Presentation Topic preferences
Will make groups + post on web soon
Next week — class in your Jammies

Last Time

Special Theory of Relativity
only valid for inertial frames of reference

Assume
- Speed of light, c, constant for all observers in all inertial reference frames
- “Physics” invariant
Time dilation

\[ \Delta t' = \Delta t \gamma \]

\[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} > 1 \]

measured time is shortest in proper frame of reference

frame where event is at rest
Length Contraction

$S' \longrightarrow \nu$

$\Delta x' = \frac{\Delta x}{\gamma}$

Length is greatest when measured in proper frame of reference.
Classical physics (Galilean transformations)

\[ x' = x - vt \]

\[ y' = y \]

\[ z' = z \]

\[ t' = t \]
Relativistic Transformations:

\[ d = \frac{x'}{\gamma} \]

\[ x' = d \gamma \]

\[ d = x - vt \]

\[ \frac{x'}{\gamma} = x - vt \]

\[ x' = \gamma(x - vt) \]
Galilean Transformation

\[ x' = d' - vt' \]

\[ d' = x \quad t' = t \]

\[ x' = x - vt \]

Relativistically

\[ x = \text{distance from } O \text{ to } A \text{ in } S \]

\[ d' = \frac{x}{\gamma} \]

\[ x' = d' - vt' \]

\[ x' = \frac{x}{\gamma} - vt' \]

\[ x = \gamma (x' + vt') \]
\[ x' = \gamma (x' + vt') \quad x' = \gamma (x - vt) \]

\[ x = \gamma \left[ \gamma (x - vt) + vt' \right] \]

\[ t = \gamma \left( t' - \frac{v}{c^2} x' \right) \quad t' = \gamma \left( t - \frac{v}{c^2} x \right) \]
Lorentz Transformations

\[ x' = \gamma (x - vt) \]
\[ y' = y \]
\[ z' = z \]
\[ t' = \gamma (t - \frac{v}{c^2}x) \]

\( v \rightarrow \text{small} \Rightarrow \gamma = 1 \quad \frac{v}{c} \rightarrow 0 \text{ get Gal. Trans.} \)
\( s' \Rightarrow v = 0.7c \quad \delta = \frac{s}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = 1.4 \)

Event 1 \( t = 0, \ t' = 0 \) origins coincide

Spaceship passes Asteroid

Event 2 Laser Flashes at \( x = 3 \ km, \ t = 5 \mu s \)

What does event 2 look like in frames' \( \frac{km}{s} \)?

\[
x'_2 = \delta (x_2 - vt_2) = (1.4) \left[ 3 - 0.7 \left(3 \times 10^5 \right) 5 \times 10^{-5} \right] = 2.73 \ km
\]

\[
t'_2 = \delta \left( t_2 - \frac{v}{c^2} x_2 \right) = 1.4 \left[ 5 \times 10^{-6} - (0.7) 3 \right] = -2.8 \mu s
\]
Velocity Transformations

\( \vec{v} \) = velocity of object in \( S \)

\( \begin{align*}
\text{in } S & \rightarrow U_x = \frac{dx}{dt} \\
& \quad \quad U_y = \frac{dy}{dt} \\
& \quad \quad U_z = \frac{dz}{dt}
\end{align*} \)

\( \begin{align*}
\text{in } S' & \rightarrow U'_x = \frac{dx'}{dt'} \\
& \quad \quad = \frac{\gamma(dx - vdt)}{\gamma(dt - \frac{v}{c^2}dx)} \\
& \quad \quad = \frac{\gamma(dx - v)}{\gamma(1 - \frac{v}{c^2} \frac{dx}{dt})}
\end{align*} \)

\( U'_x = U_x - V \frac{U_x - V}{1 - \frac{v}{c^2} U_x} \)

\( \begin{align*}
U'_y = \frac{dy'}{dt'} & = \frac{dy}{\gamma(dt - \frac{v}{c^2}dx)} \\
& = \frac{U_y}{\gamma(1 - \frac{U_x V}{c^2})}
\end{align*} \)
\[ U_3' = \frac{U_3}{\sqrt{1 - \frac{U_x v}{c^2}}} \]

Suppose flying to LA
Pilot says speed is \( \frac{5}{8} c \)

\[ \Rightarrow \frac{dx}{dt} = v \text{ speed on ground} \]

Your proper time \( \tau = \tau' \)

\[ \eta = \frac{dx}{d\tau} \text{ - mean speed } \]

\[ \text{Proper velocity} \]

Velocity relevant if you have to decide whether to eat on plane
\[ \eta = \frac{dx}{d\tau} \quad \text{transform like } dx \]

\[ dt = \gamma \, d\tau \]

\[ \eta = \frac{dx}{d\tau} = \gamma \frac{dx}{dt} = \gamma v \quad \text{proper velocity} \]

Momentum cons. should hold relativistically

Newtonian:

\[ m_a v_a + m_b v_b = m_c v_c + m_d v_d \]

\[ v_a \rightarrow \]

\[ v_a \rightarrow \]
\[ s \xrightarrow{L} v \]
\[ v_a \rightarrow \frac{\gamma (v_a - v)}{\delta (1 - \frac{v}{c^2} v_a)} \]

Use \( p = m \gamma \) → Momentum cons holds

if

\( p_x, p_y, p_z, (?) \)

define 4th entity

\( x, y, z, t \)

\[ E = \gamma mc^2 = mc^2 \gamma \]

\( \equiv \text{Relativistic energy} \)

\[ E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \]

\[ E = mc^2 \left( 1 - \frac{v^2}{c^2} \right)^{-\frac{1}{2}} \]
\((1 + x)^{1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \cdots\)

\[E = mc^2 + \frac{1}{2}mv^2 + \text{h.o.t.}\]

\[
\uparrow \quad \uparrow \quad \uparrow \\
\text{rel} \quad \text{const} \quad \text{KE}
\]

particle not moving

\[E = mc^2\]