

### Final Exam (December 17, 1999)

If you *object* to having your grades posted on the class website under your social security number, check here  .

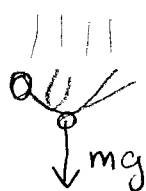
I will post those with no check in the box.

*Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless noted otherwise.*

#### Problem 1 (10 pts):

Some say if Dante were alive now, he would describe hell in terms of taking a university course in physics. The vision brought to mind by some of the comments I've heard is that of the devil standing over the pit of hell gleefully dropping young, innocent, and hardworking students into the abyss. *The speed of sound in the pit is 340 m/s*

- a) (4 pts) Suppose the opening to the pit of hell is at the surface of the earth. The devil drops students into the pit one-by-one. Ignoring air friction, how far down the pit, relative to the devil, is each student 10 s after being dropped by the devil?



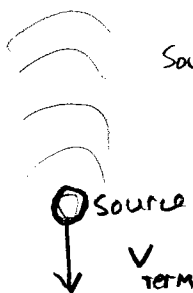
*Student falls w/ const. accel of 9.8 m/s<sup>2</sup>*

$$y_{\text{final}} - y_{\text{initial}} = v_{0y}t + \frac{1}{2}(-9.8)t^2$$

$$\text{dist from devil} = \frac{9.8}{2}(100) = 490 \text{ m}$$

- b) (6 pts) Let us suppose that air friction cannot be ignored. A student accelerates downward until the force due to air friction is equal (and opposite in direction) to that due to gravity. At that point, the student has reached a "terminal" (pardon the pun) velocity and continues downward without accelerating any further. After reaching terminal velocity, a falling student screams at a frequency of 855 Hz. The grinning devil hears the scream and judges it to be at 835 Hz. What is the terminal velocity of the student?

Observer (devil)



*Sound frequency emitted is doppler shifted down due to Motion*

*Symmetry of situation is such that can use same formula used w/ stationary source + receding observer*

$$f' = \frac{f}{1 + \frac{v_{\text{term}}}{v_{\text{sound}}}}$$

**Problem 2 (10 pts):**

A hockey puck of mass 6 kg has been rigged to explode, as part of a practical joke. Initially the puck moves at 1 m/s along the positive x direction toward the goalie. Moments before reaching the goalie, the puck explodes into three pieces. The first piece has a mass of 2 kg and moves with a speed of 2.5 m/s along a line that makes an angle of 53.1 degrees with the positive x axis. The second piece has a mass of 2 kg and moves with a speed of 1.5 m/s along a line that makes a 59 degree angle with the x axis, in the direction opposite that of the first piece. Assume the puck and the parts of the puck after the explosion glide without friction along the ice.

(a) (6 pts) What is the mass, direction and angle of the third part of the puck after the explosion?

use Momentum conservation in 2 dimensions

From II

$P_{ix} = \sum P_{fx}$

I  $(6)(1) = (2)(2.5) \cos 53.1 + M_3 v_3 \cos \theta + (2)(1.5) \cos 59$

II  $0 = \sum P_{fy} \Rightarrow (2)(2.5) \sin(53.1) + M_3 v_3 \sin \theta - (2)(1.5) \sin 59$

$-4 + 2.57 = 1.4 = M_3 v_3 \sin \theta$

↓

$\frac{1.4}{1.46} = \tan \theta$

$\Rightarrow \theta = 43.8^\circ$

From I  $6 - 3 - 1.54 = M_3 v_3 \cos \theta$

$1.46 = M_3 v_3 \cos \theta$

Mass conserved  $\Rightarrow M_1 + M_2 + M_3 = 6$

$\Rightarrow M_3 = 2 \text{ kg}$

$\frac{1.46}{2 \cos 43.8}$

$= v_3 = 1.01 \text{ m/s}$

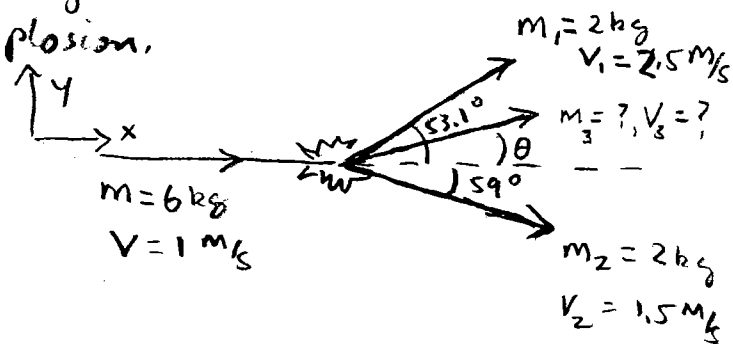
(b) (4 pts) Is energy conserved in this event? Why or why not?

NO! Explosion is inelastic.

-or-

Yes! Energy is always conserved.

IT is an inelastic process and some translational KE gained by conversion of chemical energy in explosion.



View From Above

**Problem 3 (10 pts):**

A 150 g block of ice at  $-5^{\circ}\text{C}$  sits in an insulated bucket. 20 g of steam at  $100^{\circ}\text{C}$  is injected into the bucket.

Possibly useful information:

- Mechanical equivalent of heat = 4.186 J/cal
- Volume of ideal gas at standard temp and pressure = 22.41 liters/mol
- Specific heat capacity of water = 4190 J/kg $\cdot$ K
- Specific heat capacity of ice = 2000 J/kg $\cdot$ K
- Specific heat capacity of ethanol = 2428 J/kg $\cdot$ K
- Heat of fusion of water,  $L_f = 334 \times 10^3$  J/kg
- Heat of vaporization of water,  $L_v = 2256 \times 10^3$  J/kg
- Number of shopping days before Christmas = 7

(a) (6 pts) What is the final temperature in the bucket?

$$\text{Heat energy to warm ice to } 0^{\circ}\text{C} = (.15 \text{ kg}) (2000 \frac{\text{J}}{\text{kg}\cdot\text{K}}) (5) = 1500 \text{ J}$$

$$\text{Heat energy given off as steam condenses} = (2256 \times 10^3 \frac{\text{J}}{\text{kg}}) (.02 \text{ kg}) = 45120 \text{ J}$$

$$\text{Heat energy to melt ice} = (.15 \text{ kg}) (334 \times 10^3 \frac{\text{J}}{\text{kg}}) = 50,100 \text{ J}$$

$$\text{Energy necessary to cool water from steam from } 100^{\circ}\text{C to } 0^{\circ}\text{C} = (.02 \text{ kg}) (4190) (100) = 8380 \text{ J}$$

$$\left( \begin{array}{c} \text{steam to} \\ \text{water} \end{array} \right) + \left( \begin{array}{c} \text{water} \\ \text{cool from} \\ 100 \rightarrow T \end{array} \right) = \left( \begin{array}{c} \text{warm ice} \\ 5 \rightarrow 0 \end{array} \right) + \left( \begin{array}{c} \text{melt} \\ \text{ice} \end{array} \right) + \left( \begin{array}{c} \text{warm water} \\ 0^{\circ} \rightarrow T \end{array} \right)$$

$$45120 + (.02)(4190)(100 - T) = 1500 + 50,100 + (.15)(4190)(T - 0)$$

$$8380 + 84T = 6480 + 629T$$

$$1900 = 545T$$

(b) (4pts) How much, if any, ice is left?

There is NO ice left

$$T = 3.5^{\circ}$$

Scores

- 1. \_\_\_/10
- 2. \_\_\_/10
- 3. \_\_\_/10
- 4. \_\_\_/10
- 5. \_\_\_/10
- 6. \_\_\_/8
- 7. \_\_\_/10
- 8. \_\_\_/10
- 9. \_\_\_/12
- 10. \_\_\_/10
- EC \_\_\_/8

Total \_\_\_/108

**Problem 4 (10 pts, zero/half/full credit):**

Estimate the speed of the planet Neptune through space.  
(Hint: assume the sun is at rest and that Neptune's orbit is circular.)

Possibly useful information:

- Distance from Sun to Neptune =  $4.5 \times 10^{12}$  m
  - Distance from Sun to Earth =  $1.5 \times 10^{11}$  m
  - Mass of Sun =  $1.99 \times 10^{30}$  kg
  - Mass of Neptune =  $1.03 \times 10^{26}$  kg
  - Mass of Earth =  $5.97 \times 10^{24}$  kg
  - Radius of Neptune =  $2.48 \times 10^7$  m
  - Radius of the Sun =  $6.96 \times 10^8$  m
  - Radius of the Earth =  $6.38 \times 10^6$  m
  - Number of former Spice Girls who are pregnant = 2
  - Average cream filling thickness in Oreo cookies = 1.9 mm
- $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$

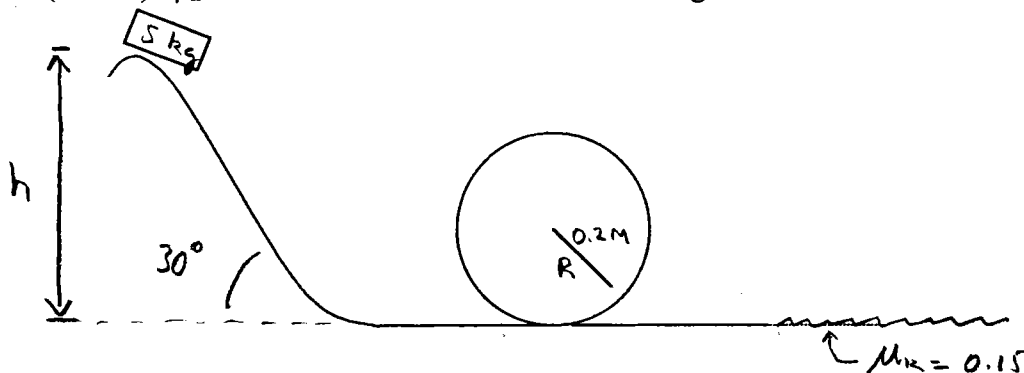
$$\frac{G M_{\text{neptune}} M_{\text{sun}}}{R_{\text{neptune-sun}}^2} = M_{\text{neptune}} \frac{V_{\text{neptune}}^2}{R_{\text{neptune-sun}}}$$

$$V = \sqrt{\frac{G M_{\text{sun}}}{R_{\text{neptune-sun}}}} = \sqrt{\frac{(6.67 \times 10^{-11})(1.99 \times 10^{30})}{4.5 \times 10^{12}}}$$

$V = 5431 \text{ m/s}$

**Problem 5 (10 pts):**

A block of mass  $M=5$  kg is released from rest and slides down a frictionless ramp of height  $h$  above the floor. The ramp makes an angle of  $30$  degrees with the horizontal. At the bottom of the track the block slides along the horizontal floor, around a vertical loop (loop-the-loop). The loop in the track has a radius of  $0.2$  meters. As the block passes the top of the loop, it never loses contact with the track. Once past the loop, the block encounters a region in the track where there is a rough surface (friction).  $\mu_k$  between the block and the track in this region is  $0.15$ .



(a) (4 pts) What is the minimum value for  $h$ , the starting height of the block above the floor?

min  $v$  at top of loop for circular motion

$$m \frac{v^2}{R} = mg \Rightarrow v = \sqrt{Rg} \text{ at top}$$

$$E_{\text{top}} = \frac{1}{2} m v^2 + (2R)mg$$

TOTAL                  KE                  PE

$$= \frac{1}{2} m (Rg) + 2Rmg = \frac{5}{2} mRg$$

$$E_{\text{top loop}} = E_{\text{top Ramp}} = mgh$$

TOTAL                  TOTAL

$$\frac{5}{2} mRg = mgh$$

$$h = \frac{5}{2} R = .5 M$$

(b) (3 pts) Assuming the value of  $h$  in part (a), what is the speed of the block at the bottom of the ramp?

$$\frac{1}{2} m v^2 = mgh = mg \frac{5}{2} R$$

$$v = \sqrt{5gR} = 3.1 \text{ m/s}$$

(c) (3 pts) How far does the block travel along the part of the track with friction before it comes to a stop?

$$E_{\text{TOT}} = \frac{1}{2} M v^2 = \frac{1}{2} M (5gR) = (\text{dist}) Mg \mu_k$$

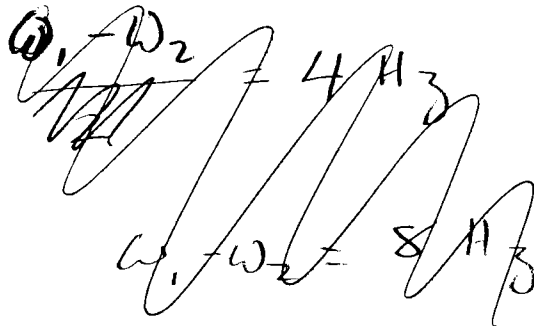
$$\frac{5}{2} R = \text{dist} \mu_k$$

$$\frac{\frac{5}{2} (0.2)}{(0.15)} = (\text{dist}) = 3.33 \text{ M}$$

**Problem 6 (8 pts) zero/half/full credit:**

A violinist is tuning the A string on her violin by listening for beats when this note is played simultaneously with a tuning fork of frequency 440 Hz. She hears a beat frequency of 4 Hz. She notices that, when she increases the tension in the string slightly, the beat frequency decreases. What was the frequency of the mistuned A string?

Beat frequency



$$f_1 - f_2 = 4 \text{ Hz}$$

∴ mistuned A string at ~~444~~ or 436 Hz

If increase  $T \rightarrow$  raises frequency  
 $\omega_1 - \omega_1$  becomes less

∴ Mistuned string has  $\nu = 436 \text{ Hz}$

due to omission in lecture  
Also accept incorrect answer

$$\frac{\omega_1 - \omega_2}{2} = 4 \text{ Hz}$$

$$\rightarrow \omega_1 - \omega_2 = 8 \text{ Hz}$$

String at 448 or 432 Hz

Mistuned string  $\nu$  at 432 Hz

**Problem 7 (10 pts) – full/half/zero credit:**

Erving Von Humbolt, famed Professor of Pre-Columbian Artifacts has discovered a musical instrument he believes was once used by native peoples in what is now southeast Paraguay for “some serious jammin’, rockin’, and gettin’ down” during adolescent mating rituals. Unfortunately, the instrument he has discovered is broken. He comes to you for help in understanding what sounds the instrument might have made. Please help him out!

The instrument has one string. That string is tied at one end and constrained to move freely up and down a thin rod on the other end (refer to the figure). Please determine the correct expression for the frequency of the  $n^{\text{th}}$  harmonic of the string in terms of the length ( $L$ ), tension ( $T$ ), and the mass/length ( $\mu$ ) of the string. You must show your work to get credit.

(a)

$$v_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$$

where  $n=1,3,5, \dots$

(b)

$$v_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$$

where  $n=1,2,3, \dots$

(c)

$$v_n = \frac{n}{2L} \sqrt{\frac{gT}{\mu}}$$

where  $n=1,2,3, \dots$

(d)

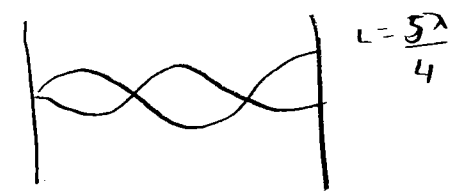
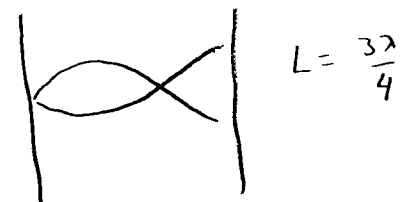
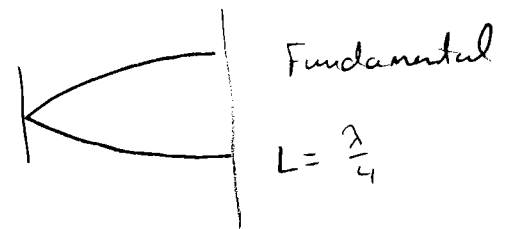
$$v_n = \frac{n}{4L} \sqrt{\frac{T}{\mu}}$$

where  $n = 1,3,5, \dots$

(e)

$$v_n = \frac{n}{4L} \sqrt{\frac{T}{\mu}}$$

where  $n=1,2,3, \dots$



Handwritten derivation:

$$v = \lambda_n v_n$$

$$\lambda_n = \frac{v}{v_n}$$

$$L = \frac{n v}{4 v_n}$$

$$v_n = \frac{n v}{4 L} = \frac{n}{4 L} \sqrt{\frac{T}{\mu}} \quad n=1,3,5$$

Handwritten general formula:

$$L = \frac{n \lambda_n}{4} \quad n=1,3,5, \dots$$

**Problem 8 (10 pts) – full/half/zero credit:**

Consider the masses configured on a frictionless inclined plane as shown in the figure. The pulley is frictionless, has a moment of inertia,  $I$ , and a radius  $R$ . The plane is inclined by an angle  $\theta$  with respect to the horizontal. Determine the correct expression for the acceleration of  $m_1$  in terms of the other quantities. Take the positive direction for the acceleration of  $m_1$  to be down. You must show your work to get credit.

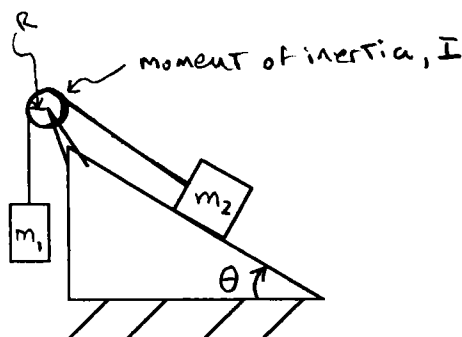
(a) 
$$a = \frac{g(m_1 - m_2 \sin(\theta))}{(m_1 + m_2)}$$

(b) 
$$a = \frac{g(m_1 - m_2 \sin(\theta))}{\left(\frac{I}{R^2} + m_1 + m_2\right)}$$

(c) 
$$a = \frac{g(m_1 - m_2)}{(m_1 + m_2)}$$

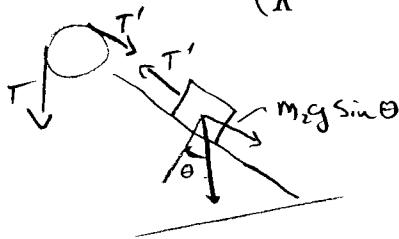
(d) 
$$a = \frac{g(m_1 - m_2 \sin(\theta))}{(m_1 + m_2)}$$

(e) 
$$a = \frac{g(m_1 - m_2)}{\left(\frac{I}{R^2} + m_1 + m_2\right)}$$



$$m_1 a = m_1 g - T$$

$$m_2 a = T' - m_2 g \sin \theta$$



$$I \alpha = R T - R T' = R(T - T')$$

$$a = R \alpha \quad \therefore \alpha = \frac{a}{R}$$

$$\frac{I a}{R} = R(T - T') = R(m_1 g - m_1 a - m_2 a - m_2 g \sin \theta)$$

$$\left[ \frac{I}{R^2} + m_1 + m_2 \right] a = m_1 g - m_2 g \sin \theta$$

$$a = \frac{m_1 g - m_2 g \sin \theta}{\frac{I}{R^2} + m_1 + m_2}$$



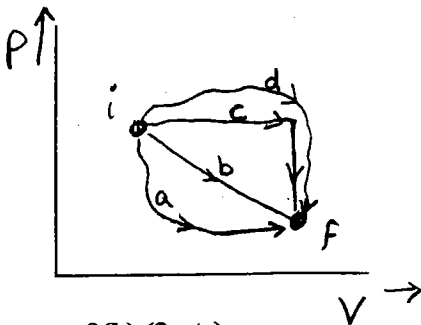
**Problem 9 (12 pts) – no partial credit:**

**9(a) (6 pts, no partial credit)**

The figure below shows four paths on a p-V (pressure-volume) diagram along which a gas can be taken from state *i* to state *f*. Rank the paths according to

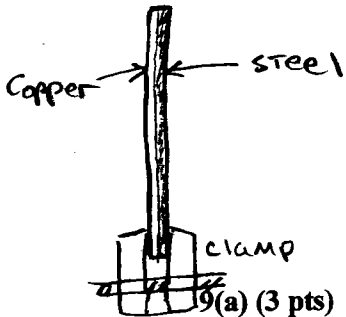
- (a) (3 pts) the change in internal energy of the gas  $a = b = c = d$   $dQ = dU + dW$   
 (b) (3 pts) the work done by the gas  $d > c > b > a$   
 (c) (3 pts) the magnitude of the heat transfer  $d > c > b > a$

Rank them greatest to least. They could be equal. For example, answers for each part should look something like the following:  $1 > 2 = 4 > 3$



**9(b) (3 pts)**

A flat, thin strip is made of a sandwich of two metals. The metal on the left is copper, which has a coefficient of linear expansion of  $1.7 \times 10^{-5}/C^\circ$ , and the metal on the right is steel, which has a coefficient of linear expansion of  $1.2 \times 10^{-5}/C^\circ$ . The bottom of the strip is clamped so the strip stands straight up at a temperature of  $20 C^\circ$ . In what direction does the strip lean at  $40 C^\circ$ ? Why?



Copper expands + contracts more w/ respect to change in temperature

Raise  $T \rightarrow$  copper expands more than steel

Bend is to the Right

**9(a) (3 pts)**

Why do divers (as in a diver on the swim team) often tuck themselves into a ball-like shape when executing several turns in a dive?

IT reduces the moment of inertia of diver which increases angular velocity of turning because momentum is conserved.

$$I_1 \omega_1 = I_2 \omega_2 \quad \text{angular}$$

$$L_1 = L_2$$

**Problem 10 (10 pts) – zero/half/full credit:**

*The matter in stars is in equilibrium between the gravitational force pulling in radially and the "radiation pressure" of energy released by thermonuclear reactions pushing out. When they run out of nuclear fuel in the center (core), the radiation pressure is reduced and they collapse.*

Under gravitational collapse, the radius of a spinning spherical star of uniform density shrinks by a factor of 2, with the resulting increased density remaining uniform throughout as the star shrinks. What will be the ratio of the final angular speed  $\omega_2$  to the initial angular speed  $\omega_1$ ? Select the correct answer below. You must show your supporting work to get credit.

$\omega_2/\omega_1 =$  (a) 2 (b) 0.5 (c) 4 (d) 0.25 (e) 1.0

Angular Momentum is Conserved

$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

$$I \text{ for Solid sphere} = \frac{2}{5} MR^2$$

$$\frac{2}{5} MR_i^2 \omega_i = \frac{2}{5} MR_f^2 \omega_f$$

$$R_i^2 \omega_i = \left(\frac{1}{2} R_i\right)^2 \omega_f$$

$$\frac{\omega_f}{\omega_i} = 4$$

**Extra credit (8 pts, 2 pts each) – no partial credit:**

(a) What subject in P113 did you find the most difficult to understand or master?

(b) What subject in P113 did you find the least difficult to understand and master?

(c) What subject in P113 did you find most interesting?

(d) What subject in P113 did you find least interesting?

Anything okay  
if Answered

W/ Topics  
we

*Travel safely and have a wonderful holiday! ... and try not to bore Grandma with a demo of standing waves in the wine glasses and discussions about the equilibrium tension in strings of Xmas tree lights!*

covered