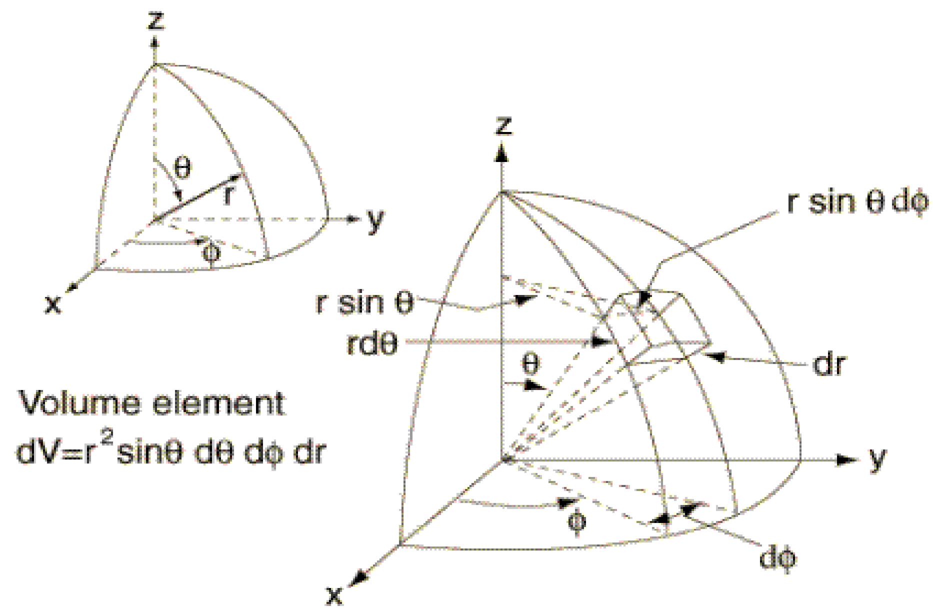
Physics 142 - September 18, 2008

Last Time -

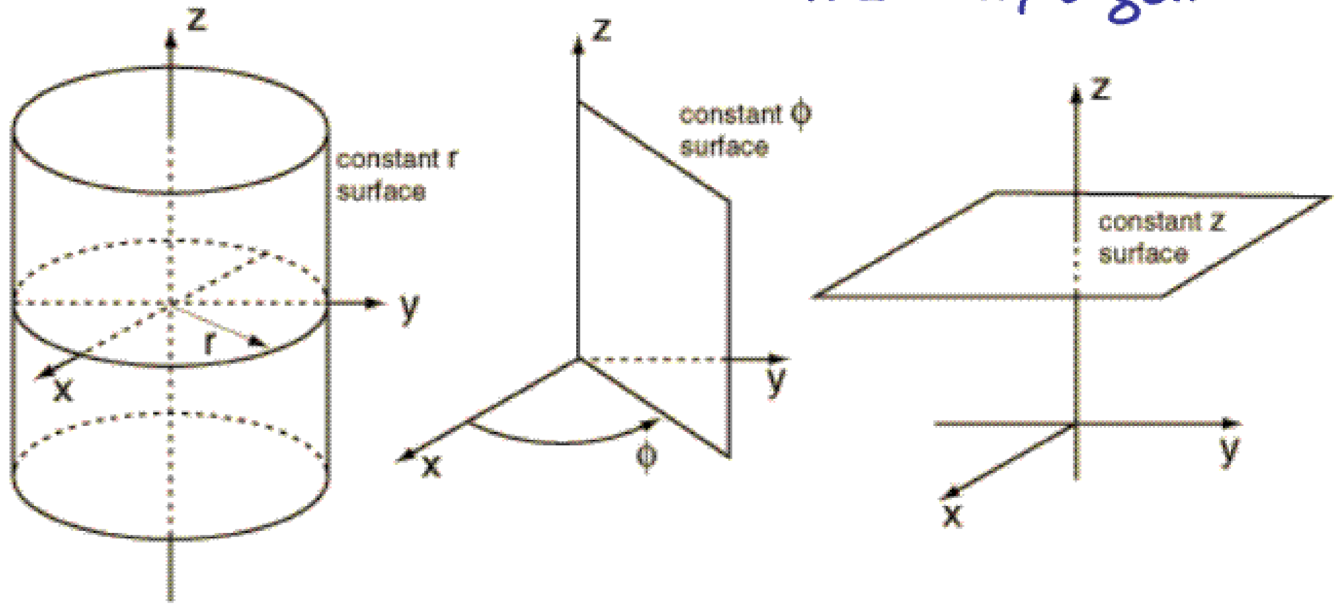
Gauss' Law $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{Encl}}{\epsilon_0}$

Always True ... Most useful under certain conditions of symmetry

Curvilinear Coordinates



$dv = r\,d\phi\,dz\,dr$

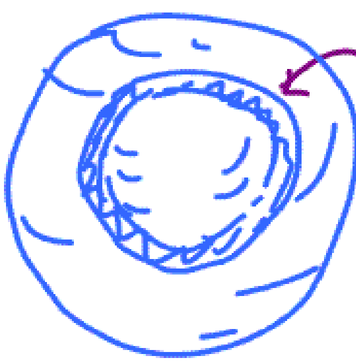


I'll limit us to problems in 1 variable

Usually that means radial dependence

Effectively integrates out Angular dependence

sphere



Shell

$dv = 4\pi r^2 dr$

cylinder l

cylindrical shell

$dv = 2\pi r l\,dr$

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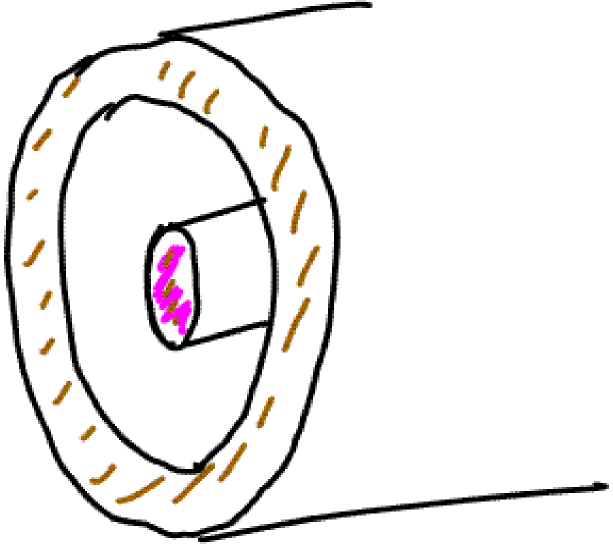


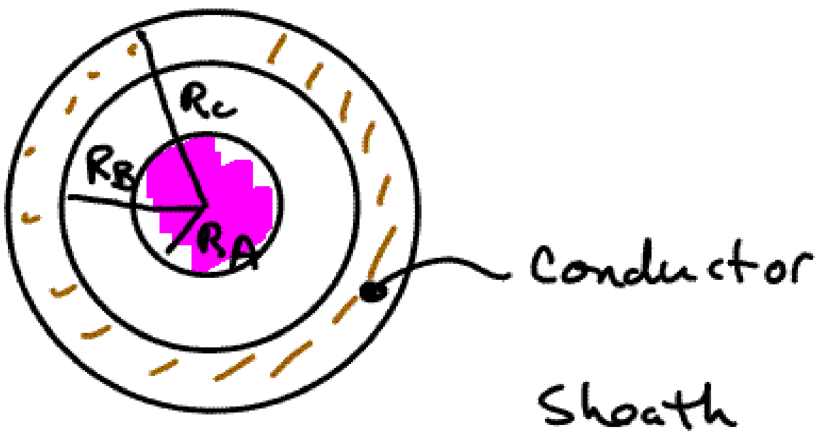
noncondocting core

radius R_A

has $+\lambda$

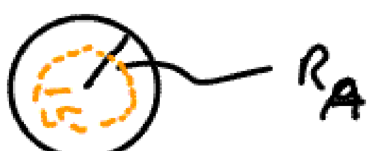
distributed

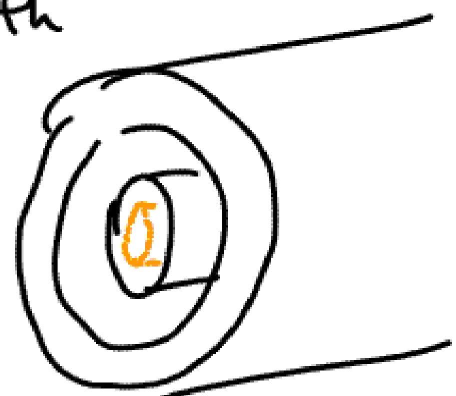
A $\rho(r) = ar \quad r < R_A$



Find $\vec{E}$ in all space

$r < R_A$





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$\vec{E}$ radially outward by symmetry

$dv = 2\pi r L\, dr$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

endcaps do Not contribute $\vec{E} \perp d\vec{A}$

$$|\vec{E}| \int dA = |\vec{E}|\, 2\pi r L$$

pipe shell

$A = 2\pi r L$

Q_{encl} ?

$$\int \rho\, dv$$

$$= \int_0^r a r\, 2\pi r L\, dr$$

B

$$= a 2\pi L \int_0^r r^2 dr = \frac{a 2\pi L r^3}{3}$$

$$Q_{encl} = \frac{a 2\pi L r^3}{3}$$

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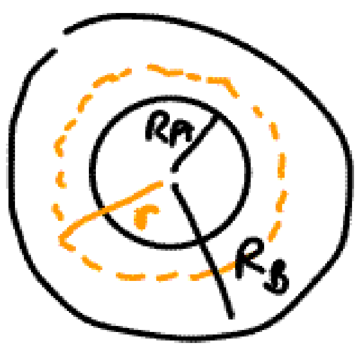
$$|\vec{E}|\, 2\pi r L = \frac{a\, 2\pi L r^3}{3\epsilon_0}$$

A $\quad \vec{E} = \dfrac{a r^2}{3\epsilon_0}$ radially out $\quad r < R_A$

$R_A < r < R_B$



$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$|\vec{E}|\, 2\pi r L = \frac{1}{\epsilon_0} \frac{a\, 2\pi L R_A^3}{3}$$

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$R_A < r < R_B$

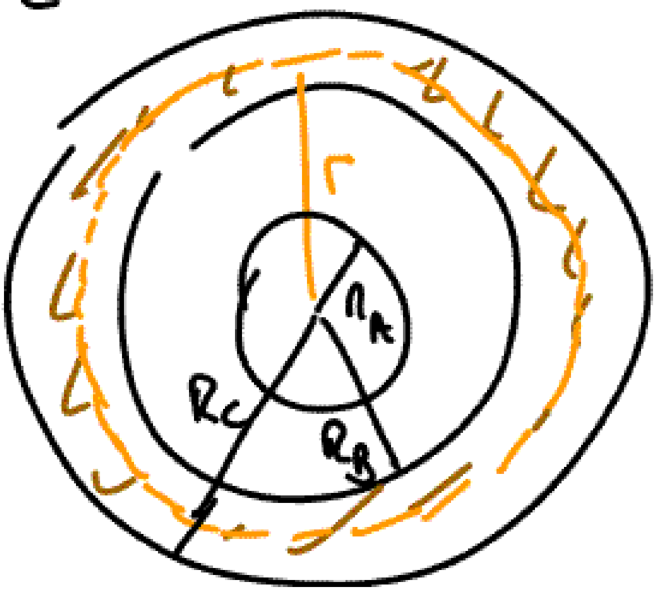
$r < R_A$

A $\vec{E} = \frac{a}{\epsilon_0 3} \frac{R_A^3}{r}$

radially outward

B $\vec{E} = \frac{a r^2}{3 \epsilon_0}$

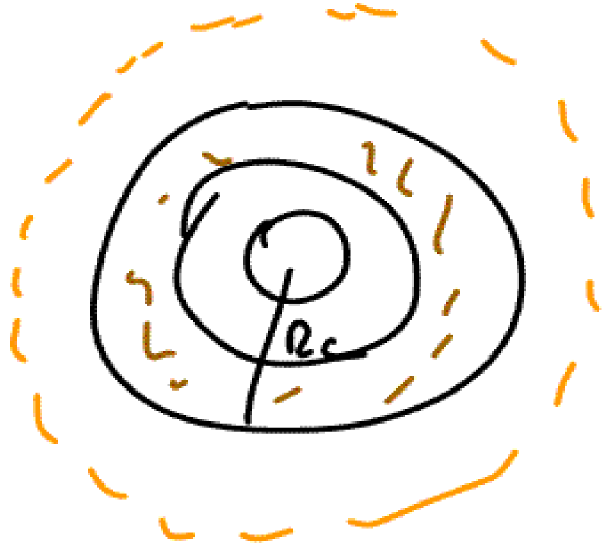
$R_B < r < R_C$



$\vec{E} = 0$

because region inside conductor

$r > R_c$



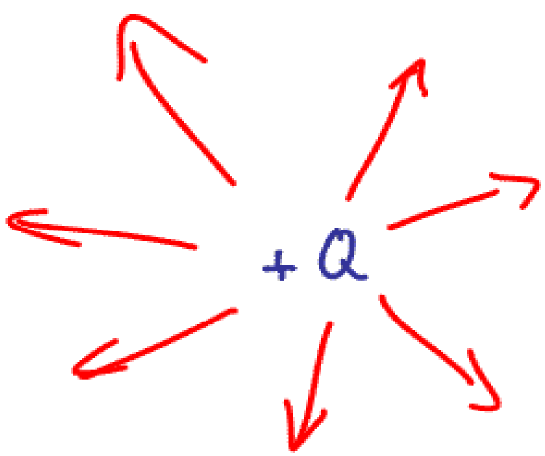
Ans. is
same as in $2^{\underline{ND}}$ region $(R_A < r < R_B)$
because symmetry And Q_{encl}
are the same

$r > R_c$

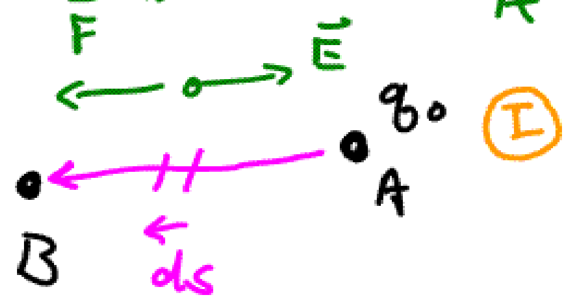
$$\vec{E} = \frac{\alpha}{\epsilon_0 3}\frac{R_A^3}{r}$$ radially outward

Recall how useful Energy considerations are for mechanics

Electric field + Energy



$\vec{F} \equiv$ Force I exert to move q_0 from A to B



How much work do I do to do this?

$$W = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B F\,ds = \int_A^B q_0 E\,ds = -\int_{R_A}^{R_B} q_0 E\,dr$$

$$= -q_0 \int_{R_A}^{R_B} \frac{kQ}{r^2}\,dr = -q_0 kQ \left[-\frac{1}{r}\right]_{R_A}^{R_B}$$

$$= q_0 kQ \left[\frac{1}{R_B} - \frac{1}{R_A}\right]$$ Net $+$ quantity

$$\Delta V = \frac{W}{q_0} \equiv \text{potential difference} \qquad \frac{\text{work}}{\text{charge}}$$

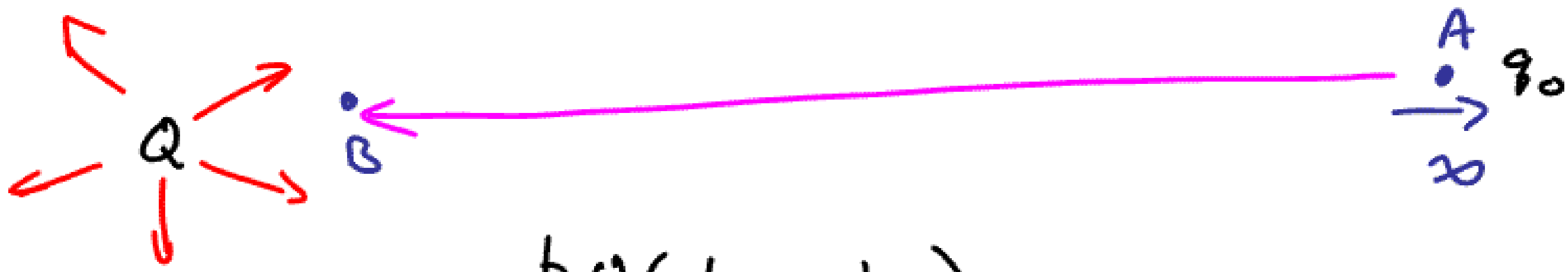
$$= -\frac{\Delta U}{q_0}$$

$U \equiv$ potential energy of system

$$\Delta V \equiv V_B - V_A \equiv V_{AB}$$

$$\text{unit} = \frac{\text{Joules}}{\text{Coulomb}} \equiv \text{Volt}$$

define potential at $\infty \to 0$



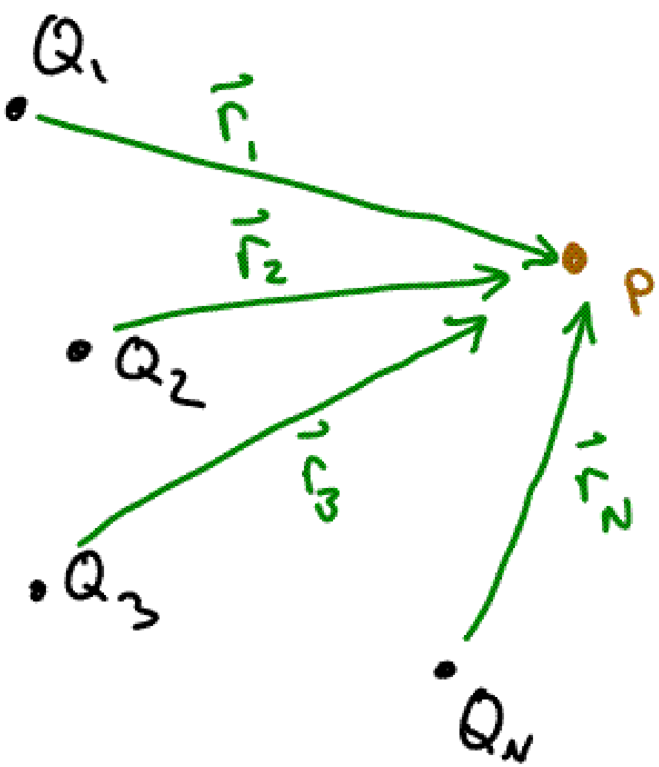
$$\frac{W}{q_0} = kQ\left(\frac{1}{R_B} - \cancel{\frac{1}{R_A}}\right)$$

0 at $R_A \to \infty$

$V(r) = \dfrac{kQ}{r}$ potential of a point charge

note potential is a Scaler

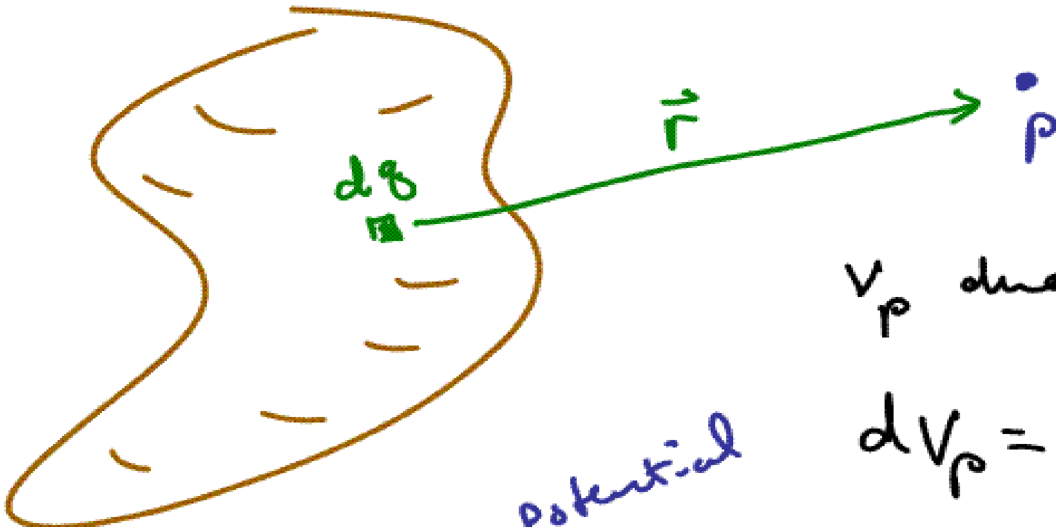
Potential of distribution of discrete charges:



$$V_P = \sum_N V_i = \sum_{i=1}^{N} \frac{kQ_i}{r_i}$$

note:
This is a
Scaler
Sum

Potential of continuous charge distribution:



V_p due dq

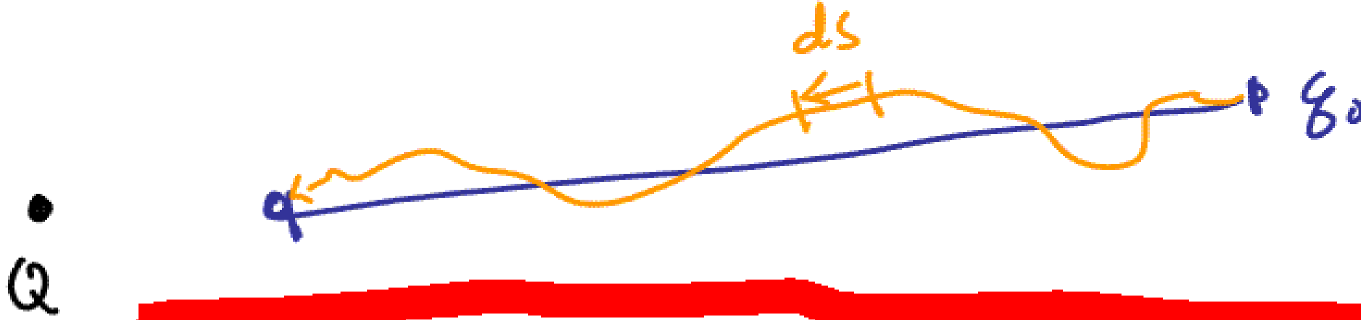
$$dV_p = \frac{k\,dq}{r} \quad (A)$$

potential

$$V_p = \int_{\text{volume}} \frac{k\,dq}{r}$$

$\rho\, dv$
volume

Scaler ... way easier to calculate than a vector such as $\vec{E}$



Electrostatics is a Conservative force

Potential Differences are Path independent

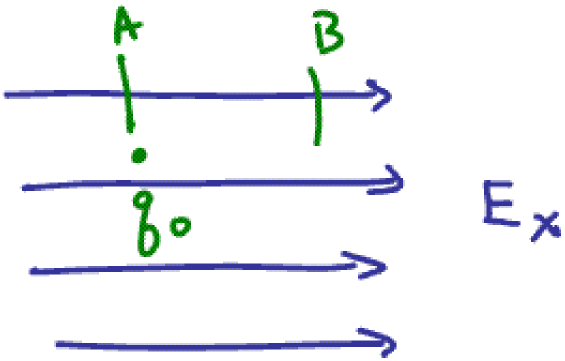
(just like gravitation ...

recall $PE \sim mgh$)

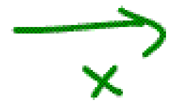
Δ in Potential goes as work to move charge from one point to the other + dot product

$$W \sim \int \vec{F} \cdot d\vec{s} \sim \int \vec{E} \cdot d\vec{r}$$

means only radial part is important



$$V = \frac{W}{q_0}$$



q_0 goes from A to B

work system does $\int \vec{F} \cdot d\vec{x} = q_0 \int E\,dx$

$$dV = -E_x\,dx$$

potential

$$E_x = -\frac{dV}{dx}$$

potential

Arbitrary direction – s

$$E_s = -\frac{dV}{ds}$$

in 3d $\quad V(x,y,z) \quad$ need is $E_x^{(x,y,z)}, E_y, E_z$

$$E_x = -\frac{dV}{dx} \qquad E_y = -\frac{dV}{dy} \qquad E_z = -\frac{dV}{dz}$$

A

$$E_x = -\frac{\partial V(x,y,z)}{\partial x}$$

Partial Derivative $\equiv$ Normal derivative with "other" variables const.

$\parallel\!\mid$

$$\frac{\partial V}{\partial x} = \frac{dV}{dx}$$

Hold all other variables CONSTANT even though $V = f(x,y,z\cdots)$

Example

$$V(x,y,z) = 2x^2 y z^3$$

$$\frac{\partial V}{\partial x} = 4xyz^3$$

$$\frac{\partial V}{\partial y} = 2x^2 z^3$$

$$\frac{\partial V}{\partial z} = 6x^2 y z^2$$