Last Time

\[ \vec{E}_p = \int \frac{k \varrho \, d\mathbf{r}}{r^2} \hat{r} = \int \frac{k \rho(r) \, d\mathbf{r}}{r^2} \hat{r}. \]

Finding the electric field \( \Rightarrow \) Brute Force

This can be hard ... find easier way to determine \( \vec{E} \)
Gauss' Law

\[ \Phi = \oint E \cdot dA = \frac{Q_{\text{enclosed}}}{E_0} \]

Sum \( \hat{E}_n \) to \( \hat{n} \) over surface

\( \oplus \) if along \( \hat{n} \)

\( \oplus \) in against \( \hat{n} \)
Gauss' Law true in general ... most useful under selected conditions of symmetry

\[ \oint E \cdot dA = \frac{Q_{\text{enc}}}{\varepsilon_0} \]

Integral over volume inside Gaussian Surface (Not necessarily all Charge)

\[ \int \mathbf{E} \cdot d\mathbf{A} \]

Easy if \( \mathbf{E} \perp d\mathbf{A} \) or \( \mathbf{E} \parallel d\mathbf{A} \)

Easy if \( \mathbf{E} \) is constant on surface - can pull out of integral
\[ \vec{E} = 0 \text{ inside a conductor} \]

**Important**

Charge resides on outside surface of conductor

\[ \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0} \]

\[ \vec{E} = 0 \Rightarrow Q_{\text{enc}} = 0 \]
Curvilinear Coordinate Systems

Could do a generalized curvilinear coordinate system...

We will only use Spherical, Polar, and Cylindrical systems.

\[ s = r \theta \]
\[ ds = r \, d\theta \]
Find Area of disk

Integrate $dA$ around $\theta$ and then along $r$ → double integral.

$$dA = r \, d\theta \, dr$$

$$\int_0^R \int_0^{2\pi} dA = \int_0^R 2\pi r \, dr = \frac{2\pi R^2}{2} = \pi R^2$$
cylindrical coordinates

Some figures in this section from:
http://hyperphysics.phy-astr.gsu.edu/hbase/sphc.html

Also Griffiths, Intro to Electromagnetism
**Spherical polar coordinates**

Volume element:

$$dV = r^2 \sin \theta \, d\theta \, d\phi \, dr$$

$$d\phi = r \sin \theta \, d\phi.$$
Sphere Area = \int_0^{2\pi} \int_0^\pi dA = \int_0^{2\pi} \int_0^\pi r^2 \sin \theta d\theta d\phi

\int_0^\pi r^2 \sin \theta d\theta \int_0^{2\pi} d\phi = 2\pi r^2 \int_0^\pi \sin \theta d\theta = 4\pi r^2

Griffiths — Intro. to Electrodynamics
\[ dA = 2\pi r \, dr \]
\[ \int dA = \int_0^R 2\pi r \, dr \]
\[ = \pi R^2 \]

Spherical shell
\[ dv = 4\pi r^2 \, dr \]
\[ \int_0^R dv = \int_0^R 4\pi r^2 \, dr = \frac{4\pi R^3}{3} \]
\[ g = kr^3 \text{ for } r < R \]
\[ = 0 \text{ for } r \geq R \]

**Total charge**

\[
\int_S dv = \int_0^R g \, 4\pi r^2 \, dr
\]

\[
= \int_0^R k \, r^3 \, 4\pi r^2 \, dr
\]

\[
= 4\pi k \frac{R^6}{6}
\]
Example

Cylindrical symmetry
\( \vec{E} \) radially out

\[ \Phi \vec{E} \cdot \hat{n} \, dA = \frac{\text{Gauss}}{\varepsilon_0} \]

\[ \vec{E} \cdot \hat{n} = 0 \]

Area of Gaussian surface
\[ = 2\pi rL + (2) \pi r^2 \]

\( + \lambda = \text{chg/lenth} \)

\( \text{as line charge} \)

Find Field at Point R

Endcaps don't contribute
\[ \oint \vec{E} \cdot \hat{n} \, dA = |E| 2\pi r L = \frac{LA}{\varepsilon_0} \]

\[ |E| = \frac{\lambda}{\varepsilon_0 2\pi r} \]

\( \vec{E} \) Field a distance \( r \) from an ideal charge

\( L \) is the length of the charged wire.