

Physics 142 - October 18, 2007

- Presentation Topic preferences
- Will make groups + post on web soon
- Next week - class in your Jammies

Last Time

Special Theory of Relativity

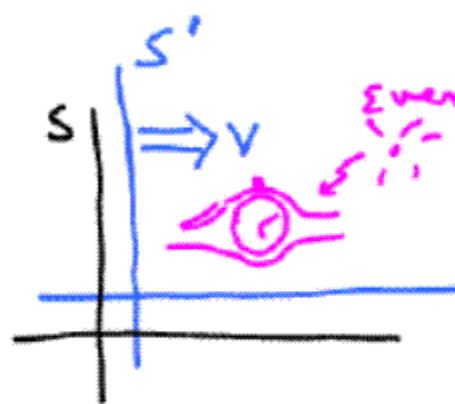


only valid for inertial frames of reference

Assume

- Speed of light, c , constant for all observers in all inertial reference frames
- "Physics" invariant

Time dilation



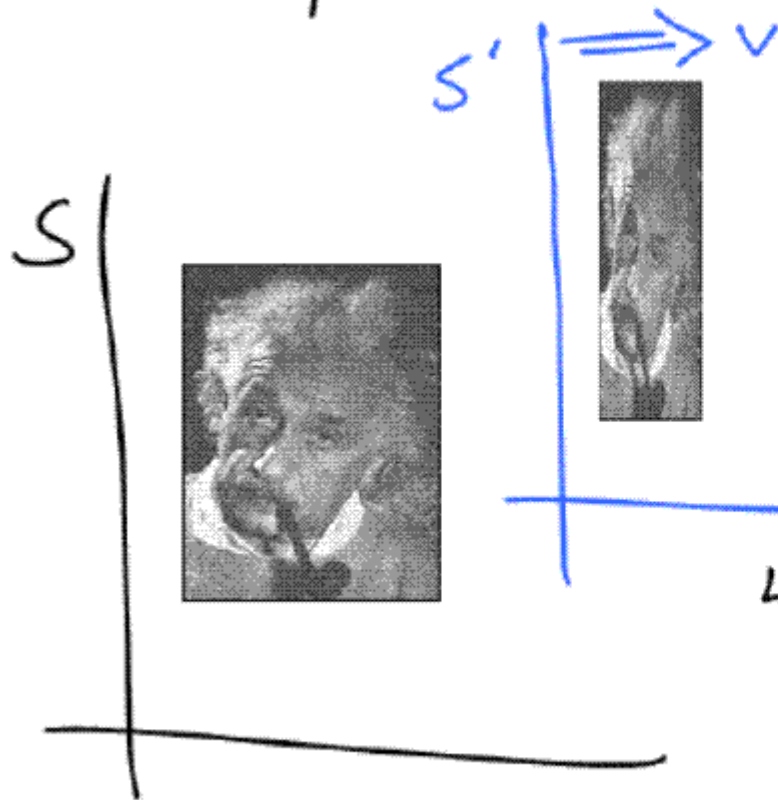
$$\Delta t' = \Delta t \gamma$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} > 1$$

measured time is
shortest in
proper frame
of reference

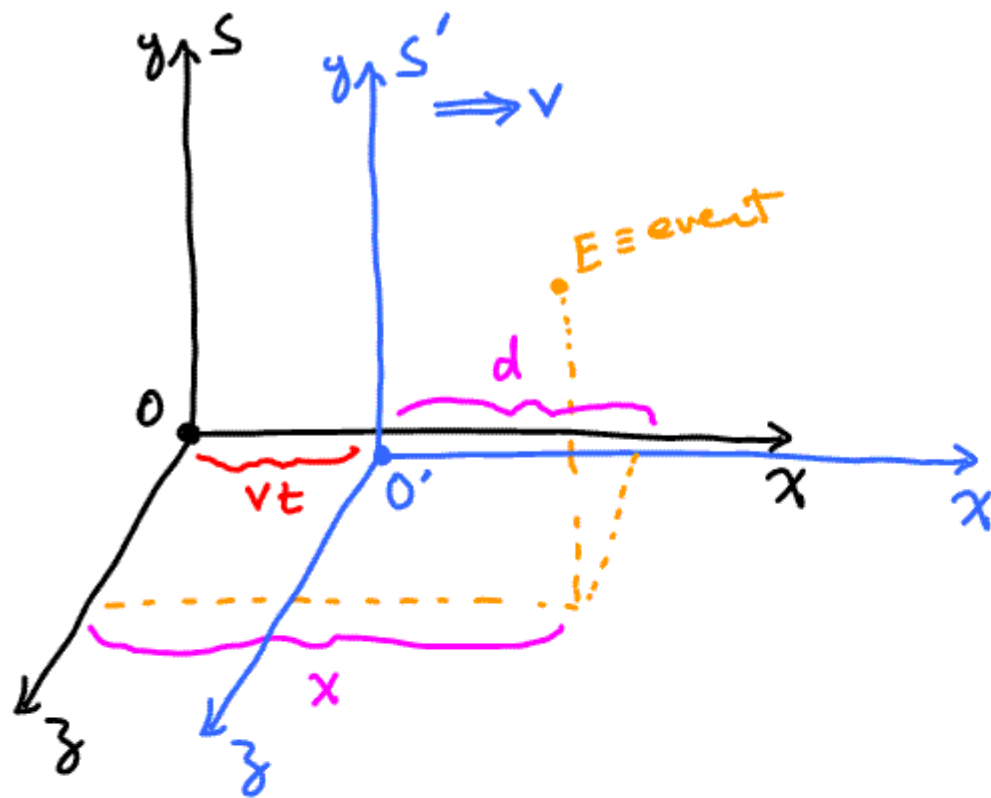
frame where event is at rest

Length Contraction



$$\Delta x' = \frac{\Delta x}{\gamma}$$

Length is greatest when
measured in
Proper Frame
of Reference



at $t=0, t'=0$
 two systems
 overlap
 $O=O'$

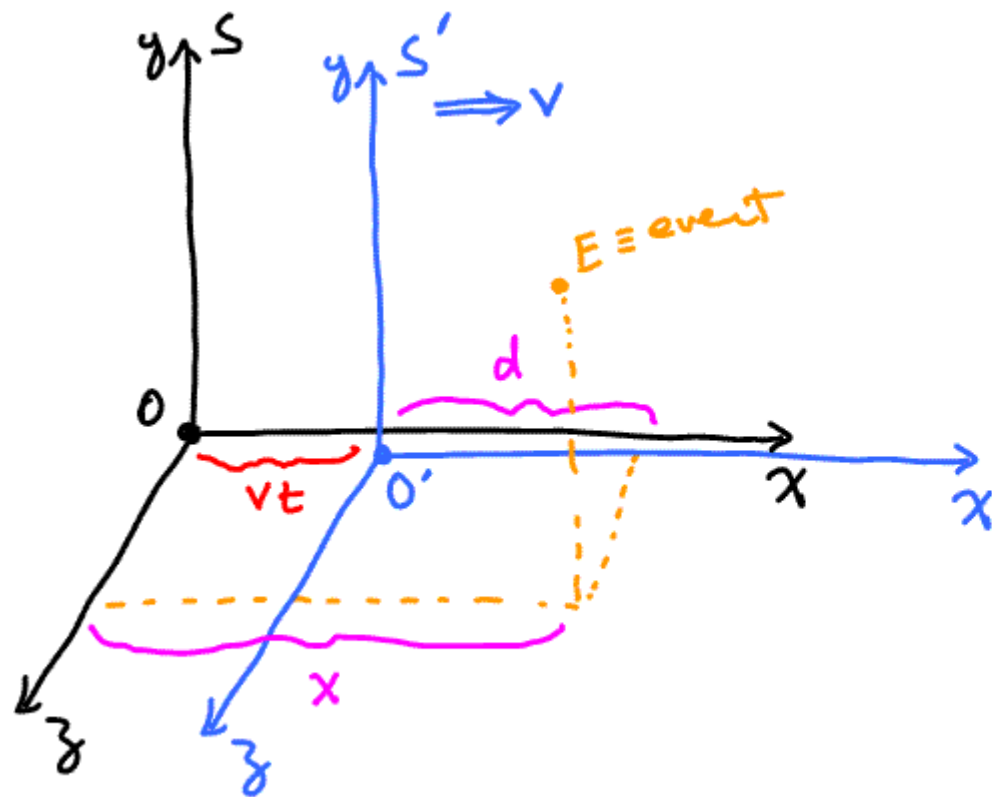
classical physics (Galilean transformations)

$$x' = d = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$



Relativistic Transf.

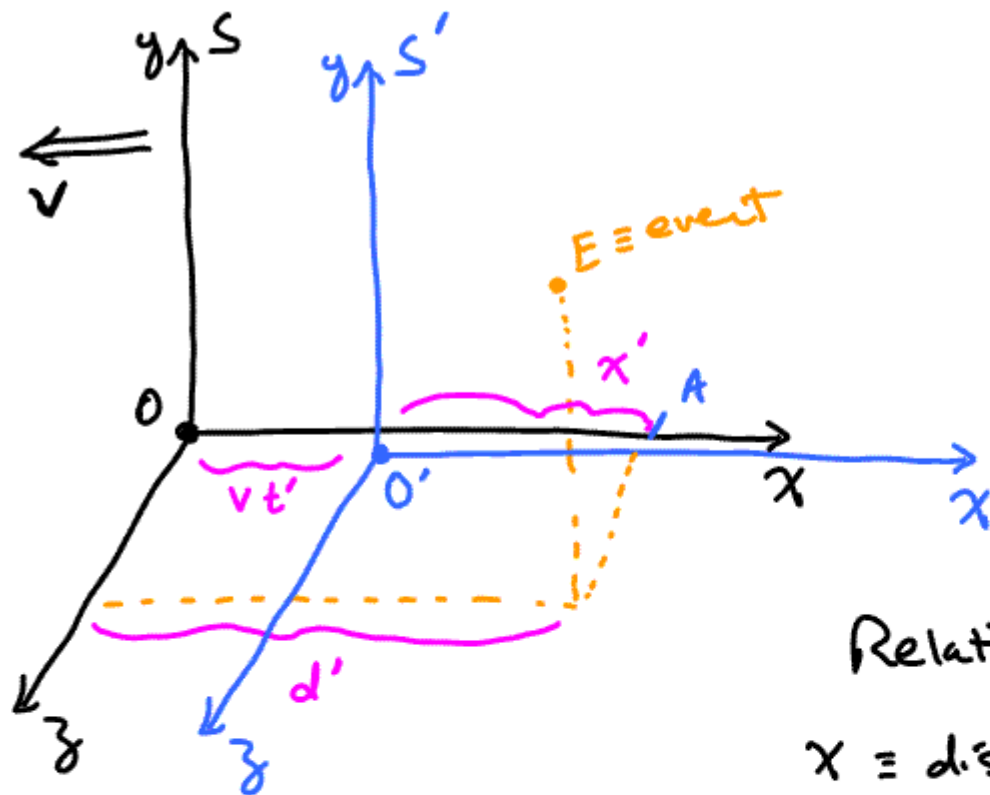
$$d = \frac{x'}{\gamma}$$

$$x' = d\gamma$$

$$d = x - vt$$

$$\frac{x'}{\gamma} = x - vt$$

$$x' = \gamma(x - vt)$$



Galil. TRANS.

$$x' = d' - vt'$$

$$d' = x \quad t' = t$$

$$x' = x - vt$$

Relativistically

$x \equiv$ dist from O to A in S

$$d' = \frac{x}{\gamma}$$

$$x' = d' - vt'$$

$$x' = \frac{x}{\gamma} - vt'$$

$$x = \gamma(x' + vt')$$

$$x = \gamma(x' + vt')$$
$$x' = \gamma(x - vt)$$

↓ ← sub. in

$$x = \gamma[\gamma(x - vt) + vt']$$

↓ bit of algebra

$$t = \gamma\left(t' - \frac{v}{c^2}x'\right)$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

Lorentz
TRANSFORMATIONS

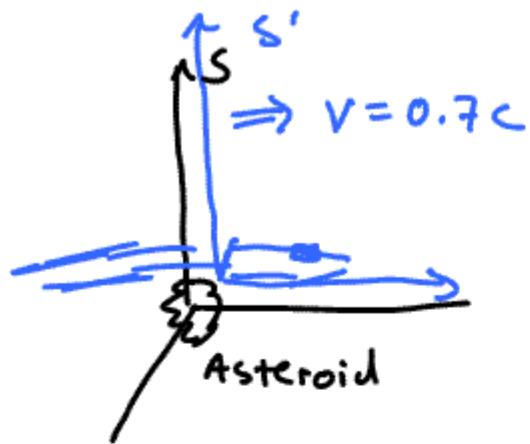
$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$v \rightarrow \text{small} \Rightarrow \gamma = 1$ $\frac{v}{c} \rightarrow 0$ get Gal. TRANS.



$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = 1.4$$

Event 1 $t=0, t'=0$ origins coincide

Spaceship passes Asteroid

Event 2 Laser flashes at $x_2 = 3 \text{ km}, t_2 = 5 \mu\text{s}$
in S

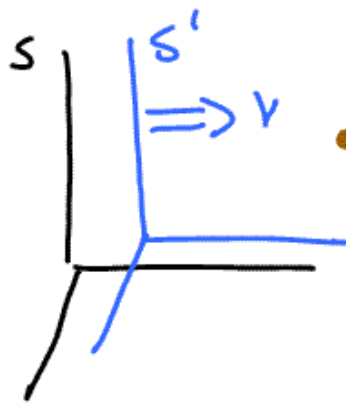
what does event 2

look like in frames' ? km/s

$$x_2' = \gamma(x_2 - vt_2) = (1.4) \left[3 - .7 (3 \times 10^5) 5 \times 10^{-6} \right] = 2.73 \text{ km}$$

$$t_2' = \gamma \left(t_2 - \frac{v}{c^2} x_2 \right) = 1.4 \left[5 \times 10^{-6} - (.7) 3 \right] = -2.8 \mu\text{s}$$

Velocity Transformations



$\vec{U} \equiv$ velocity of object in S

$$\text{in S} \rightarrow U_x = \frac{dx}{dt}$$

$$U_y = \frac{dy}{dt}$$

$$U_z = \frac{dz}{dt} \leftarrow U_x$$

in S'

$$U_x' = \frac{dx'}{dt'} = \frac{\gamma(dx - v dt)}{\gamma(dt - \frac{v}{c^2} dx)} = \frac{\gamma(\frac{dx}{dt} - v)}{\gamma(1 - \frac{v}{c^2} \frac{dx}{dt})}$$

$$\boxed{U_x' = \frac{U_x - v}{1 - \frac{v}{c^2} U_x}}$$

$$U_y' = \frac{dy'}{dt'} = \frac{dy}{\gamma(dt - \frac{v}{c^2} dx)} = \boxed{\frac{U_y}{\gamma(1 - \frac{U_x v}{c^2})}}$$

$$U_3' = \frac{U_3}{\gamma(1 - \frac{U_x v}{c^2})}$$

Suppose flying to LA

Pilot says Speed is $5/8 c$

$$\hookrightarrow \frac{dx}{dt} = v \quad \text{Speed on Ground}$$

Your proper time $\tau \equiv T$

$$\eta = \frac{dx}{dT} \quad \leftarrow \text{mes on plane}$$

Proper velocity

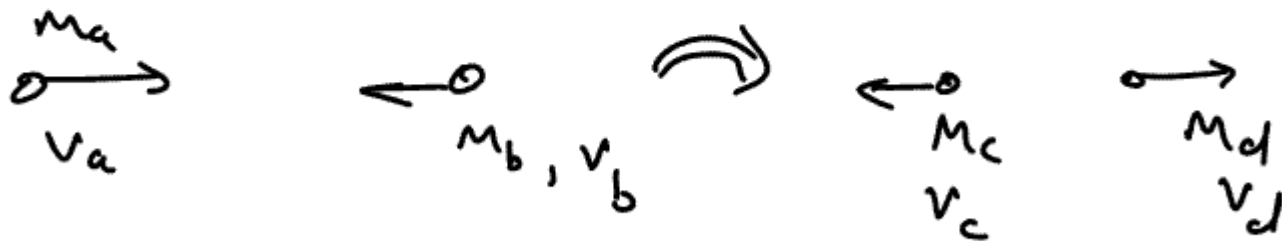
Velocity relevant if you have to decide whether to eat on plane

$$\eta = \frac{dx}{d\tau} \quad \text{transform like } dx$$

$$dt = \gamma d\tau$$

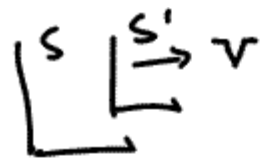
$$\eta = \frac{dx}{d\tau} = \gamma \frac{dx}{dt} = \gamma v \quad \text{proper velocity}$$

Momentum cons. should hold relativistically



Newton $m_a v_a + m_b v_b = m_c v_c + m_d v_d$

$$v_a \rightarrow$$



$$v_a \rightarrow \frac{\gamma (v_a - v)}{\gamma (1 - \frac{v}{c^2} v_a)}$$

use $p \equiv m\eta \rightarrow$ Momentum cons holds

$$p_x, p_y, p_z, (?)$$

if
define 4th entity

$$x, y, z, t$$

$$E = \gamma m c^2 = m c^2 \eta$$

\equiv Relativistic
energy

$$E = \frac{m c^2}{\sqrt{1 + (\frac{v}{c})^2}}$$

$$E = m c^2 (1 - (\frac{v}{c})^2)^{-1/2}$$

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \dots$$

$$E = mc^2 + \frac{1}{2}mv^2 + \text{h.o.t.}$$

↑ ↑ ↑
rel const KE
energy

particle not moving

↳ $E = mc^2$