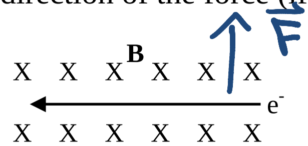
P142 University of Rochester NAME Soln key - SLM
S. Manly Fall 2008

Exam 2 (November 13, 2008)

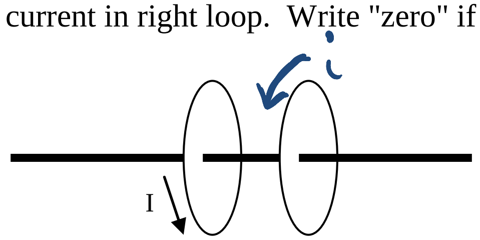
Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

<u>Problem 1 (10 pts, answer for each part is correct or incorrect - 2 pts/part):</u>

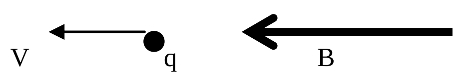
(a) An electron moves in the direction shown into a uniform magnetic field. Clearly indicate on the sketch the direction of the force (if any) on the electron. Write "zero" if there is no force.



(b) Consider the two coaxial circular conducting loops shown below. There is a current in the left loop that is decreasing in magnitude with time. Indicate the direction (if any) of the current in right loop. Write "zero" if there is no current in the second loop.

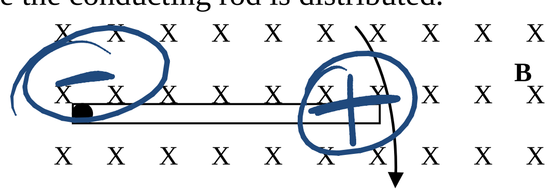


(c) A positive charge moves as shown in a uniform magnetic field. Clearly indicate on the sketch the direction of the force (if any) on the electron. Write "zero" if there is no force.

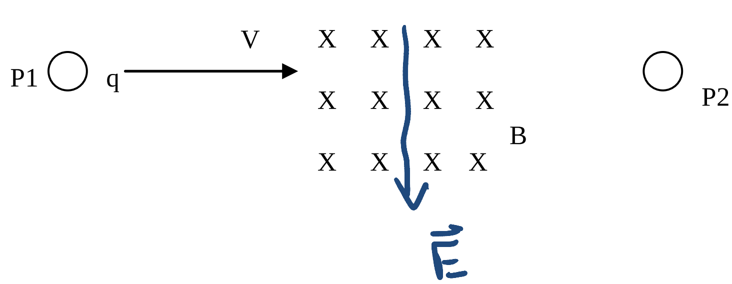


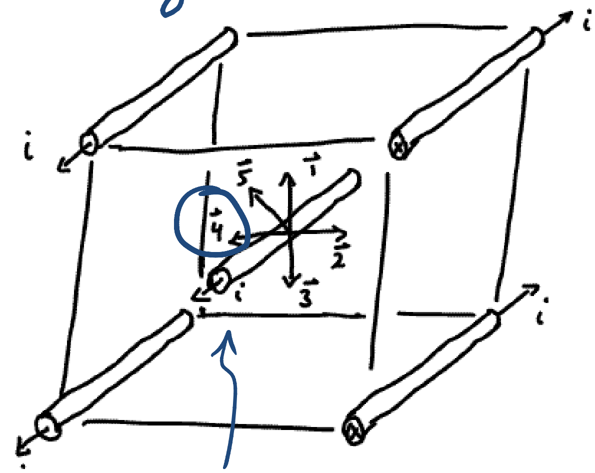
zero

(d) A conducting rod with no net charge is spinning in a uniform magnetic field around an axis through one end as shown below. Indicate roughly on the drawing how the charge inside the conducting rod is distributed.



(e) A positive charge is moving initially with a velocity V directly from point P1 to point P2. It enters a region with a uniform magnetic field B and a uniform electric field E. The direction of the magnetic field is shown. Indicate clearly what the direction of the electric field must be in order for the charged particle to reach point P2.



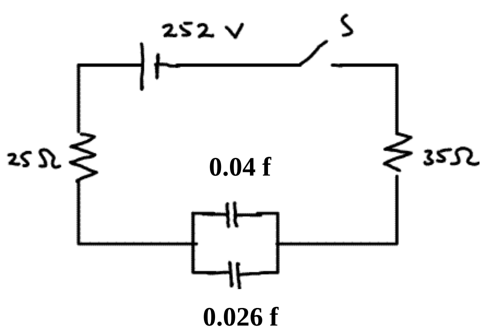


wire moves to left

Problem 2 (5 pts):

Each of four wires arranged along the parallel edges of a cube carry a current i. A fifth parallel current-carrying wire passes through the center of the cube. If the fifth wire were free to move, circle the vector that best represents the direction the fifth wire would move.

Problem 3 (15 pts, show work):



In the circuit to the left, how much energy is stored in the 0.026 farad capacitor 2 seconds after the switch S is closed?

RC circuit

$\tau = (60)(.066) \approx 4$

$R_{TOT} = 35 + 25 = 60\,\Omega$

$C_{TOT} = .04 + .026 = .066\text{ F}$

$$Q_{TOT}(t) = C_{TOT}\mathcal{E}\left(1 - e^{-t/RC}\right) = (.066)(252)\overbrace{\left[1 - \underbrace{e^{-2/4}}_{.6}\right]}^{.4} \approx 6.7\text{ Coul.}$$

Voltage Drop across capacitors (ΔV)

$Q_{TOT} = C_{TOT}(\Delta V)$

$\frac{6.7\text{ coul}}{.066\text{ F}} \approx 102\text{ Volts}$

Energy in 0.026 F capacitor at $t = 2$ s

$U(t=2) = \frac{1}{2}C\Delta V^2$

$= \frac{1}{2}(.026)(102)^2$

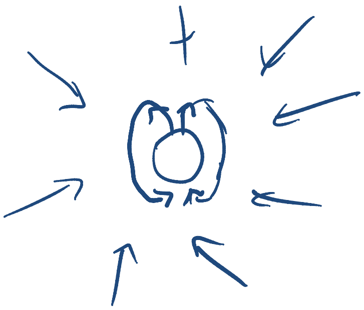
$\boxed{U_{t=2} = 1353\text{ Joules}}$

1)	10/10
2)	5/5
3)	15/15
4)	10/10
5)	20/20
6)	20/20
7)	20/20
tot	100/100

Wow! :)

Problem 4 (10 pts) :

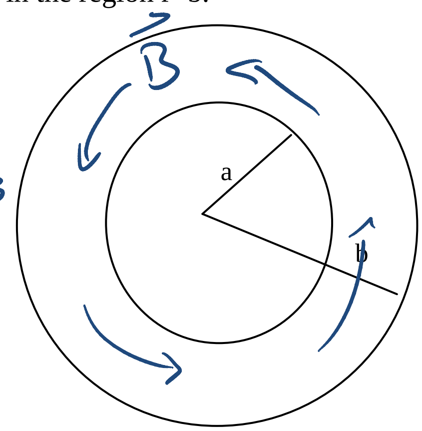
Briefly explain why the cosmic ray flux in the atmosphere over the north pole is greater than the cosmic ray flux in the atmosphere over Florida. (Think of the cosmic rays as energetic charged particles from distant astrophysical sources that are coming toward the earth with a uniform distribution over the full solid angle of the sky.)



Cosmic rays near the poles approach earth along Earth's B-field lines and are NOT deflected much by earth's magnetic field. Cosmic rays approaching earth along the equator are deflected (by Lorentz force law) $F \sim q\vec{v} \times \vec{B}$

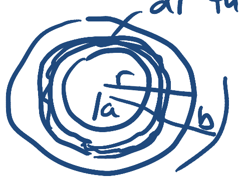
Problem 5 (20 pts, show all work) :

A thick conducting pipe with inner radius a and outer radius b carries a current coming out of the page. The current density is not uniform. It varies as Kr^2, over the conducting region of the cross section of the pipe, where K is a constant. There is no current for r<a. There is a current density that varies as $j(r)=Kr^2$ from r=a to r=b (out of the paper). There is no current in the region r>b. Determine the magnetic field in all space.



$K = \frac{\text{Tesla}}{m^3}$

By Ampere's law $B=0$ for $r<a$ ← No enclosed current



for $a<r<b$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$$

$$B\,2\pi r = \mu_0 \int_a^r K r^2\, 2\pi r\, dr = \mu_0 K \frac{2\pi}{4}(r^4 - a^4)$$

$$\vec{B} = \frac{\mu_0 K}{4}\left[\frac{r^4 - a^4}{r}\right]$$ units ok counterclockwise

$r>b$

$$B\,2\pi r = \mu_0 \int_a^b K r'^2\, 2\pi r'\, dr' = \mu_0 K \frac{2\pi}{4}(b^4 - a^4)$$

$$\vec{B} = \frac{\mu_0 K}{4}\frac{[b^4 - a^4]}{r}$$ counterclockwise

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Problem 6 (20 pts, show all work) :

Consider two concentric solenoids of length h, one with radius r_1 and n_1 turns per unit length and one with radius r_2 and n_2 turns per unit length, where $r_1<r_2$.

a) Derive an expression for the mutual inductance of the solenoids in terms of the variables given in the problem.

$B_{of\,2} = \mu_0 n_2 i_2$ $\qquad \phi_1 = B_2 \pi r_1^2 n_1 h = \mu_0 n_2 i_2 \pi r_1^2 n_1 h$

$\phi_1 = M_{12} i_2$ $\qquad \boxed{\therefore M_{12} = \mu_0 n_1 n_2 h \pi r_1^2}$

b) Now derive an expression for the mutual inductance of the same two solenoids as seen by a physicist flying past the solenoids at a constant velocity, v, along a path parallel to the axis of the solenoids. Assume v is a very large fraction of the speed of light.

what changes is h, n_1, n_2 $\qquad \gamma = \dfrac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} > 1$

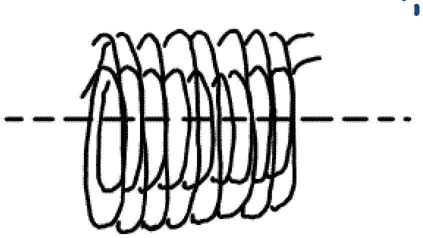
let N_1, N_2 = # turns in Sol. 1, 2, respectively

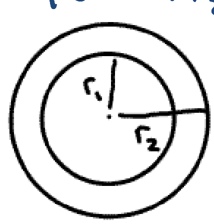
$n_1 = N_1/h \qquad n_2 = N_2/h$

h is solenoid length in rest frame of solenoids

In another frame $h' = h/\gamma$ (contraction)

$\gamma > 1 \qquad n_1' = n_1\gamma \qquad n_2' = n_2\gamma$





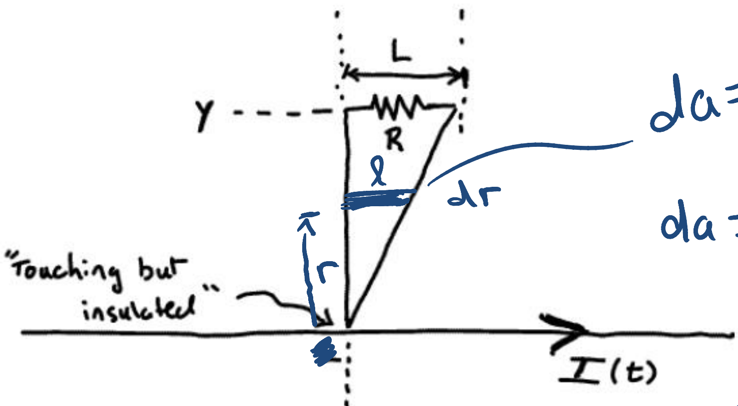
so,

$M_{12}' = \mu_0 n_1\gamma n_2\gamma \dfrac{h}{\gamma} \pi r_1^2$

$\boxed{M_{12}' = \gamma M_{12}}$

Problem 7 (20 pts, show all work) :

A triangular conducting loop containing a resistor, R, and an infinite current-carrying wire are arranged in the plane of the paper as shown in the sketch below. Assume the tip of the triangle is touching the infinite wire, but is insulated electrically from the wire. The current in the wire, $I(t)=Kt^2$, where K is a constant with units of Amperes/s^2. Assume the current in the infinite wire is zero at t=0, calculate the total energy dissipated by the resistor after 2 seconds. That is, calculate the energy dissipated by the resistor between t=0 and t=2.



$da = \ell\, dr$ $\qquad \frac{L}{Y} = \frac{\ell}{r} \Rightarrow \ell = \frac{rL}{Y}$

$da = \frac{L}{Y} r\, dr$

1st Figure out B(r) from wire

$\int B \cdot ds = \mu_0 I$

$B(r) = \frac{\mu_0 I}{2\pi r}$

$B(r) = \frac{\mu_0 I}{2\pi r}$

determine Φ_M at fixed t

$\Phi_M = \int_0^Y B\, da = \int_0^Y \frac{\mu_0 I}{2\pi r} \frac{L}{Y} r\, dr$

$\Phi_M = \frac{\mu_0 I L}{2\pi}$

Now $\mathcal{E} = -\frac{d\Phi_M}{dt}$

$\Phi_M(t) = \frac{\mu_0 L K t^2}{2\pi}$

$\frac{d\Phi_M}{dt} = \frac{\mu_0 L K 2t}{2\pi}$

So $|\mathcal{E}| = \frac{\mu_0 L K}{\pi} t$

Power dissipated by resisor $= Vi = i^2 R = \frac{V^2}{R}$

$= \left(\frac{\mu_0 L K t}{\pi}\right)^2 / R$ P (instantaneous)

Power $\int_0^2$ prresolt bet t=0 $\quad$ ——

$\frac{1}{R}\left(\frac{\mu_0 L K}{\pi}\right)^2 \int_0^2 t^2\, dt = \frac{8}{3R}\left(\frac{\mu_0 L K}{\pi}\right)^2$