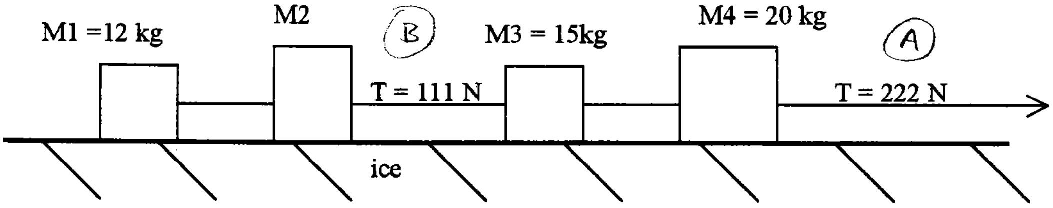
P121 University of Rochester NAME Soln key SM
S. Manly Spring 2001

Exam 2 (March 15, 2001)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (20 pts):

Four penguins are tied together by a zoo-keeper, who pulls them with a constant force along the surface of a frozen pond (assume the penguins can slide with no friction). Consider the ropes to be massless. The situation is pictured below (excuse the lack of talent in the graphical art). Determine the mass of the second penguin.



at pt A

$\Sigma F_x = ma$

① $222 = (M1 + M2 + M3 + M4)a = (47 + M2)a$

at pt B

$\Sigma F_x = Ma$

② $111 = (M1 + M2)a = (12 + M2)a$

2 eqns and 2 unknowns

÷ ① by ② $\frac{222}{111} = \frac{(47 + M_2)a}{(12 + M_2)a} \Rightarrow 2(12 + M_2) = (47 + M_2)$

$24 + 2M_2 = 47 + M_2$

$\boxed{M_2 = 47 - 24 = 23 \text{ kg}}$

P121 S. Manly

University of Rochester Spring 2001

NAME Soln key – SM

Problem 2 (20 pts):

1)	/20
2)	/20
3)	/20
4)	/20
5)	/20
6)	/20
tot	/120

(a) In a zero-gravity environment, can a rocket-propelled spaceship ever attain a speed greater than the relative speed with which the burnt fuel is exhausted? Explain why or why not.

Yes !! Ejection of gas from rocket causes a net change in momentum of the rocket with time (due to conservation of momentum in the exhaust/rocket system). This change in momentum with time is a force. So as long as there is rocket fuel being ejected there is a force accelerating the rocket. This force will continue to accelerate the rocket no matter what its speed — even if the rocket goes faster than the speed of the exhausted fuel.

(b) Suppose I create a very bizarre spring such that the potential energy function goes as Cx^4, where C is a constant (with units of N/m^4) and x is the distance from the equilibrium position of the spring. How much work is done by the spring as I compress it from x=0 to $x=X_f$?

$$W = \Delta PE = -CX_f^4$$

"–" sign comes from fact that work is done on spring ∴ spring does negative work

(c) What is the force exerted by the spring as it is compressed by an amount X_f?

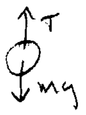
$$F = -\frac{dU}{dx} = -\frac{d}{dx}Cx^4 = -C4x^5$$

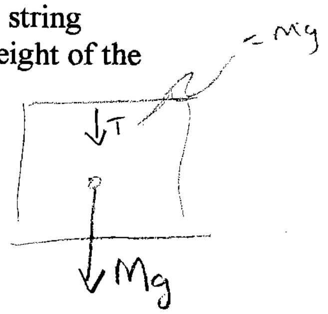
$$\Rightarrow |\text{Force at position } X_f| = C4X_f^5$$

and the direction will be such as to restore the spring to its natural equilibrium position

Problem 3 (20 pts):

On a table on the surface of the earth a small ball of mass m is suspended by a string inside a square box of mass M. The box sits on a scale which measures the weight of the system.





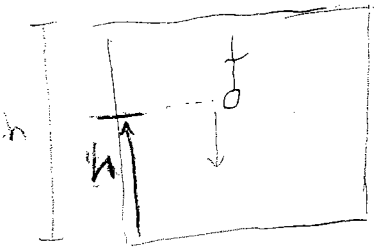
(a) What is the reading on the scale?

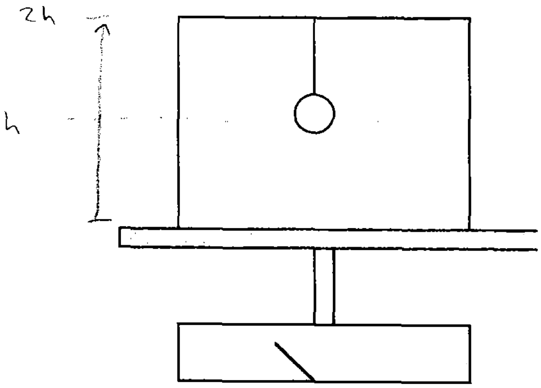
$$(M+m)g = F_{Total}$$

(b) Suppose the string is cut and the ball falls. While the ball is falling, what is the reading on the scale?

$$Mg$$

(c) While the ball is in free fall, find the acceleration of the center of mass of the ball-box system as a function of time (both magnitude and direction) in terms of M, m and g.





$$y = y_0 + v_{0y}t + \tfrac{1}{2}at^2$$

$$y = h - \tfrac{1}{2}gt^2$$ position of ball as fn of time

$$Y_{\text{Center of Mass}} = \frac{Mh + m(h - \frac{1}{2}gt^2)}{M+m}$$

$$Y_{CM} = \frac{(M+m)h - \frac{1}{2}gt^2}{(M+m)}$$

$$V_{CM} = \frac{dY_{CM}}{dt} = \frac{-gt}{M+M}$$

$$\boxed{a_{CM} = \frac{d^2Y_{CM}}{dt^2} = -\frac{g}{M+m}}$$

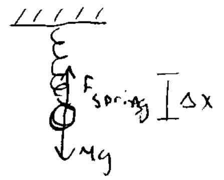
Problem 4 (20 pts):
The White House press did Bill and Monica a great disservice! I happen to know that they actually spent their time alone doing experiments with springs, inclined planes and friction (of course). When discovered, they were so embarrassed that they made up all that other stuff you heard so much about. Let's examine the physical system that so entranced Bill and Monica.

In the first experiment (not pictured) Monica hangs a 0.5 kg mass from the spring vertically. Bill carefully makes a measurement and excitedly reports to Monica that the hanging mass stretches the spring 10 cm from its natural length. Assume the spring is massless.

In the second experiment, Monica and Bill use the setup pictured below. The 0.5 kg block is free to slide on the inclined plane and is attached to the spring. The spring is firmly attached to a rod at the top of the plane. Assume the inclined plane is frictionless and makes an angle θ = 60 degrees with the vertical.

(a) Suppose the block is slowly let down the inclined plane until it stops. How far is the spring stretched beyond its natural length?

1st determine The spring constant
From the 1st experiment



$\Sigma F = 0$

$Mg = K(\Delta X)$

$(0.5)(9.8) = (K)(0.1)$

$K = 49$ N/M

$K(\Delta X)_{\text{inclined plane}} = Mg\cos\theta$

$\Delta X = Mg\cos\theta / K$

$\Delta X = \frac{(0.5)(9.8)\cos 60}{49} = 5$ CM

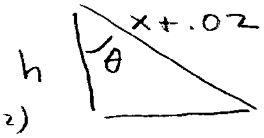
(b) Suppose initially the spring is compressed 2 cm and the block is let go. How far beyond its natural length will the spring be stretched as the mass slides down the plane?

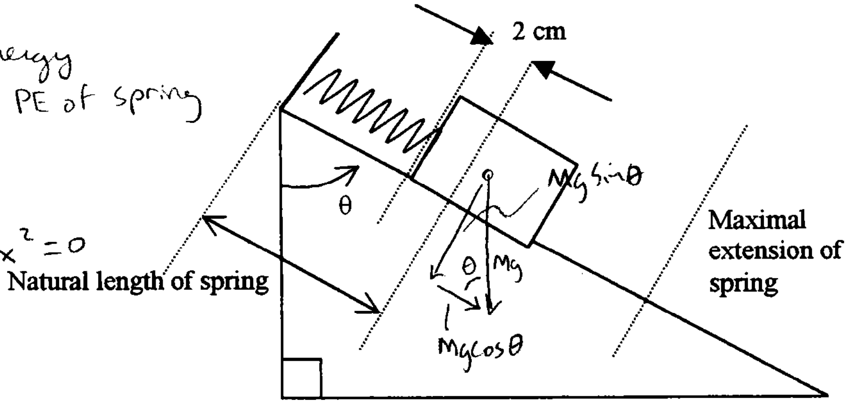
Use energy conservation

$$\underbrace{mgh + \frac{1}{2}K(.02)^2}_{\text{Initial Energy}} = \underbrace{\frac{1}{2}KX^2}_{\text{Final energy all in PE of spring}}$$

$\cos\theta = \frac{h}{X + 0.02}$

$h = \cos\theta(x + .02)$





$mg\cos\theta(x + .02) + \frac{1}{2}K(.02)^2 = \frac{1}{2}Kx^2$

$-\frac{2}{K}Mg\cos\theta\, x - Mg\frac{2}{K}\cos\theta(.02) - \frac{1}{2}K(.02)^2 + x^2 = 0$

$-0.1X - .002 - .01 + X^2 = 0$

$X^2 - (.1)X - .012 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+.1 \pm \overset{.24}{\sqrt{(.1)^2 + 4(.012)}}}{2} = \boxed{17\text{CM}}$$

-7CM

problem 4 continued:

(c) As for the stained dress … well, the stain was actually from a special glue that Monica and Bill put on the plane to provide friction. Suppose they modify the system so there is friction between the plane and the block. Let the coefficient of kinetic friction be 0.2. Now, given the initial compression of 2 cm, how far beyond its natural length will the spring be stretched as the mass slides down the plane? **Do NOT solve for this. Just set up the relevant equation.**

$$\underbrace{Mgh}_{\text{Initial gravi Energy}} + \underbrace{\tfrac{1}{2}k(0.02)^2}_{\text{Initial Energy in Spring}} = \underbrace{\tfrac{1}{2}kx^2}_{\text{PE of Spring}} + \underbrace{\overbrace{(x+.02)}^{\text{distance slides}}\,\overbrace{Mg\sin\theta}^{\text{Normal force}}\,\mu_K}_{\text{Energy lost to friction}}$$

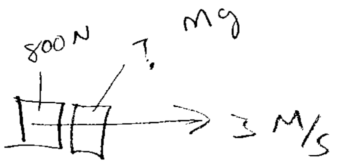
<u>Problem 5 (20 pts):</u>

Ken and Barbie are skating on ice in a straight line at 3.00 m/s. Ken wants to know Barbie's weight. Barbie becomes angry and pushes away from Ken so that she speeds up to 4.00 m/s and he slows down to 2.25 m/s in the same direction. Friction, in the physics sense, is negligible in this scene. If Ken weighs 800 N, what does Barbie weigh?

Soln on next sheet

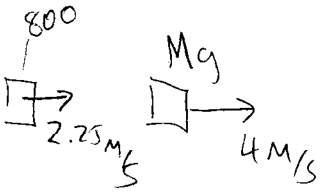
Problem 5 (20 pts):

Ken and Barbie are skating on ice in a straight line at 3.00 m/s. Ken wants to know Barbie's weight. Barbie becomes angry and pushes away from Ken so that she speeds up to 4.00 m/s and he slows down to 2.25 m/s in the same direction. Friction, in the physics sense, is negligible in this scene. If Ken weighs 800 N, what does Barbie weigh?



Initial Momentum

$(800 + Mg)\,3$



Final P

$(800)(2.25) + 4\,Mg$

P conservation $\vec{P}_{initial} = \vec{P}_{final}$

1-d problem so vector part drops out $P_{init} = P_{final}$

$$(800 + Mg)\,3 = (800)(2.25) + 4\,Mg$$

$$2400 + 3\,Mg = 1800 + 4\,Mg$$

$$600\ N = Mg$$

$$\boxed{\text{Barbie weighs } 600\ N}$$

Problem 6 (20 pts):

I happen to know a student named Ralph who drove from Rochester to Florida over spring break. He ran into a little trouble in a little place called Big Lick, North Carolina. You see, cars with New York plates aren't exactly a welcome sight in some small towns in the south and the local law enforcement officials are prone to look for an excuse to have a little fun with them.

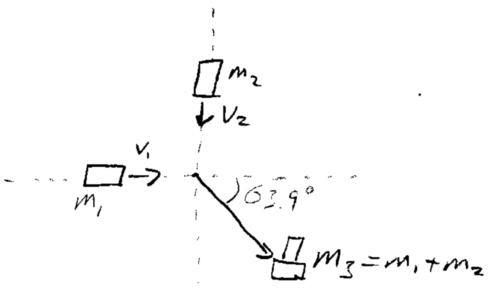
should read 55

Here's the story:
Ralph was driving due east through an intersection in his Toyota in a ~~35~~ mi/hr zone, when the local sheriff, Bubba Joe Howard, approaching the intersection from the north, hit him broadside. The two cars stuck together and skidded a distance of 22.2 m with locked wheels at an angle of 63.9 degrees to the south of east. The mass of Ralph's Toyota is 1465 kg. The mass of Bubba Joe's Chevy is 1923 kg. The coefficient of kinetic friction for the tires on the road was determined to be 0.9. Bubba Joe was not pleased that Ralph messed up his Chevy. So he ticketed Ralph for speeding. However, Ralph knew he'd been going less than the speed limit. Fortunately, he had taken physics 121 and was able to prove this to the judge along with the fact that the sheriff was speeding.

a) Please compute the speed of the two cars just before the collision.

b) Was the collision elastic or inelastic? Why?

Momentum is conserved before and just after the collision velocity of two cars together just after collision is V_3.



x-axis: $m_1 V_1 = m_3 V_3 \cos(63.9°)$ $\quad 1465\, V_1 = 3388 \cos(63.9°)\, V_3$

y-axis: $m_2 V_2 = m_3 V_3 \sin(63.9°)$ $\quad 1923\, V_2 = 3388 \sin(63.9°)\, V_3$

two equations, three unknowns: need more information.

Find the work after the collision to find V_3:

$$\frac{1}{2}(m_1+m_2)V_3^2 = (m_1+m_2)(g)(\mu_k)(d)$$

K.E. just after collision — work required to stop vehicle.

$$\frac{1}{2}(3388)V_3^2 = (3388)(9.8)(0.9)(22.2)$$

$$V_3 = 19.8 \text{ m/s}$$

Substitute into above equations:

x-axis: $1465(V_1) = 3388\cos(63.9)(19.8) \Rightarrow V_1 = 20.13 \text{ m/s} = 45.3 \text{ mi/hr}$

y-axis: $1923(V_2) = 3388\sin(63.9)(19.8) \Rightarrow V_2 = 31.32 \text{ m/s} = 70.5 \text{ mi/hr}$

Collision is inelastic because cars stick together. Show by comparing KE before and after: $\frac{1}{2}(1465)(20.13)^2 + \frac{1}{2}(1923)(31.32)^2 \neq \frac{1}{2}(3388)(19.8)^2$