Physics 142 - October 28, 2008

Previously

$$\vec{B} = \int_{\text{current}} \frac{i\,d\vec{l} \times \hat{r}}{r^2}$$

Biot-Savart

See java Applet

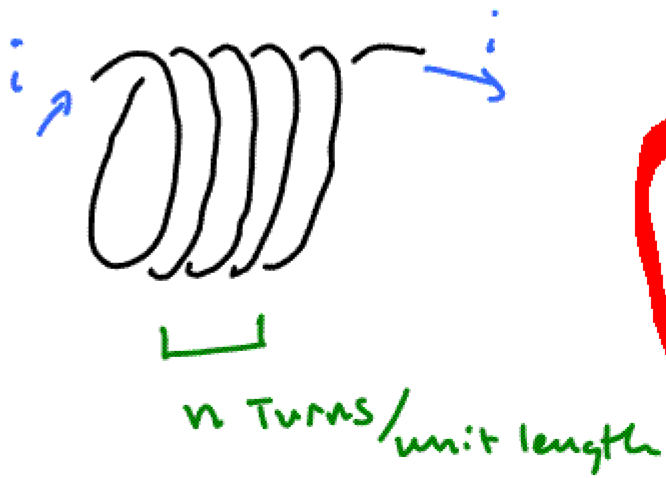
Electrostatics

Gauss' law

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

magnetostatics

$$\boxed{\text{Ampere's law} \qquad \int_{\substack{\text{closed}\\\text{curve}}} \vec{B} \cdot d\vec{s} = \mu_0 I_{enclosed}}$$

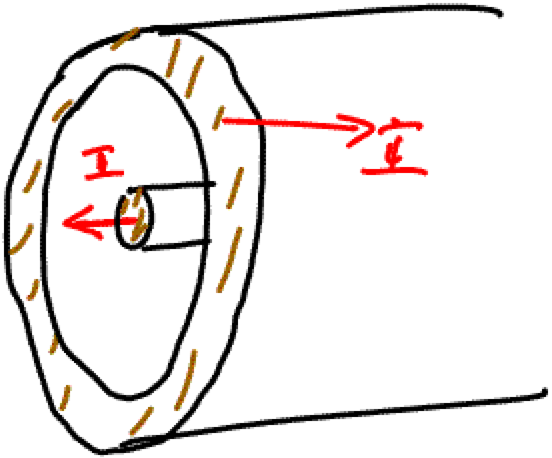


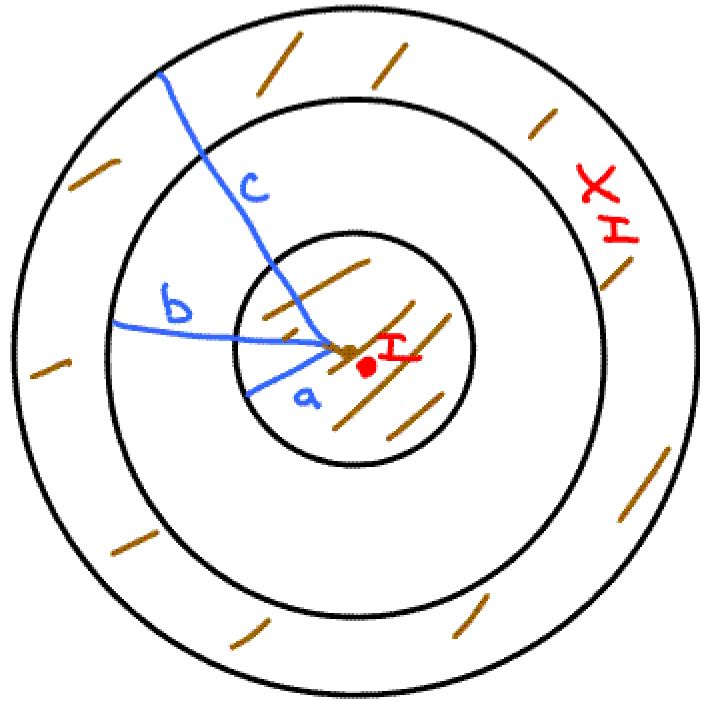
$$\left|\vec{B}_{\substack{\text{inside}\\\text{solenoid}}}\right| = \mu_0 n i$$

$$\vec{B}_{\substack{\text{outside}\\(\infty)\text{ solenoid}}} = 0$$

more Right-hand rules ~

Coaxial cable (long)





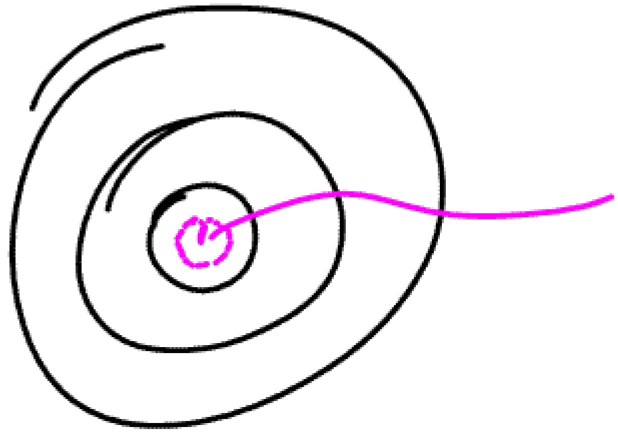
Assume I is uniform across both inner + outer conductors.

Find $\vec{B}$ in all space

use Ampere's law

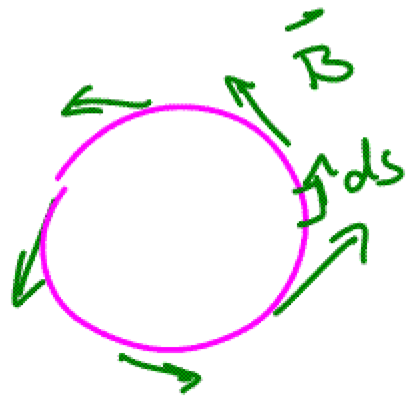
$$\oint_{\text{curve}} \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$$

$r < a$



coaxial, circular
Amperian loop
radius $r < a$

$\perp$ To Axis



I is out

$$\int \vec{B}\cdot d\vec{s} = |\vec{B}(r)|\int_0^{2\pi r} ds$$

LH side of Ampere's law

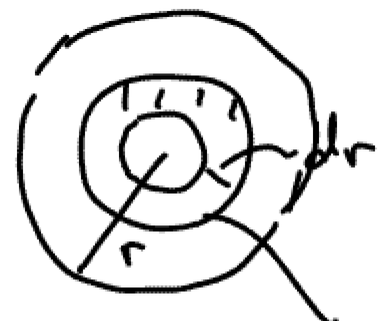
$$= |\vec{B}(r)|\, 2\pi r$$

RH side

I spread out

$\vec{j}(r) \equiv$ current density $\equiv$ constant
(special to this case)

$j = \dfrac{I}{\pi a^2} \equiv$ constant



$I_{encl} = \int j\,dA = \int_0^r j\,2\pi r\,dr \qquad 2\pi r\,dr = dA$

$= j\,2\pi \dfrac{r^2}{2} = j\pi r^2 = \dfrac{I}{\pi a^2}\pi r^2$

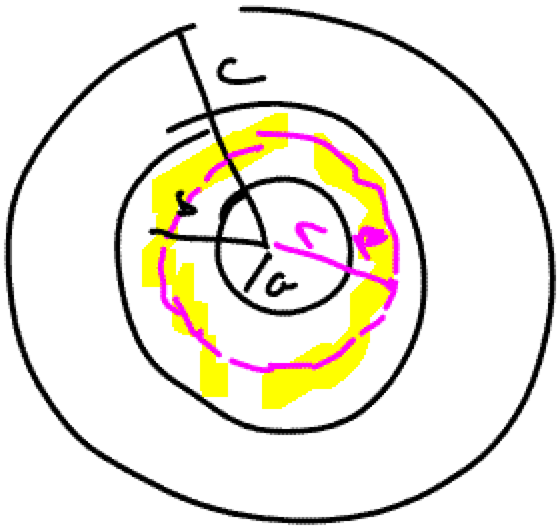
$I_{encl} = I\,r^2/a^2$

$$I_{encl} = j\pi r^2 = \frac{I}{\pi a^2}\pi r^2 = I\frac{r^2}{a^2}$$

$$\int \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$$

$$|\vec{B}|2\pi r = \mu_0 I \frac{r^2}{a^2}$$

$$|\vec{B}|_{r<a} = \frac{\mu_0 I r}{2\pi a^2}$$

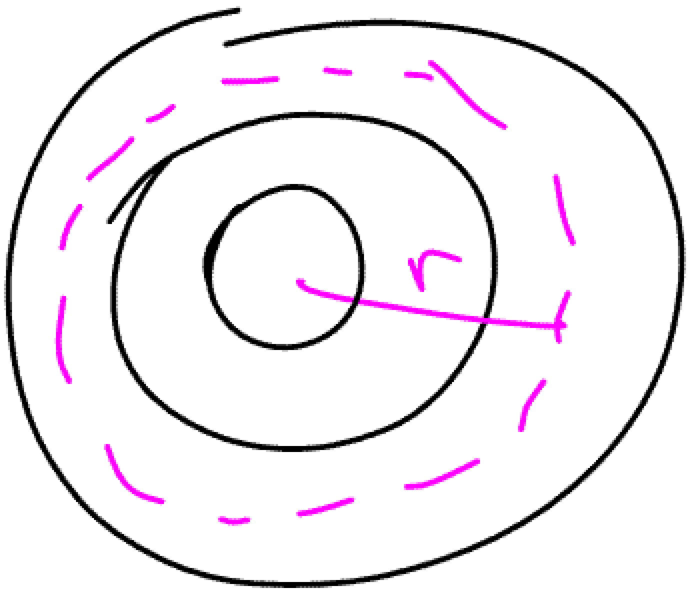


$b > r > a$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{encl}$$

$$|\vec{B}|\, 2\pi r = \mu_0 I$$

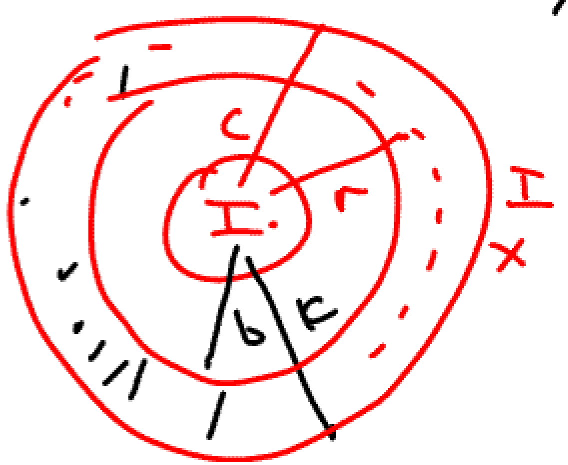
$$|\vec{B}|_{b>r>a} = \frac{\mu_0 I}{2\pi r}$$



$b < r < c$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$$

$$|\vec{B}|\,2\pi r = \mu_0 \overbrace{I_{encl}}^{\text{harder}}$$



Inner contrib = I

outer contrib → calc

$I_{outer} \Rightarrow \text{const } j$

$$|j_{outer}| = \frac{I}{\pi c^2 - \pi b^2}$$

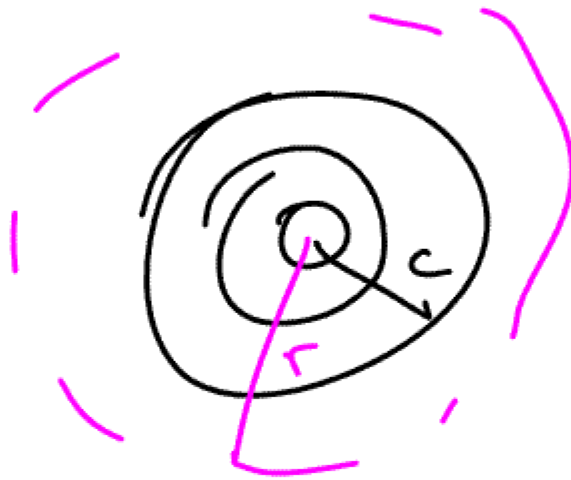
$$I_{encl} = I_{(inner)} - \int j_{outer} \, dA$$

$$I_{encl} = I_{(inner)} - j_{outer} (\pi r^2 - \pi b^2)$$

$$I_{encl} = I - I \left[\frac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2} \right]$$

$$|\vec{B}|\,2\pi r = I\left\{1-\left[\frac{\pi r^2-\pi b^2}{\pi c^2-\pi b^2}\right]\right\}$$

$$|\vec{B}|_{c>r>a} = \frac{\mu_0 I}{2\pi r}\left\{1-\left[\frac{\pi r^2-\pi b^2}{\pi c^2-\pi b^2}\right]\right\}$$



Amperian loop

$r>c$

$$\int \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$$

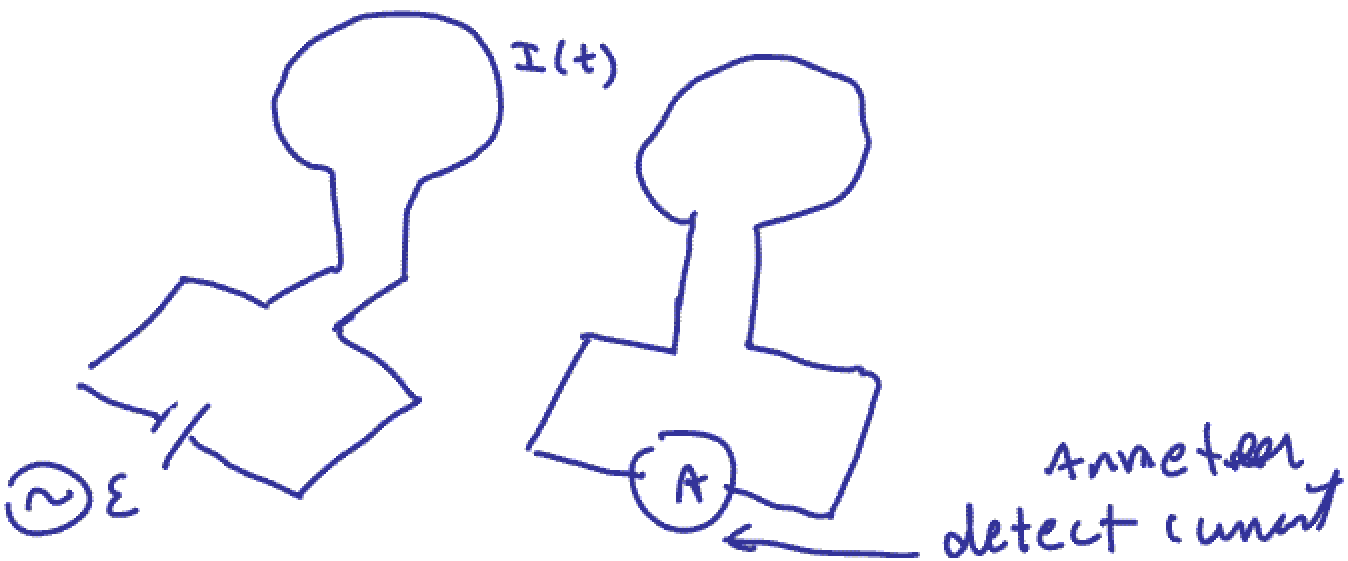
$$|\vec{B}|\,2\pi r = \mu_0\, 0$$

$$|\vec{B}|_{r>c} = 0$$

Magnetic Induction

1830's Michael Faraday (England)
Joseph Henry (US)

changing magnetic field "induces"
a changing electric field



Induction

steady currents

magnetostatics

B is not changing

Kirchoff

$$\sum V \Big|_{\substack{\text{closed}\\\text{loop}}} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

Kirchoff's law in free space

Changing Magnetic Field (in time)

$$\mathcal{E} = \int_{\text{loop}} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_M}{dt}$$

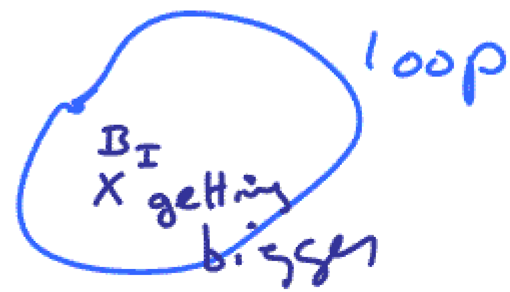
induced EMF

$$\Phi_M = \int_{\text{loop}} \vec{B} \cdot d\vec{A}$$

True in $\begin{cases} \text{material} \\ + \\ \text{Free space} \end{cases}$

Lenz's Law - An induced current in a closed conducting loop will appear in such a way as to oppose the change that created it

I



II