Physics 142 - November 18, 2008

- Exams graded ...
- Presentation group meetings

$$B = \mu_0 \left(1 + \chi_M \right) B_{free}$$

Magnetic Susceptibility

External B field

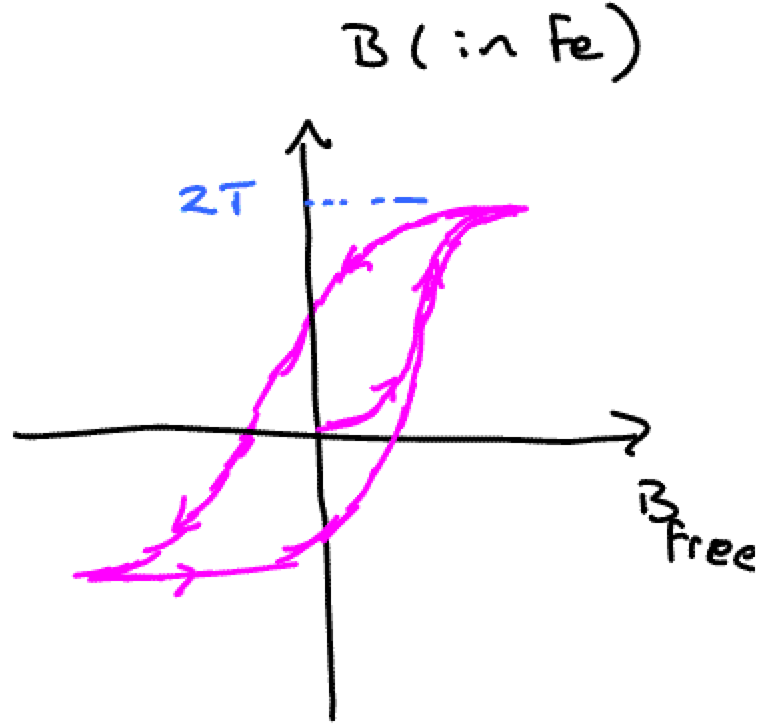
relative permeability

$\mu_0 K_M \sim$ permeability $\to \mu$

$K_M \gtrsim 1$ Paramagnetism

$\gg 1$ Ferromagnetism

< 1 Diamagnetism



Hysteresis loop

Last Time

Integral form of Maxwell's equations

Gauss $\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$

No magnetic monopole $\oint_S \vec{B}\cdot d\vec{A} = 0$

Ampere $\oint_c \vec{B}\cdot d\vec{l} = \mu_0 \int_s \vec{j}\cdot d\vec{A} + \boxed{\mu_0 \epsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A}}$

new term - "Maxwell's Displacement current"

Faraday $\oint_c \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}$

Will now derive "differential" form ...

Divergence of vector field $\vec{V}$ (cartesian coordinates)

$$\vec{\nabla}\cdot\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

Measures divergence or convergence of field


+ Divergence


− Divergence


0 Divergence

Can prove to yourself

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

Degree of "Divergence" or "convergence"

$$\vec{\nabla}\cdot\vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(V_x\hat{i} + V_y\hat{j} + V_z\hat{k}\right)$$

use $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0$

vector calculus

Gauss' Theorem
Green's theorem
divergence theorem
$$\rightarrow \int_{Vol} (\vec{\nabla} \cdot \vec{V})\, dv = \int_{Surf} \vec{V} \cdot d\vec{A}$$

for vector field $\vec{V}$

Gauss

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \rightarrow \int_S \vec{E} \cdot d\vec{A} = \int_V (\vec{\nabla} \cdot \vec{E})\, dv = \frac{\int_V \rho\, dv}{\epsilon_0}$$

$$\boxed{\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0}$$

No magnetic monopole

$$\oint_S \vec{B} \cdot d\vec{A} = 0 \longrightarrow \int_S \vec{B} \cdot d\vec{A} = \int_V (\vec{\nabla} \cdot \vec{B})\, dv = 0$$

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$$

Curl of vector field

$$\text{curl}\,\vec{V} \equiv \vec{\nabla}\times\vec{V} \longrightarrow \underline{\underline{\text{vector}}}$$

cartesian

$$\vec{\nabla}\times\vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} = \hat{i}\left(\frac{\partial}{\partial y}V_z - \frac{\partial}{\partial z}V_y\right) - \hat{j}\left(\frac{\partial}{\partial x}V_z - \frac{\partial}{\partial z}V_x\right) + \hat{k}\left(\frac{\partial}{\partial x}V_y - \frac{\partial}{\partial y}V_x\right)$$

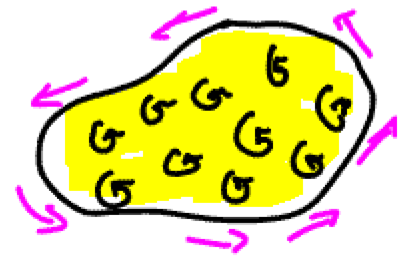
large curl

0 curl

degree of "circulation" of vector field about point

Stoke's Theorem

$$\oint_C \vec{v}\cdot d\vec{l} = \oint_S (\vec{\nabla}\times\vec{v})\cdot d\vec{A}$$



$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\oint_S \vec{B}\cdot dA$$

$$\oint_S (\vec{\nabla}\times\vec{E})\cdot d\vec{A} = -\frac{d}{dt}\oint_S \vec{B}\cdot d\vec{A}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}}$$

$$\oint_{\ell} B \cdot d\ell = \mu_0 \oint_S \vec{j} \cdot \vec{dA} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \oint_S \vec{E} \cdot \vec{dA}$$

$$\oint_S (\vec{\nabla} \times \vec{B}) \cdot \vec{dA} = \mu_0 \oint_S \vec{j} \cdot \vec{dA} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \oint_S \vec{E} \cdot \vec{dA}$$

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \vec{E}}$$

region w/ NO current

$$\vec\nabla\times\vec B = \mu_0\epsilon_0 \frac{\partial\vec E}{\partial t}$$

Take curl of Both sides

$$\underbrace{\vec\nabla\times(\vec\nabla\times\vec B)}_{\text{and it was said ...}} = \mu_0\epsilon_0 \frac{\partial(\vec\nabla\times\vec E)}{\partial t}$$

$$\vec\nabla\times\vec\nabla\times\vec B = \vec\nabla(\underbrace{\cancel{\vec\nabla\cdot\vec B}}_{\vec\nabla\cdot\vec B = 0}) - \underbrace{\vec\nabla\cdot(\vec\nabla\vec B)}_{\text{Known as the Laplacian } \nabla^2\vec B}$$

Laplacian of Scalar field T

$$\nabla^2 T = \vec{\nabla} \cdot (\vec{\nabla} T) = \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \cdot \left(\hat{i} \frac{\partial T}{\partial x} + \hat{j} \frac{\partial T}{\partial y} + \hat{k} \frac{\partial T}{\partial z} \right)$$

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

Laplacian of vector field $\vec{v}$

$$\nabla^2 \vec{v} = (\nabla^2 v_x)\hat{i} + (\nabla^2 v_y)\hat{j} + (\nabla^2 v_z)\hat{k}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \mu_0\epsilon_0 \frac{\partial(\vec{\nabla}\times\vec{E})}{\partial t}$$

from another Maxwell eqn

$$\vec{\nabla}\cdot(\vec{\nabla}\vec{B}) =$$

$$\vec{\nabla}^2\vec{B} = \mu_0\epsilon_0 \frac{\partial(\nabla\times E)}{\partial t} = \mu_0\epsilon_0 \frac{\partial}{\partial t}\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$\nabla^2\vec{B} = -\mu_0\epsilon_0 \frac{\partial^2\vec{B}}{\partial t^2}$$

$$\nabla^2 B_x = -\mu_0\epsilon_0 \frac{\partial^2 B_x}{\partial t}$$

$$\nabla^2 B_y = -\mu_0\epsilon_0 \frac{\partial^2 B_y}{\partial t^2}$$

$$\nabla^2 B_z = -\mu_0\epsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

$$\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} = -\mu_0 \epsilon_0 \frac{\partial B_x}{\partial t^2}$$