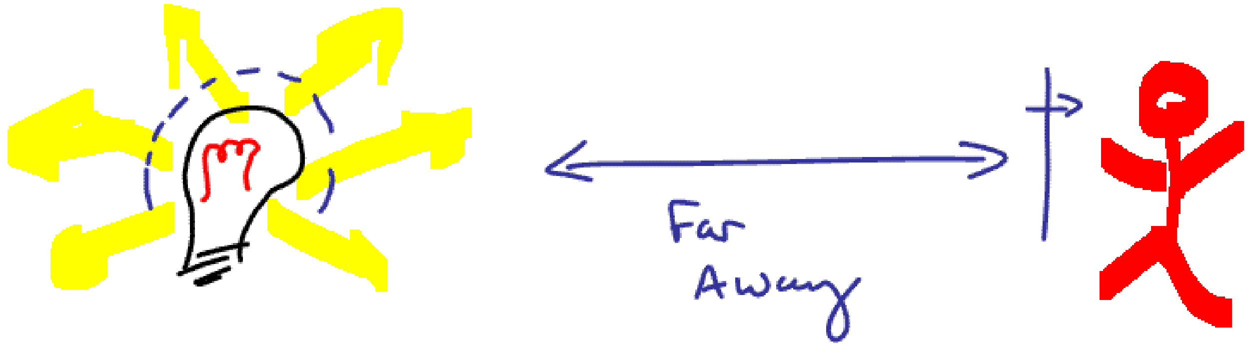
$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

1d wave prop in x

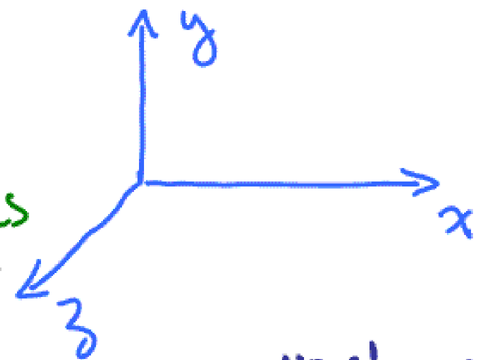
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

wave eqns for $\vec{E}, \vec{B}$ w/ velocity of propagation

$$\sqrt{\frac{1}{\mu_0 \epsilon_0}} = c$$



Impose Boundary Conditions



$\vec{E} = \vec{E}(x,t)$

No charge $\rightarrow \rho = 0$

$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$

$\vec{\nabla} \cdot \vec{E} = 0$

$$\frac{\partial E_x}{\partial x} + \underbrace{\cancel{\frac{\partial E_y}{\partial y}}}_{0} + \underbrace{\cancel{\frac{\partial E_z}{\partial z}}}_{0} = 0$$

So E_x is const for all $x \rightarrow E_x = 0$

E_y or E_z might be nonzero

E is Transverse to Direction of Propagation

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

Equivalent to this

$E = E(x,t)$ only

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t}$$

$$-\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) = -\frac{\partial B_y}{\partial t}$$

$$\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = -\frac{\partial B_z}{\partial t}$$

Impose Boundary Condition

$\vec{E} = E(x,t)\,\hat{j}$

"polarization"

Time dependent B field

$\perp$ to $\vec{E}$

B_x is constant in time

B_y is constant in time

$\vec{E}, \vec{B}$ are Transverse and Mutually $\perp$

$$E_y(x,t) = E_{oy}\cos(kx - \omega t + \phi)$$

$\frac{2\pi}{\lambda}$

$\frac{2\pi}{T}$

Phase (initial condition)

$$B_z = -\int \frac{\partial E_y}{\partial x}\, dt$$

set $\phi = 0$ for now

$$B_z = \int k E_{oy} \sin(kx - \omega t)\, dt$$

$$B_z = \frac{k}{\omega} E_{oy} \cos(kx - \omega t)$$

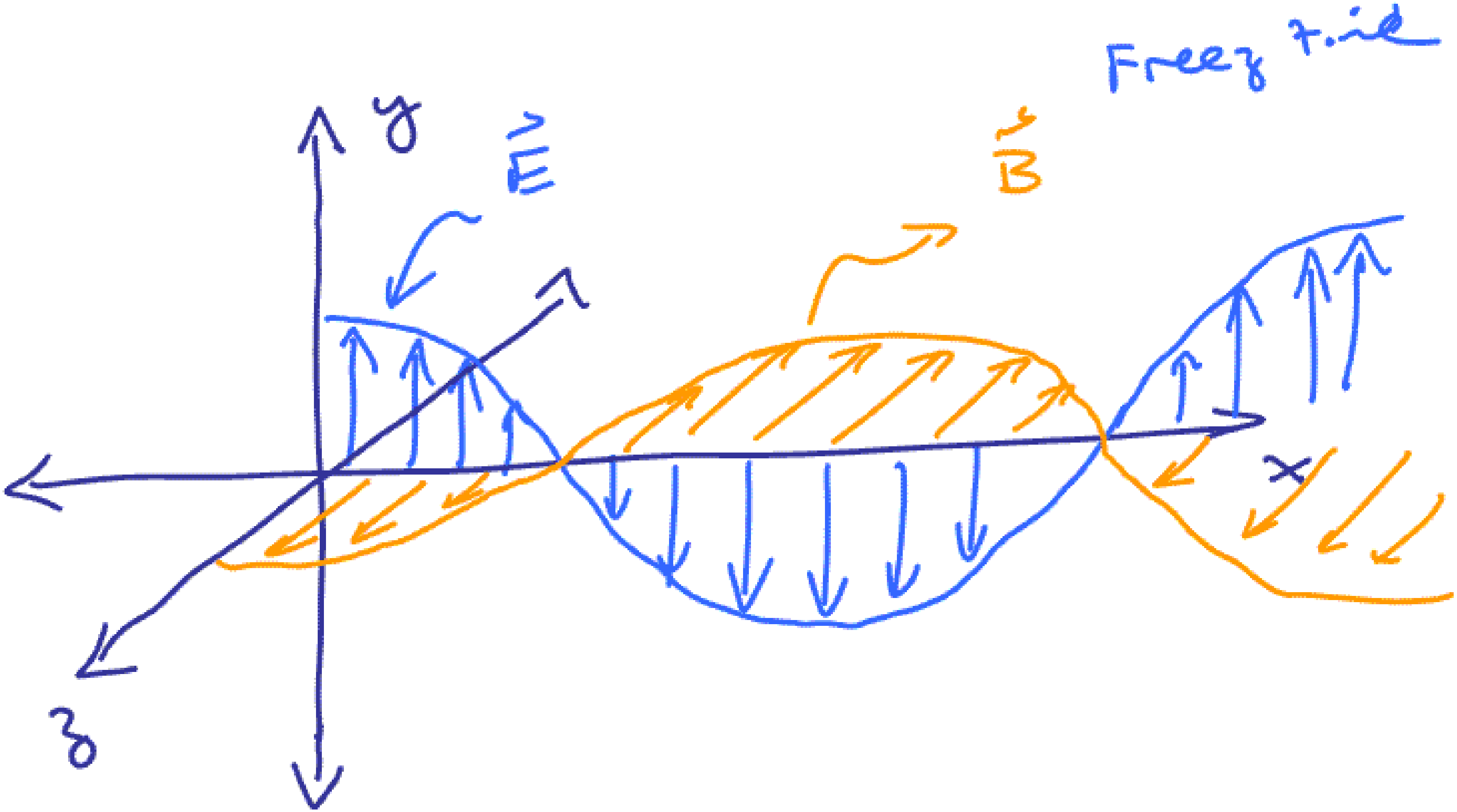
$$\frac{k}{\omega} = \frac{\frac{2\pi}{\lambda}}{\frac{2\pi}{T}} = \frac{T}{\lambda} = \frac{1}{c}$$

$$B_z = \frac{1}{c} E_y$$

E, B are coupled, time dependent

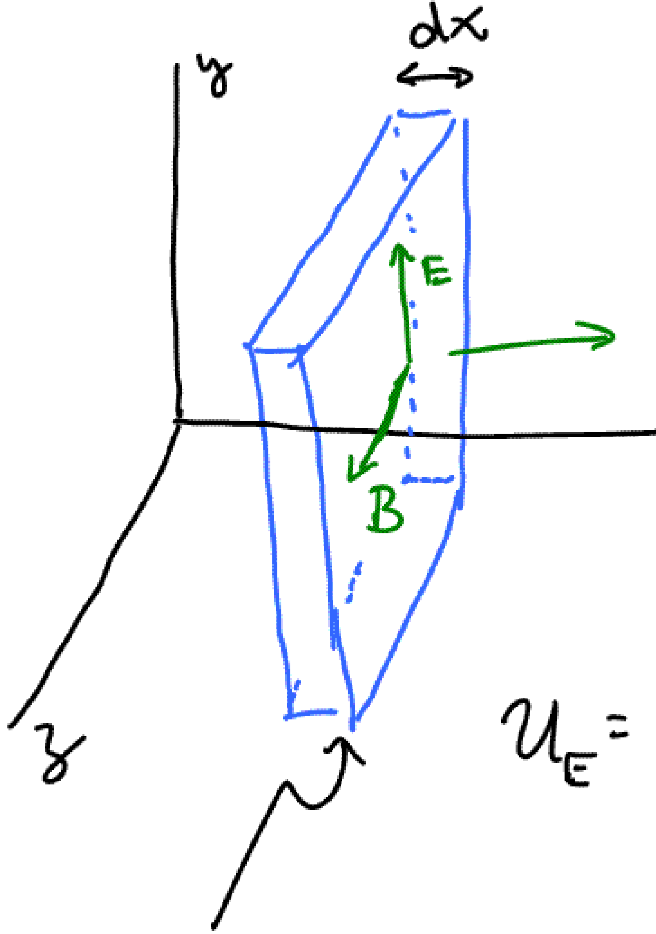
Mutually $\perp$ in phase in

$|\vec{E}| = c\,|\vec{B}|$ Time + space



Freez time
y
$\vec{E}$
$\vec{B}$
x
z

Energy Flow in EM waves



$$u_E = \frac{\epsilon_0}{2} E^2 \qquad u_B = \frac{1}{2\mu_0} B^2$$

What is Energy in volume

$$dU = (u_E + u_B) \underbrace{\text{volume of box}}_{(\text{Area})\,dx}$$

$$dU = \left(\frac{\epsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2\right)(\text{Area})\,dx$$

$$E = cB \qquad \epsilon_0 = \frac{1}{\mu_0 c^2} \qquad c\ \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$dU = \left[\frac{1}{2\mu_0 c^2}EcB + \frac{1}{2\mu_0}B\frac{E}{c}\right](\text{Area})\,dx$$

ddd diff energy moves thru box

in time $dx/c = dt$

$$dU = \left(\frac{1}{\mu_0 c}EB\right)\text{Area}\,dx$$

$$\frac{dU}{dt} = \frac{EB}{\mu_0} \text{ Area}$$

$$\frac{dU}{dt}\frac{1}{\text{Area}} = \frac{EB}{\mu_0} = \frac{\text{Watts}}{m^2}$$

$$\equiv \text{Intensity}$$

Energy flux