Physics 142 - Sept. 11, 2008

Last Time

Electrostatic Force — Coulomb's Law

Force of Q on q

$Q \bullet \xrightarrow{\vec r} \bullet q \qquad \vec F = \frac{kQq}{r^2}\hat r$

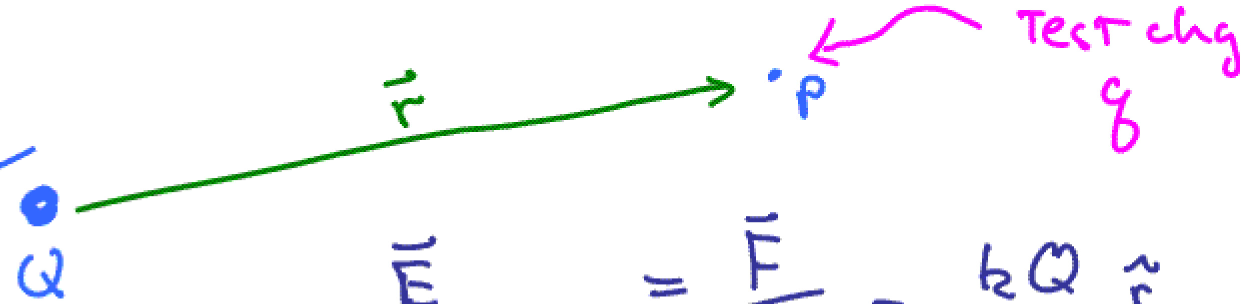
discrete

continuous

$$\underset{\substack{\text{on } q\\ \text{due to}}}{d\vec F} = \frac{kq\,dQ}{|\vec r_q - \vec r|^2}\,\widehat{(\vec r_q - \vec r)}$$

$$\boxed{\vec F_q = \int_{\substack{\text{vol}\\ \text{of charge}}} \frac{kq\,\rho(\vec r)\,dv}{|\vec r_q - \vec r|^2}\,\widehat{(\vec r_q - \vec r)}}$$

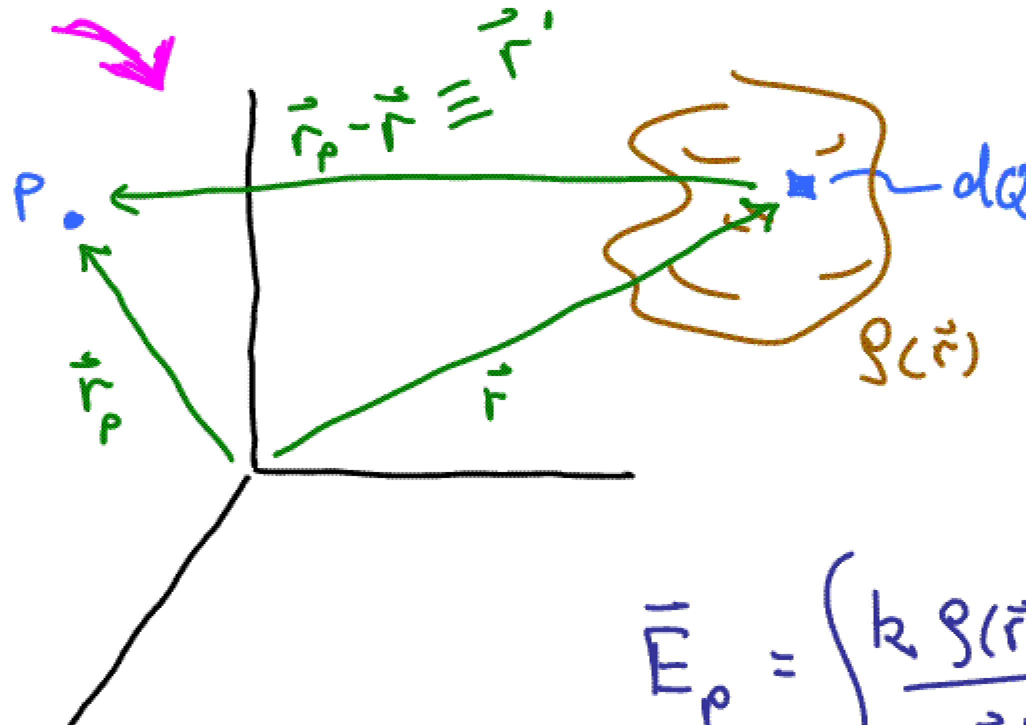
Electric Field



discrete

$$\bar{E}_{\text{due to } Q} = \frac{\bar{F}}{q} = \frac{kQ}{|\vec{r}|^2}\hat{r}$$

continuous

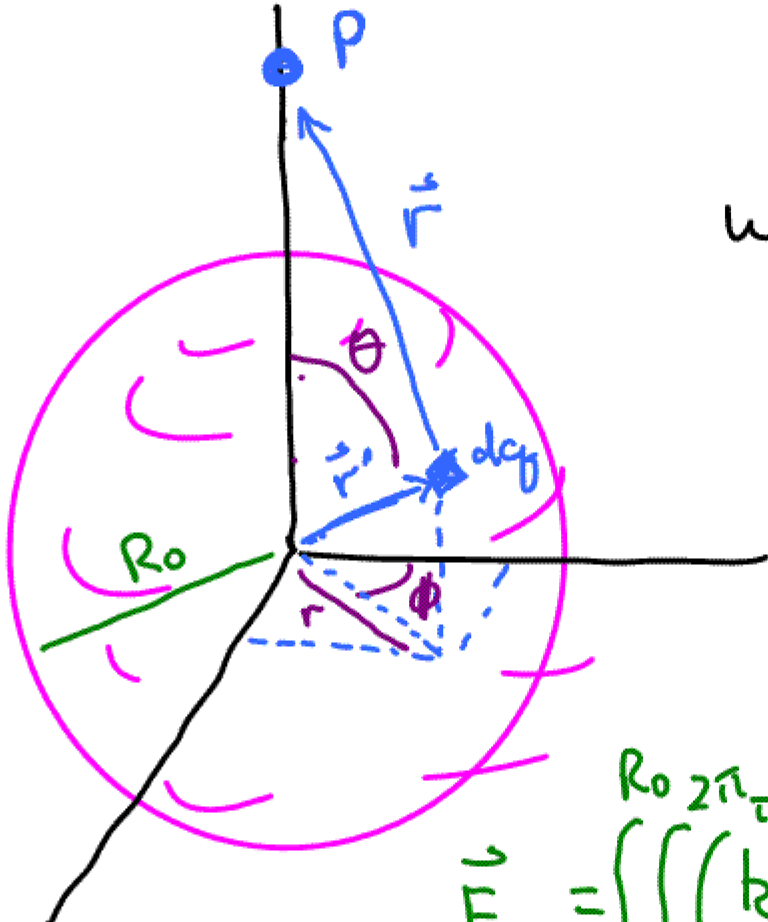


$$\bar{E}_p = \int \frac{k\,\rho(\vec{r})\,\hat{r}'}{|\vec{r}\,'|^2}\,dv$$

charge Q is dist evenly in sphere

$$\rho(\vec{r}') = \frac{Q}{\frac{4}{3}\pi R_0^3} = \text{const}$$

What is $\vec{E}_p$?

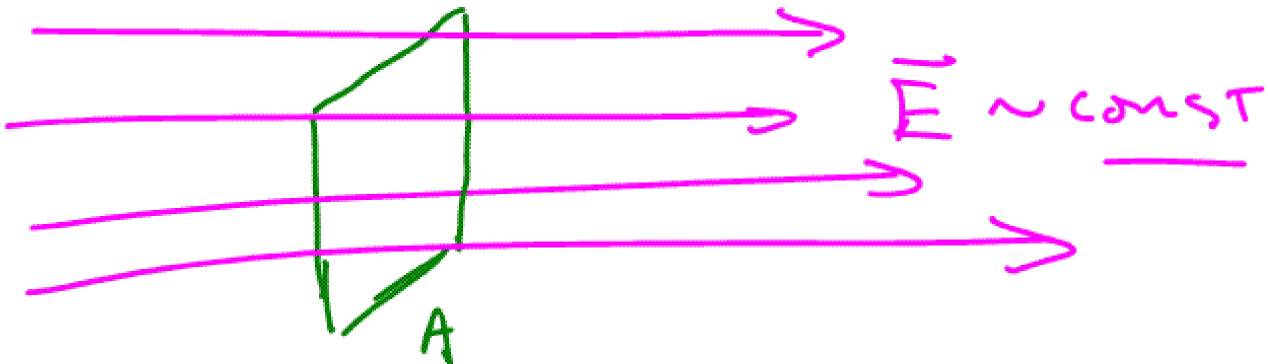


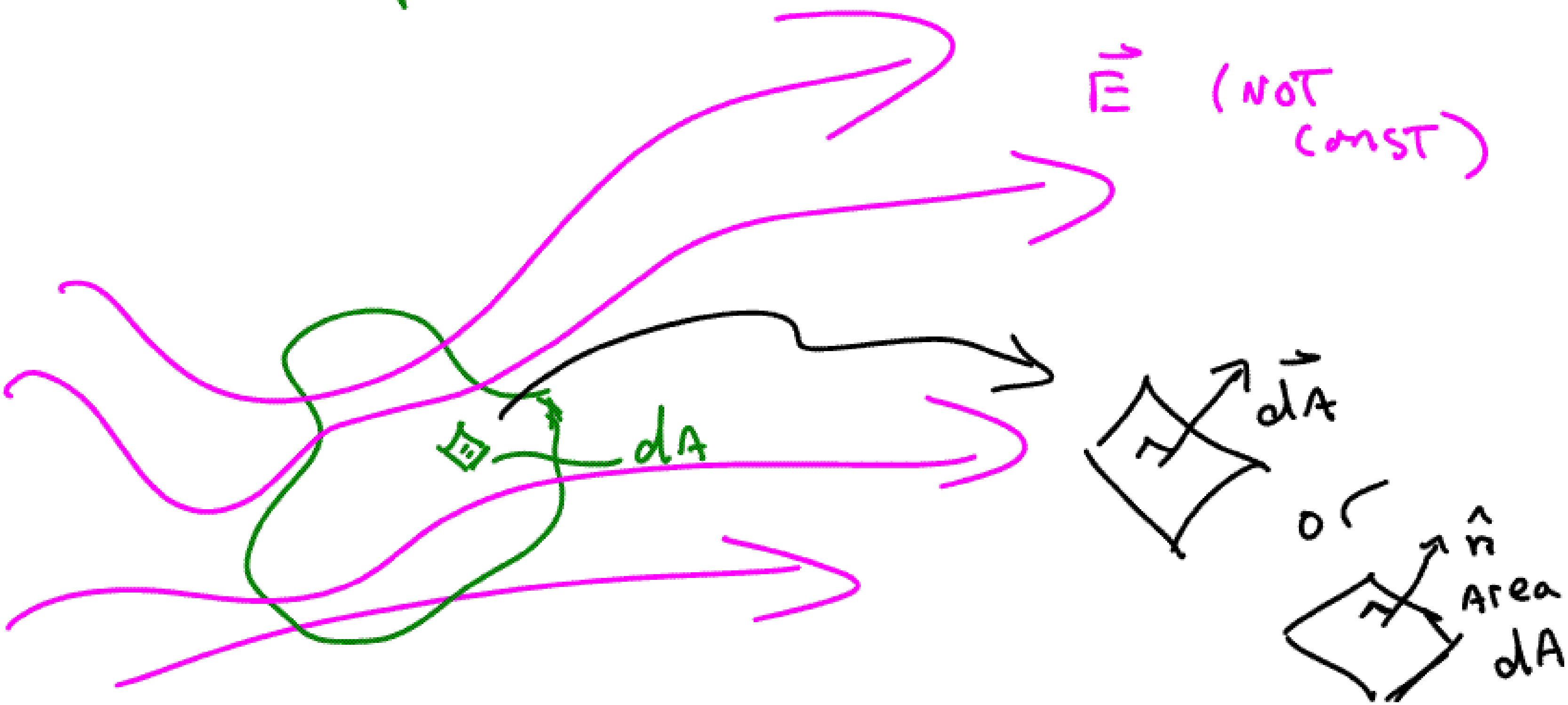
$$\vec{E}_p = \int_{\substack{\text{chg}\\\text{dist}}} \frac{k\rho\, dv\, \hat{\vec{r}}}{|\vec{r}|^2}$$

$$\vec{E}_p = \int_0^{R_0}\int_0^{2\pi}\int_0^{\pi} \frac{k\rho\, r'^2 \sin\theta\, d\theta\, d\phi\, dr}{r^2}\,\hat{r}$$

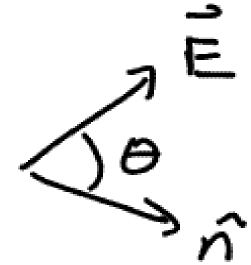
Holy Crap!

Electric Flux $\equiv \phi = A|\vec{E}|$



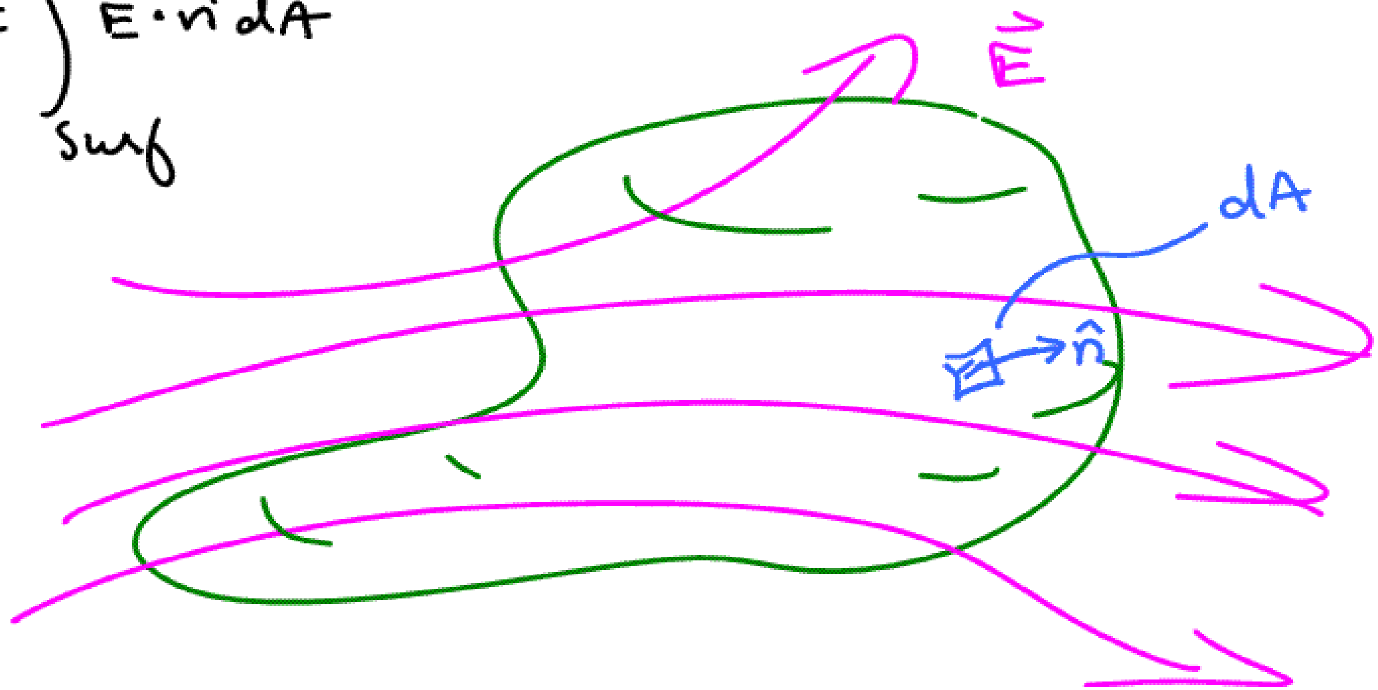


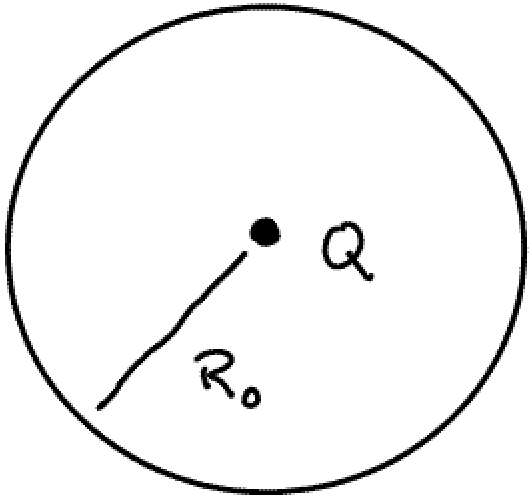
$$\Phi = \int_{\text{surface}} \vec{E} \cdot \hat{n} \, dA$$



$$\vec{E} \cdot \hat{n} = |\vec{E}||\hat{n}| \cos\theta$$

$$\Phi = \int_{\text{surf}} \vec{E} \cdot \hat{n} \, dA = \oint \vec{E} \cdot \hat{n} \, dA$$





Pt. chg at origin of spherical surface

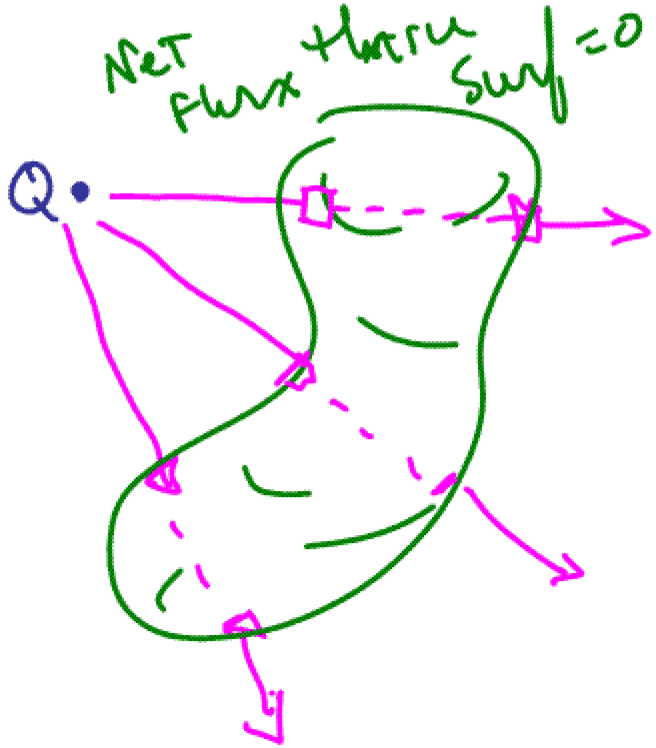
→ what is ϕ thru surface?

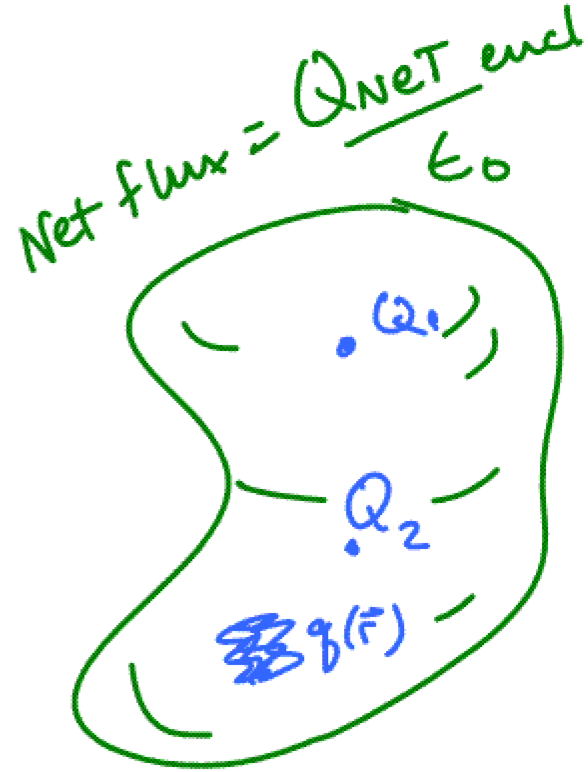
$$\phi_{surf} = \oint \vec{E}\cdot \underbrace{d\vec{A}}_{\equiv \hat{n}\, dA} = |\vec{E}(R_0)| \oint dA = |\vec{E}|\, 4\pi R_0^2$$

$$\phi_{surf} = |\vec{E}|\, 4\pi R_0^2 = \frac{kQ}{R_0^2}\, 4\pi R_0^2 = \frac{Q}{\epsilon_0}$$

$$k \equiv \frac{1}{4\pi\epsilon_0}$$

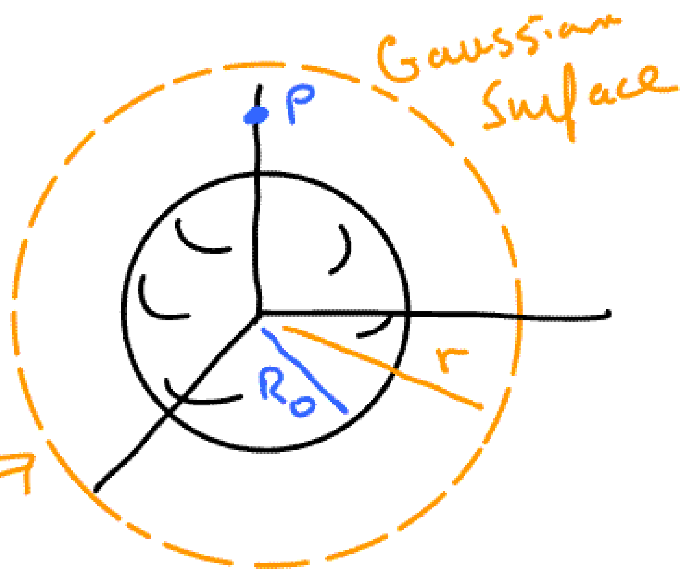
is enclosed





$$\boxed{\phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}}$$

Gauss' Law



Spherical dist

const ρ

Total chg Q

What is $\vec{E}_P$?

what is $\vec{E}$ everywhere?

evaluate Gauss' law

determine $\vec{E}$ $r > R_0$

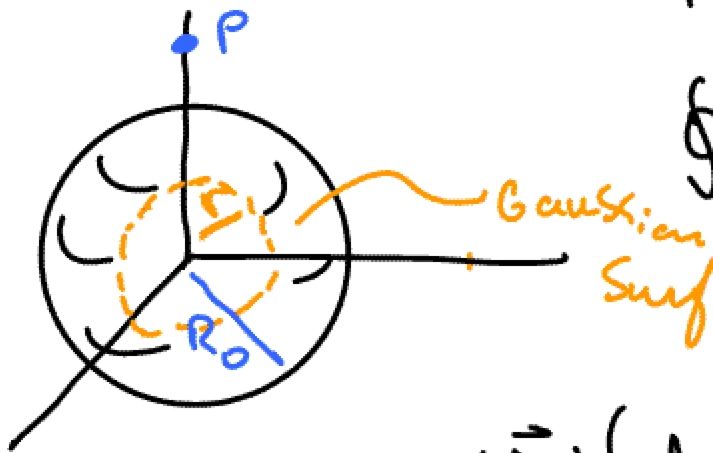
From symmetry $\vec{E}$ radial

$|\vec{E}|$ on spherical surf centered at origin is const

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$|\vec{E}| \int dA = \frac{Q_{TOT}}{\epsilon_0}$$

$$|\vec{E}|_{r > R_0} = \frac{Q_{TOT}}{4\pi\epsilon_0 r^2}$$



$r < R_0$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$|\vec{E}| \oint dA = \frac{Q_{encl}}{\epsilon_0}$$

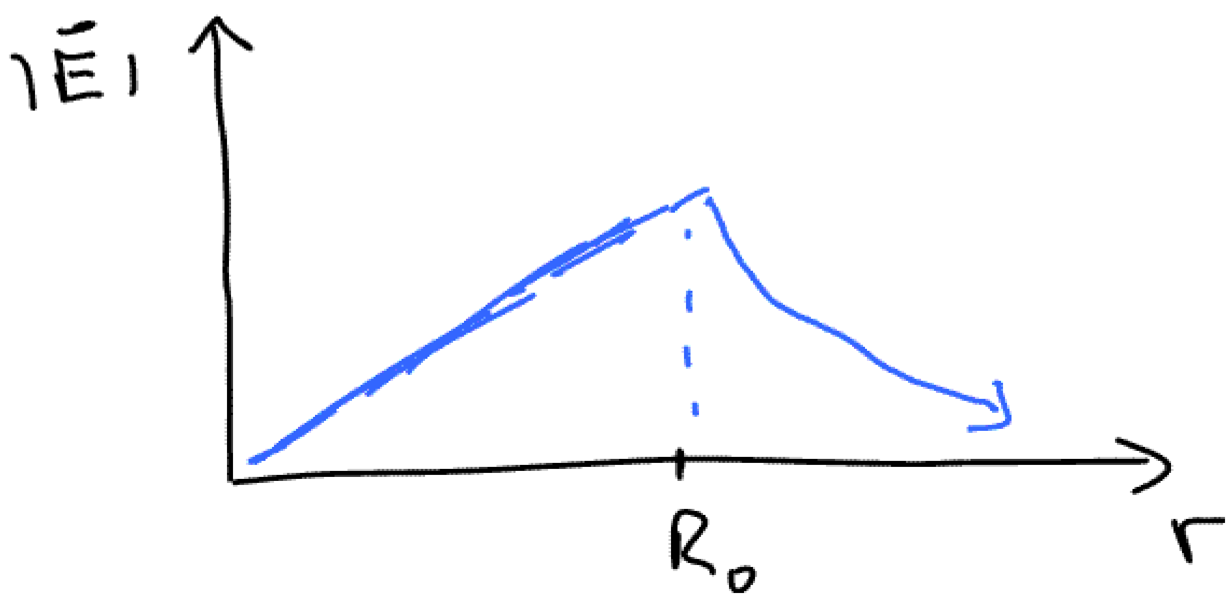
$$|\vec{E}| 4\pi r^2 = \frac{\oint \rho \, dv}{\epsilon_0} = \oint \frac{Q_{TOT}}{\frac{4}{3}\pi R_0^3} \frac{1}{\epsilon_0} dv$$

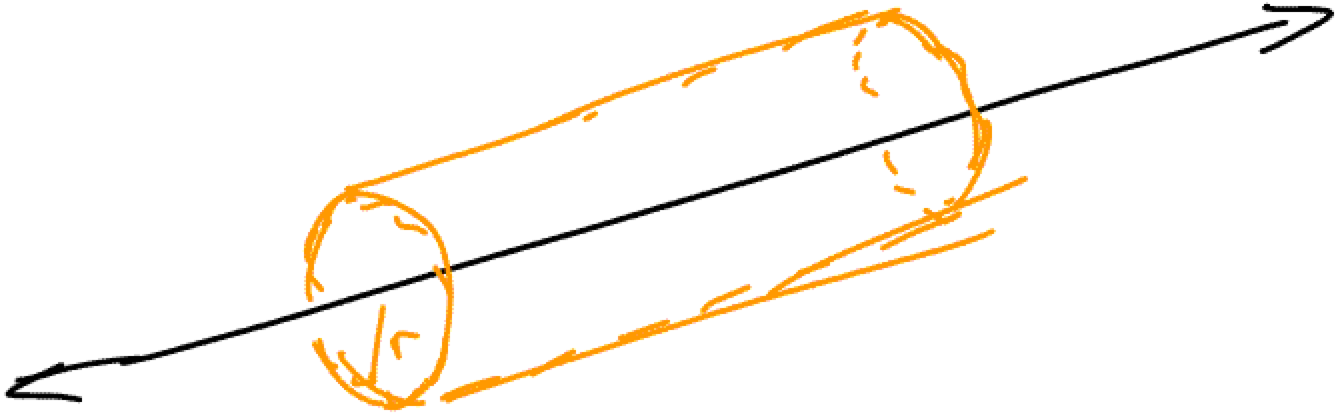
$$= \frac{Q_{TOT}}{\frac{4}{3}\pi R_0^3 \epsilon_0} \oint dv$$

small sphere

$$|\vec{E}|\,4\pi r^2 = \frac{Q_{TOT}}{\frac{4}{3}\pi R_0^3 \epsilon_0}\frac{4}{3}\pi r^3$$

$$|\vec{E}|_{r<R_0} = \frac{Q_{TOT}}{4\pi\epsilon_0}\frac{r}{R_0^3}$$





Gauss' Law useful when you have a physical situation such that $\vec{E}\cdot\vec{dA}$ is simple to evaluate. Often most useful when (by symmetry)

$$\vec{E}\cdot\vec{dA} = 0 \qquad \text{or} \qquad \vec{E}\cdot\vec{dA} = |\vec{E}|\,dA$$

And $|\vec{E}|$ = CONSTANT on Gaussian surface so that

$$\int \vec{E}\cdot\vec{dA} \longrightarrow |\vec{E}|\int dA$$