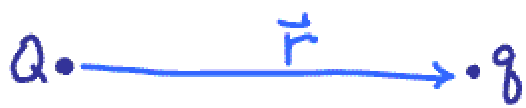
Physics 142 - September 9, 2008

- Web link issues ... now fixed
- Workshops begin this week (Today)

  Synchronization issues
- PS 1 due at end of next class

Last time

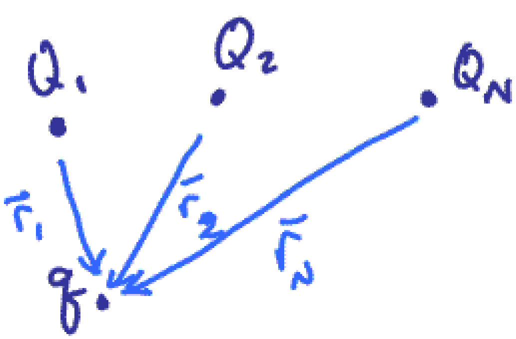
single charge



Force of Q on q

$$\vec{F} = \frac{kQq}{r^2}\,\hat{r}$$

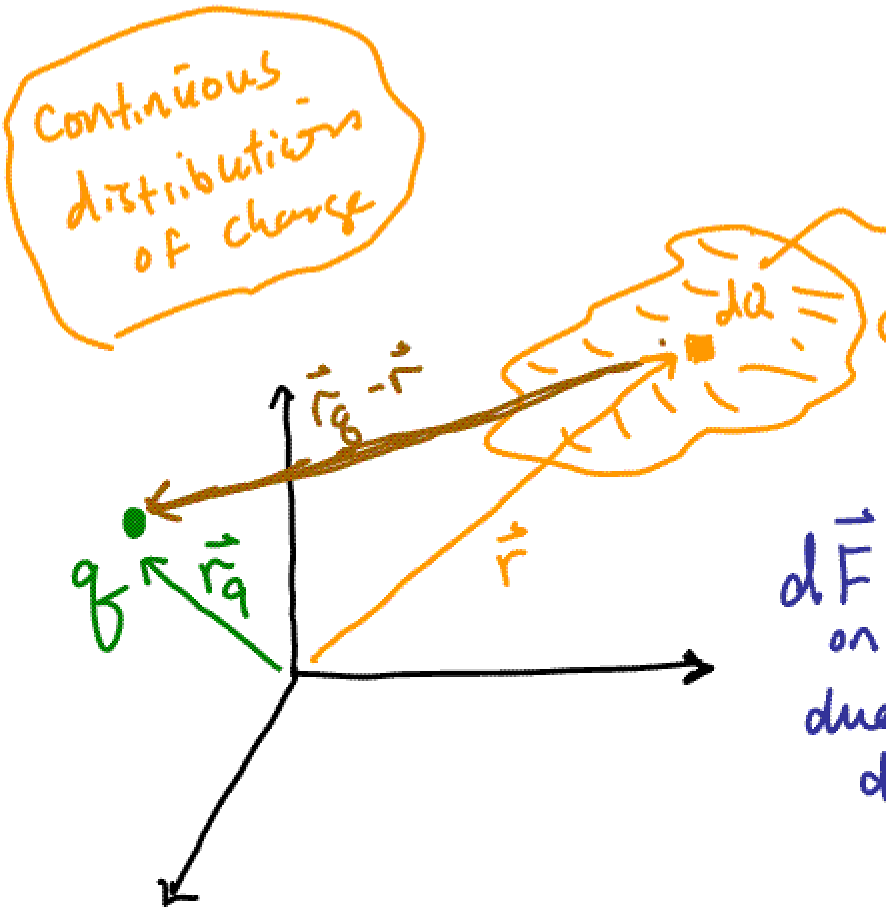
multiple discrete charges



vector superposition

Force of $Q_1, Q_2, \cdots Q_N$ on q

$$\vec{F} = \sum_i \frac{k\,Q_i\,q}{r_i^2}\,\hat{r}_i$$



$$d\vec F_{\substack{\text{on } q \\ \text{due to} \\ dQ}} = \frac{kq\,dQ}{|\vec r_q - \vec r|^2}\,\widehat{(\vec r_q - \vec r)}$$

$$dQ = \underbrace{\rho(\vec r)}_{\substack{\text{volume} \\ \text{charge} \\ \text{density}}}\,dv$$

$$\vec F_q = \int_{\text{vol of charge}} \frac{kq\,\rho(\vec r)\,dv}{|\vec r_q - \vec r|^2}\,\widehat{(\vec r_q - \vec r)}$$

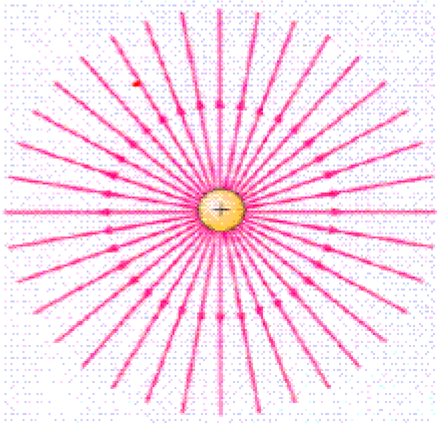
Electric Field

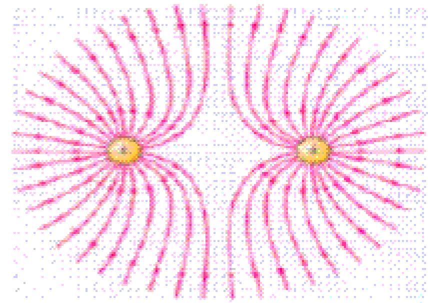
Q

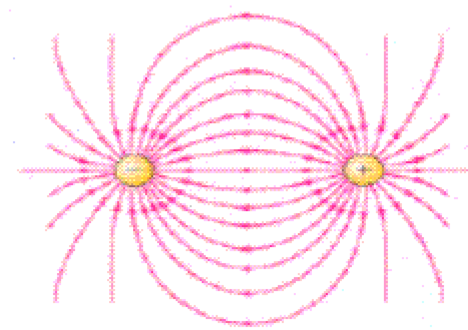
$$\vec{E}_p = \frac{\vec{F}_q}{q}$$

Imagine placing positive test charge q at point p $\rightarrow$ $\vec{F}_{on\, q} = q\vec{E}_p$

use this to visualize $\vec{E}$

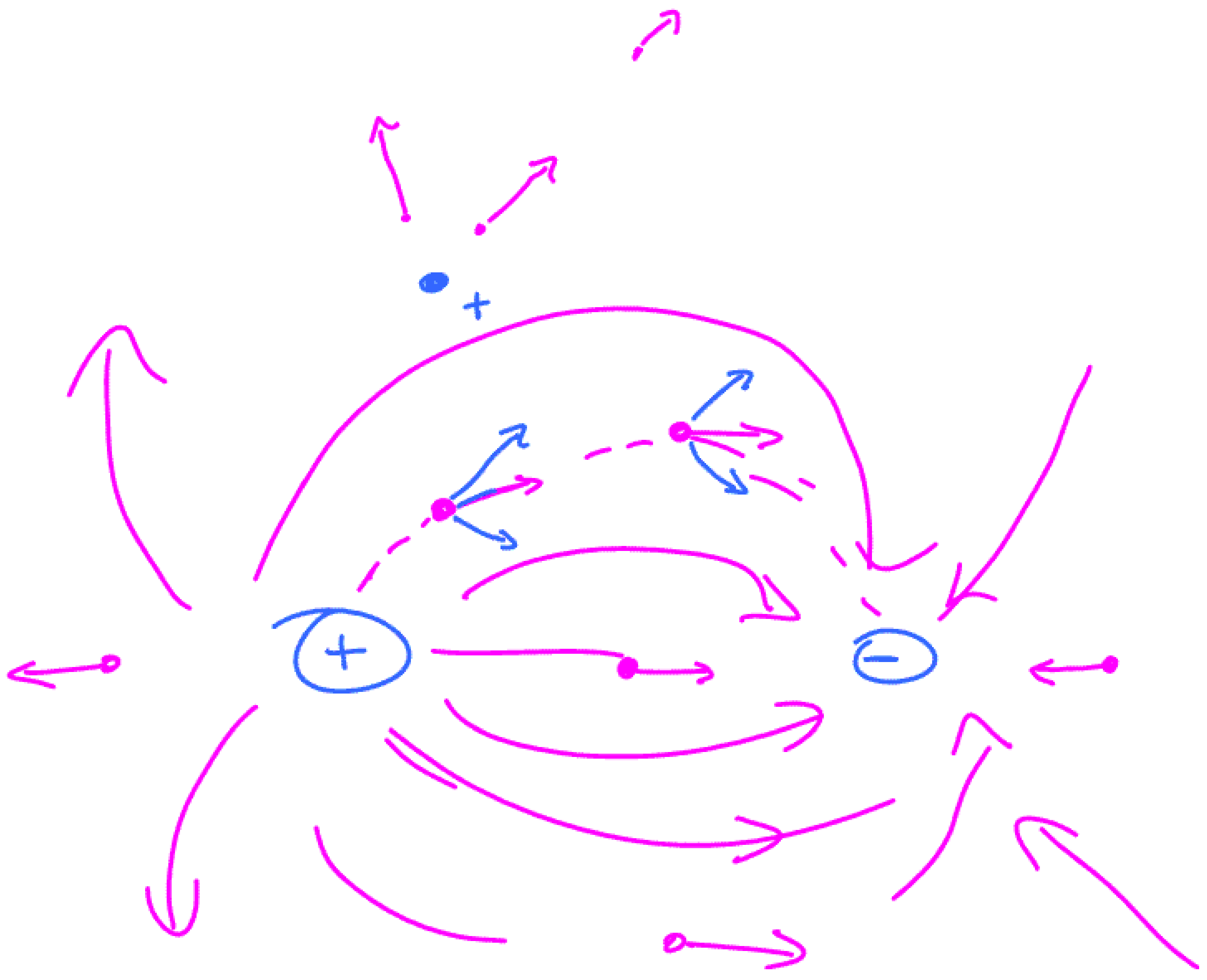




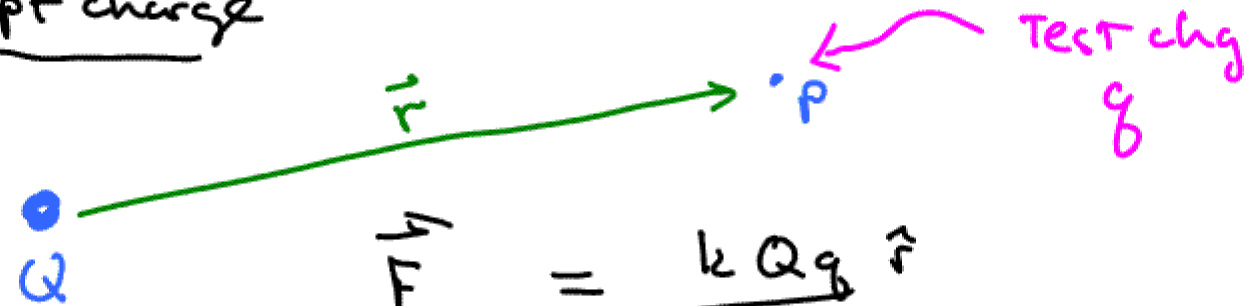


use "lines of force" to visualize the electric field

- go from $\oplus$ to $\ominus$ or have an endpoint at ∞
- lines never cross
- density of lines $\propto |\vec{E}|$
- $\vec{F}, \vec{E}$ always tangent to line of force



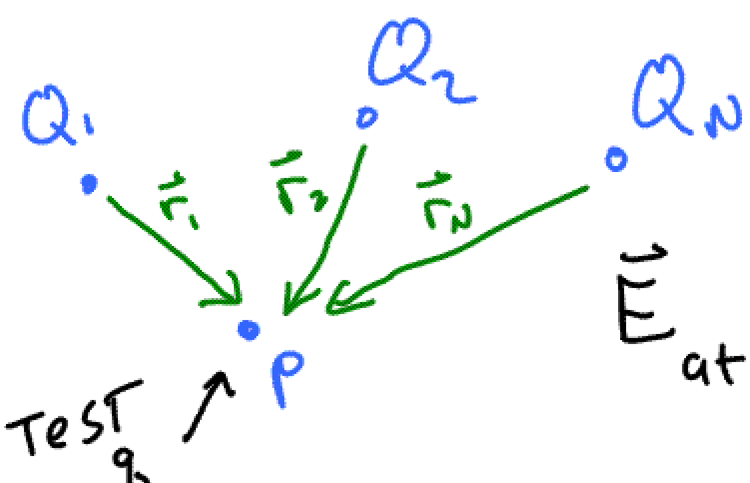
E field of pt charge



$$\vec{F}_{Q\,on\,q} = \frac{kQq}{|\vec{r}|^2}\hat{r}$$

$$\vec{E}_{\substack{due\ to\\ Q}} = \frac{\vec{F}}{q} = \frac{kQ}{|\vec{r}|^2}\hat{r}$$

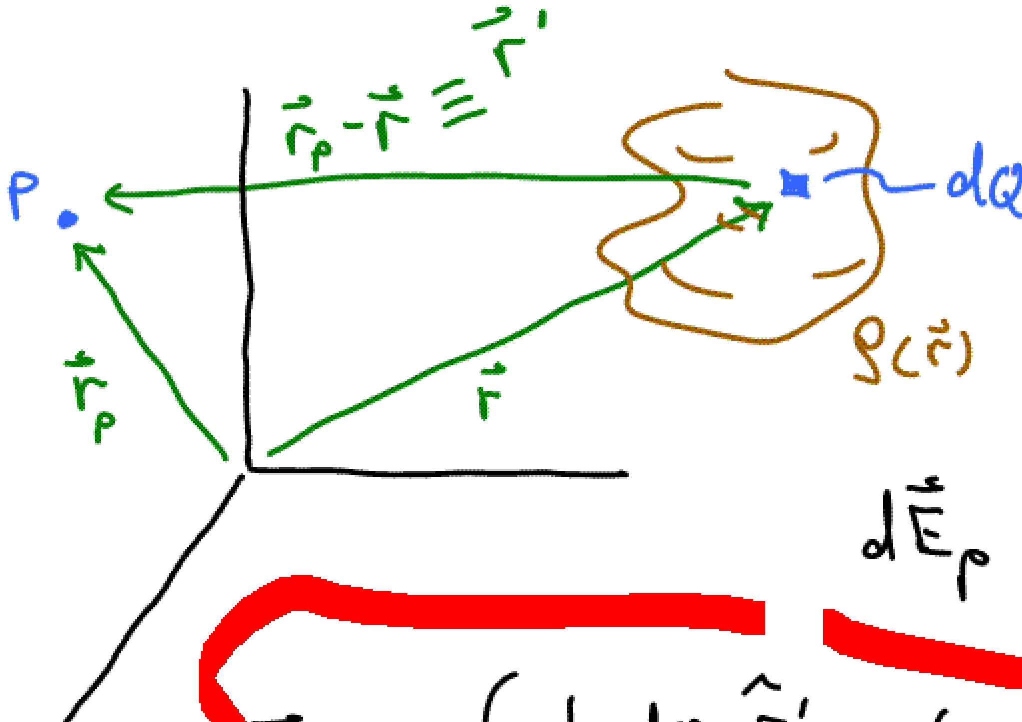
discrete charge distr.



$$\frac{\vec{F}}{q} = \sum \frac{kQ_i q}{|\vec{r}_i|^2}\hat{r}_i$$

$$\vec{E}_{at\,P} = \sum \frac{kQ_i\,\hat{r}_i}{|\vec{r}_i|^2}$$

Electric field for general chg. distribution

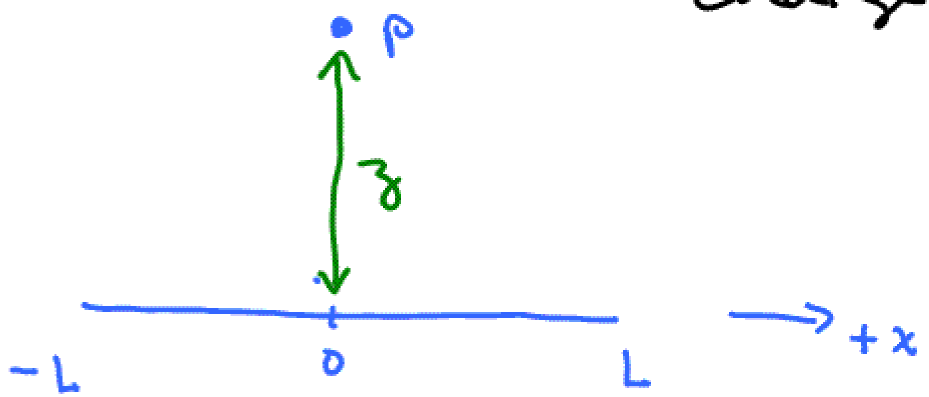


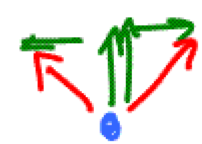
$$d\vec{E}_p = \frac{k\,dQ}{|\vec{r}\,'|^2}\,\hat{r}\,'$$

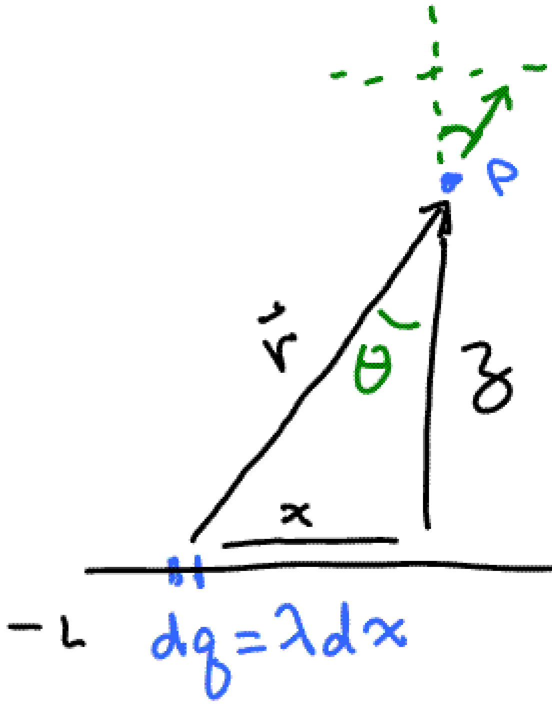
$$\boxed{\vec{E}_p = \int_{\substack{\text{chg}\\ \text{dist}}} \frac{k\,dQ}{|\vec{r}\,'|^2}\,\hat{r}\,' = \int \frac{k\,\rho(\vec{r})\,dv}{|\vec{r}\,'|^2}\,\hat{r}\,'}$$

Example

Find $\vec{E}$ at dist z above the midpoint of a line segment of length $2L$ which carries a uniform line charge of λ







$d\vec{E}_p$ due to $dq = \lambda dx$

impt part

$dE_p \cos\theta \, \hat{z}$

$dE_p \dfrac{z}{|\vec{r}|} \hat{z}$

$$\vec{E}_p = \int \frac{k\,dq}{r^2}\,\hat{r}$$

$$\vec{E}_p = 2\int_0^L \frac{k\lambda\,dx\cos\theta}{r^2}\,\hat{z}$$

$$|\vec{r}| = (x^2+z^2)^{1/2}$$

$$\cos\theta = \frac{z}{\sqrt{x^2+z^2}}$$

$$\vec{E}_p = 2\int_0^L \frac{k\lambda\,z\,dx}{(x^2+z^2)^{1/2}(x^2+z^2)}\,\hat{z}$$

$$\vec{E}_p = 2k\lambda z \int_0^L \frac{dx}{(x^2+z^2)^{3/2}}\,\hat{z}$$

$$\vec{E}_p = 2k\lambda z \left[\frac{x}{z^2(x^2+z^2)^{1/2}}\right]_0^L \hat{z}$$

$$\vec{E}_p = \frac{2k\lambda z L}{z^2(L^2+z^2)^{1/2}}\,\hat{z} = \frac{2k\lambda L}{z(L^2+z^2)^{1/2}}\,\hat{z}$$

How do we know we are right ?

- dimensional analysis

$$\frac{NT\,\cancel{m^2}}{\cancel{c^2}} \quad \frac{\cancel{c}}{\cancel{m}} \quad \cancel{m} \quad \frac{1}{\cancel{m^2}}$$

$$E \rightarrow \frac{NT}{C}$$

$$F \sim \frac{k\,qq}{r^2} \qquad \frac{NT\cdot m^2}{c^2}\ k$$

- Limiting Cases

→ $z \to \infty$ $\qquad E \sim \dfrac{Q^{(2\lambda L)}}{z^2}$ $\qquad$ L'Hopital

→ $L \to \infty$ $\qquad E \to \dfrac{2\lambda k}{z}\,\hat{z}$

As we'll see, this is the field around an infinite uniform line charge