

Physics 142 - November 4, 2008

Happy Election day

- Lecture Thursday - online PDF + MP3  
class will NOT be meeting
- EXAM 2 - Week from Thursday  
HoyT - During normal lecture time

Lecture: Oct 2 (dielectrics) - Oct 30 (inductance)

LR, LC circuits, AC circuits + beyond NOT on EXAM 2

Text: chapters 26 (dielectrics), 27 - 31, 36

but NOT section 31.6 on LR circuits

Prob Sets: P.S. 4 (probs 13-15), P.S. 5-9

Workshops: Workshops 5-9

# Induction

Faraday's Law

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = - \frac{d\Phi_m}{dt}$$

↑ Induced EMF

$$\Phi_m \equiv \int_{\text{loop}} \vec{B} \cdot d\vec{A}$$

Last  
Time

Lenz's Law - An induced current in a closed conducting loop will appear in such a way as to oppose the change that created it.

Gives direction of induced effect



$$\Phi_m = L i$$

$L \equiv$  CONSTANT OF  
Self-inductance

$$\mathcal{E} = - \frac{d\Phi_m}{dt} = - L \frac{di}{dt}$$



$\text{resistor} \equiv L$   
units Henrys

$$\mathcal{E} = - L \frac{di}{dt}$$



$$\Phi_{m(2)} = M i_{(1)}$$

$$\Phi_{m(1)} = M i_{(2)}$$

SAME "M"  
Cross-talk dictated  
by Geometry

$M \equiv$  CONSTANT OF Mutual inductance

$$\mathcal{E}_{(2) \text{ by } (1)} = - M \frac{di_{(1)}}{dt}$$

$$\mathcal{E}_{(1) \text{ by } (2)} = - M \frac{di_{(2)}}{dt}$$

unless problem is specifically about  
mutual inductance ... usually treat  
inductance in a circuit as self-inductance

Energy density in the fields:

$$u_E = \frac{\epsilon_0 E^2}{2}$$

$$u_B = \frac{B^2}{2\mu_0}$$

general — says nothing about  
circuits or sources  
or boundary  
conditions!

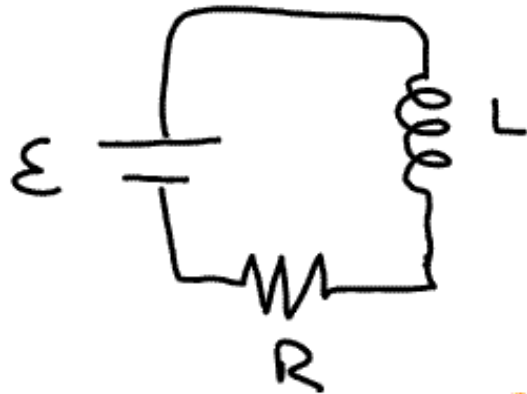
Energy in Inductor

$$U = \frac{1}{2} LI^2$$

similar to  $U_{\text{capacitor}} = \frac{1}{2} CV^2$

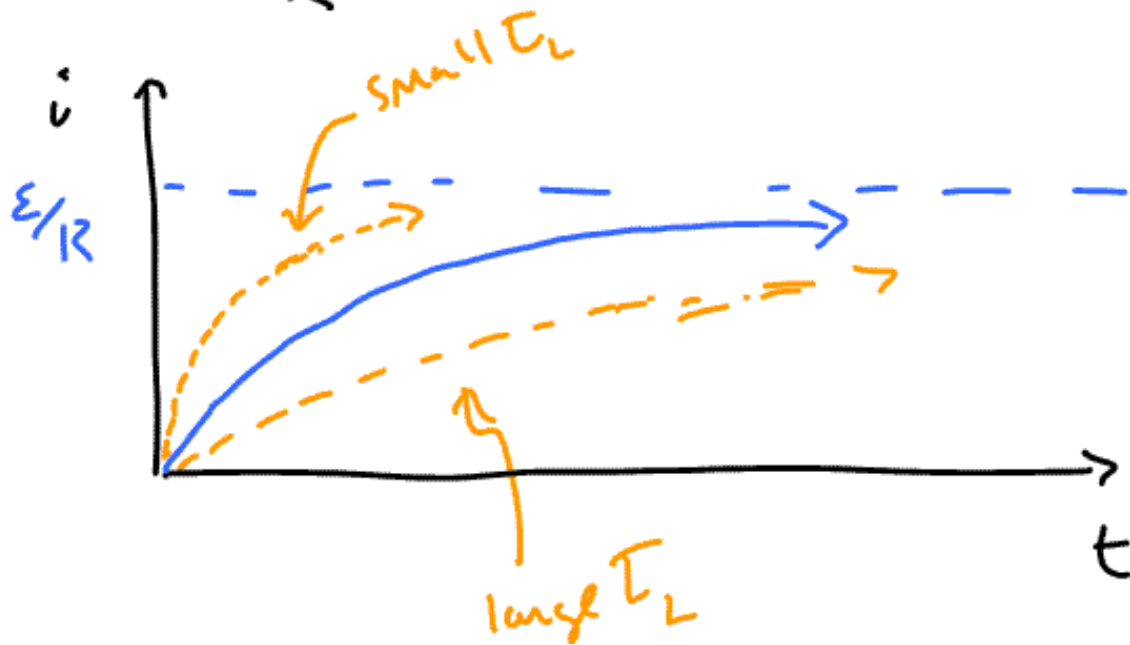
# LR circuit

$\sum V = 0$  around circuit



$$\mathcal{E} - L \frac{di}{dt} - iR = 0$$

$$i = \frac{\mathcal{E}}{R} (1 - e^{-tR/L}) = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L})$$



Inductive time  
constant

$$\equiv \tau_L = \frac{L}{R}$$

$$V = iR \quad (V \text{ across } R)$$

$$V = \mathcal{E}(1 - e^{-tR/L}) = \mathcal{E}(1 - e^{-t/\tau_L})$$

at some large  $t$  later, short  $L$  across  $R$

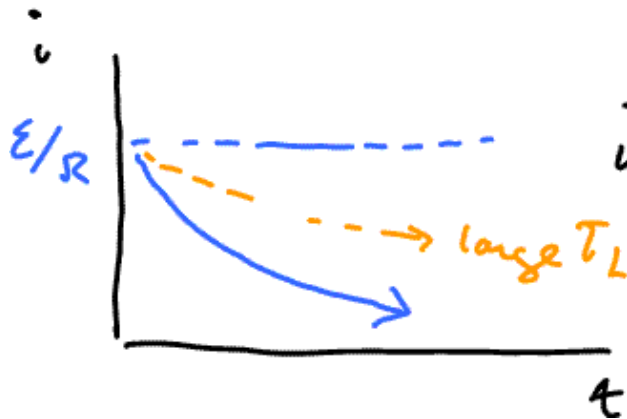


$$0 = iR + L \frac{di}{dt}$$

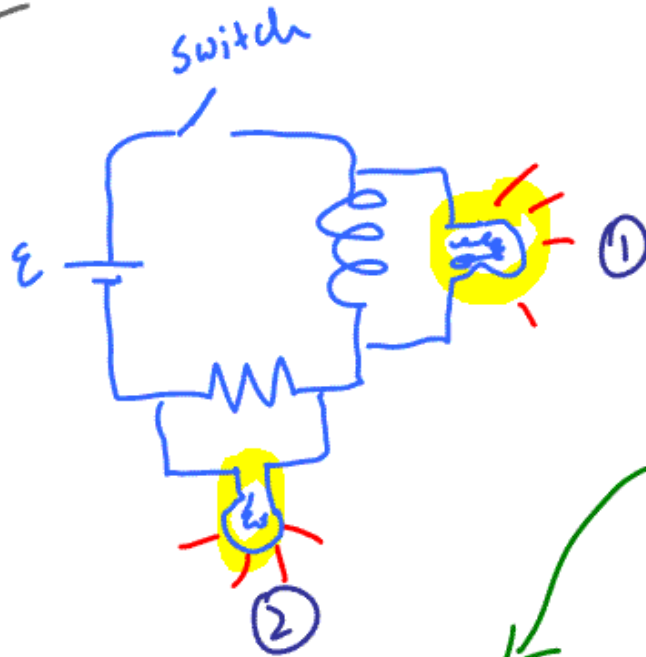
$$i = \frac{\mathcal{E}}{R} e^{-t/\tau_L}$$

$$i = i_0 e^{-t/\tau_L}$$

after  $i$   
reaches  
asymptotic  
value of  
 $\mathcal{E}/R$



Demo



did demo -

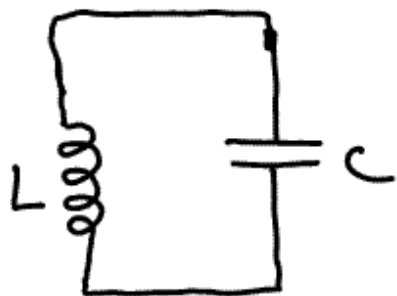
Turn switch on  
What happens to  
light bulbs?

Turn switch off  
What happens  
to light bulbs?

early time  
switch on  $i=0$ ,  $\frac{di}{dt}$  big  $\rightarrow$  ② off ① bright  
Some time later  $i$  increases ② gets brighter  
 $\frac{di}{dt}$  less ① dims

# LC circuit

imagine that capacitor is fully charged at  $t=0$



$$R \rightarrow 0$$

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{L i^2}{2}$$

$$\frac{1}{2} C V^2 = \frac{1}{2} \frac{q^2}{C}$$

$$\frac{dU}{dt} = 0 = \frac{2q}{2C} \frac{dq}{dt} + \frac{L}{2} \frac{2i}{dt} \frac{di}{dt}$$

$$0 = L \frac{di}{dt} + \frac{q}{C}$$

$$\frac{dq}{dt}$$

$$0 = L \frac{d^2 q}{dt^2} + \frac{q}{C}$$

SHM



$$m \frac{d^2 x}{dt^2} = -kx$$

$$m \frac{d^2 x}{dt^2} + kx = 0$$

↳ SHM  $\omega = \pm \sqrt{\frac{k}{m}}$

SHM in  $q(t)$  w/  $\omega \sim \pm \frac{1}{\sqrt{LC}}$

$$q(t) = Q \cos(\omega t + \phi)$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$i = \frac{dq}{dt} = -\omega Q \sin(\omega t + \phi)$$

Look at energy flow ...

$$U_E = \frac{q^2}{2C} = \frac{Q^2 \cos^2(\omega t + \phi)}{2C}$$

Energy in  
capacitor  
Stored as  
 $\vec{E}$  field

$$U_B = \frac{1}{2} L i^2 = \frac{L}{2} Q^2 \omega^2 \sin^2(\omega t + \phi)$$

Energy stored  
in B field in  
inductor

# LC Oscillations

