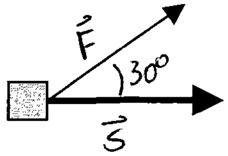
Exam 2 (October 15, 1999)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (24 pts):

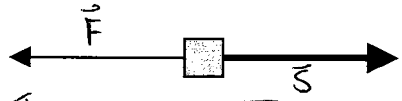
Consider a mass of 2 kg that is moved by one or more forces along the surface of a horizontal, frictionless floor. In the following diagrams, one vector represents one of the forces (of magnitude 3 N) that acts on the mass. The other vector represents the net displacement, S = 4 meters, of the mass due to the motion caused by the forces acting on it. Determine the work performed on the mass by the force shown during the movement.

(a)



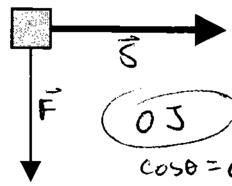
$W = \vec{F}\cdot\vec{S} = |\vec{F}||\vec{S}|\cos 30 = (3)(4)(\cos 30) = 10.4$ N m (or J)

(b)



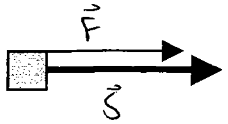
−12 J ($\cos\theta = -1$)

(c)



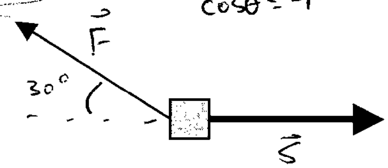
0 J ($\cos\theta = 0$)

(d)



+12 J ($\cos\theta = +1$)

(e)

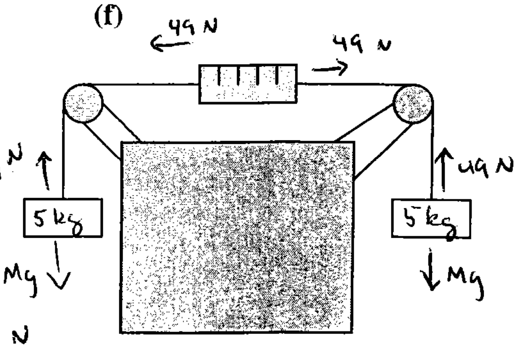


$W = -(3)(4)\cos 30° = -10.4$ J

All use $W = \vec{F}\cdot\vec{S} = |\vec{F}||\vec{S}|\cos\theta$ (θ = Angle between)

The following are diagrams of systems in equilibrium (static, not moving). They consist of one or more masses of 5 kg, connected to a set of spring scales calibrated in Newtons. Consider the strings and scales to be massless and the pulleys to be massless and frictionless. Assume there is no friction between the inclined plane and the mass. What value is read by the scales in each case?

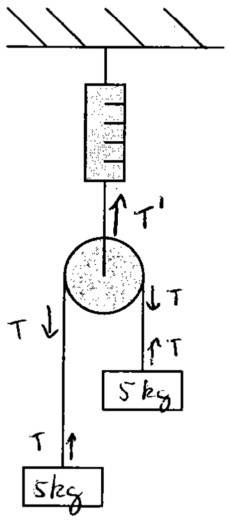
(f)



(5)(9.8) = 49 N

Scale will read Mg = (49 N) = 49 N

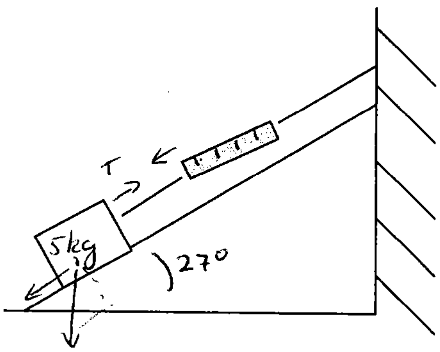
(g)



$T' = 2T = 2mg$

$T = (2)(5)(9.8) = 98$ N

(h)



$T = mg\sin\theta$

$T = mg\sin 27°$

$T = (5)(9.8)\sin 27°$

$T = 22$ N

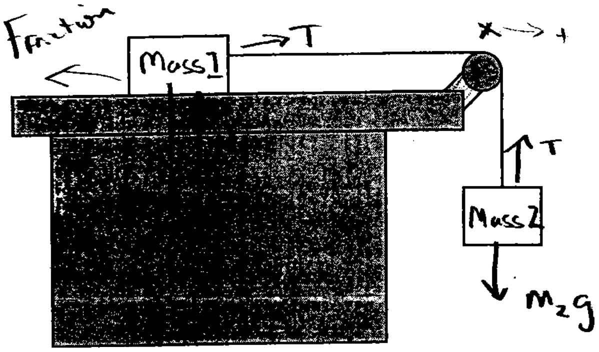
NAME Solution Set - SM

P113
S. Manly

University of Rochester
Fall 1999

Problem 2 (26 pts):

a) Consider the drawing below. Mass 1, is supported by the table. It is attached to mass 2 via a string passing over a massless, frictionless pulley. The coefficient of static friction between the table and mass 1 is 1.50. The coefficient of kinetic friction between the table and mass 1 is 1.03. Let mass 1 = 4 kg and mass 2 = 5 kg. The system starts out at rest. Describe the movement of mass 1 along the table, i.e., find the acceleration.



$N = m_1 g$ ← Assume NOT Moving for Now

$m_1 a_x = T - F_{Fr} = T - \mu_s N$

$a_x = 0 \quad \therefore T = \mu_s N \qquad \mu_{s_{Th}} = \frac{T}{N} = \frac{M_2 g}{M_1 g} = \frac{5}{4} = 1.25$

This is Threshold μ_s

if $\mu_s < \mu_{s_{Th}}$ system is Moving and must go back and use μ_k

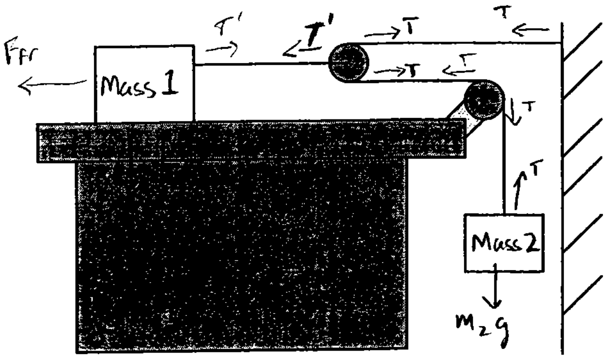
if $\mu_s \geq \mu_{s_{Th}}$ System is NOT Moving

$\mu_s = 1.50 > 1.25 = \mu_{s_{Th}}$ $\therefore$ System NOT Moving

$a_x = 0$

b) Repeat part (a) for a the new configuration of masses and pulleys pictured below. Again, assume the pulleys are massless and frictionless and the strings are massless. The other numbers remain the same.

Find μ_s Threshold to see if system is Moving.



$T' = F_{Fric} \qquad T' = 2T \quad$ w/ $a_x = 0$

$2 M_2 g = \mu_{s_{Th}} M_1 g$

$\mu_{s_{Th}} = \frac{2M_2 g}{M_1 g} = \frac{2M_2}{M_1} = \frac{10}{4} = 2.5$

$2.5 = \mu_{s_{Th}} > \mu_s = 1.5 \quad \therefore$ System is Moving

So, must use μ_k and Assume system Moving

Note: String geometry says $a_{M_2}\ \text{down} = 2a_{M_1}$ to right

$m_1 a_{M_1} = T' - F_{Fr} = 2T - F_{Fr} = 2T - \mu_k m_1 g$

$m_2 a_{m_2}(\text{down}) = m_2 g - T$

$\left\{ \begin{array}{l} M_1 a_{M_1} = 2T - \mu_k M_1 g \\ 2M_2 a_{M_1} = M_2 g - T \end{array} \right.$ → 2 eqns, 2 unknowns. Solve for T, a_{M_1}

$M_1 a_{M1} + 4M_2 a_{M_1} = 2M_2 g - \mu_k M_1 g$

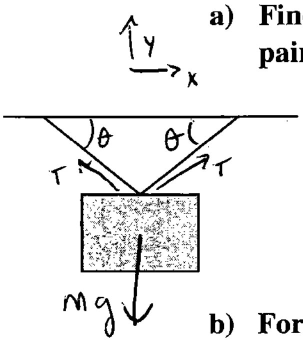
$a_{M_1} = \frac{(2M_2 - \mu_k M_1) g}{M_1 + 4M_2}$ to right

NAME Solution - SLM

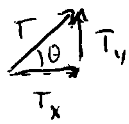
Problem 3 (25 pts):

Consider a 10 kg painting held in place by two wires of equal length (0.5 meters). Each wire makes an angle of θ with the horizontal, as shown in the figure.

a) Find the general equation for the tension in the wires as a function of the weight of the painting and the angle, θ.



$\Sigma F_y = 0 = 2T_y - Mg$ $\qquad 2T\sin\theta = Mg$

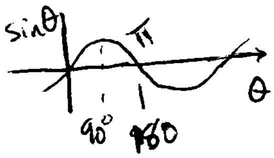


$T_y = T\sin\theta$

$$T = \frac{Mg}{2\sin\theta}$$

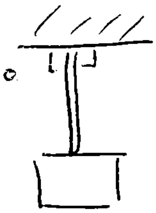
Tension in each wire is the same by symmetry

b) For what angle is the tension the least?



Tension is least when $\sin\theta$ is greatest

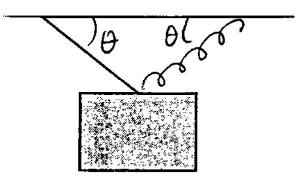
This happens at $\sin\theta = 1$ or $\theta = \pi/2 \Rightarrow \theta = 90°$

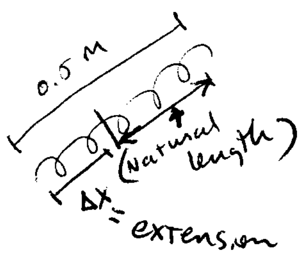


c) Assuming the angle ~~in part (b)~~ is 35°, solve for the tension in each wire using your answer to part (a).

$$T = \frac{(10\ kg)(9.8\ m/s^2)}{2\sin 35°} = 85\ N$$

d) Now, let one wire be replaced by a spring with spring constant = ~~0.5 N/m~~ ~~200 N/m~~ 1000 N/m, while the angle remains ~~that found in part (b)~~ 35°. Refer to the new drawing. What is the natural length of this spring?





$|F| = k\Delta x$

$85\ N = $ ~~200 N/m~~ $1000\ N/m\ \Delta x$

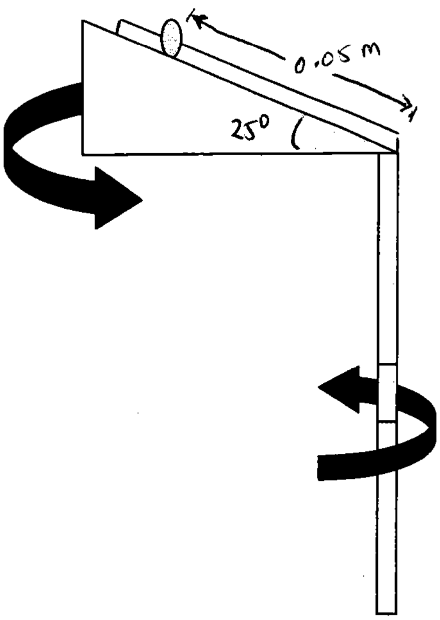
$\Delta x =$ ~~[illegible]~~ $.085\ m = 8.5\ cm$

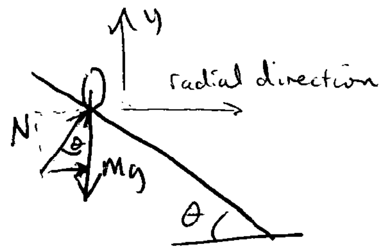
$\therefore$ Natural length $= 0.5 - 0.085 = .415\ m$

NAME Solution Set SM

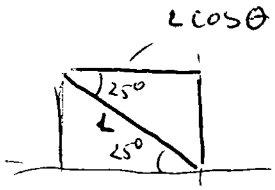
Problem 4 (25 pts):

My kids have a toy that consists of a rod that is held vertically. Firmly attached to this rod at one end is a wedge with an opening angle from the horizontal of θ=25 degrees. On top of this wedge is a thin wire with a bead of mass m on it. The wire is parallel to the surface of the wedge. When the rod is spun at a constant speed, the bead is observed to rise up a distance L=0.05 meters along the wedge surface and remain there. What is the speed of the mass?





bead moves in circle of radius $L\cos\theta$



For circular motion

$$F_{radial} = m\frac{v^2}{R}$$

$$F_{radial} = N\sin\theta$$

To find N note that in y direction

$$\Sigma F_y = 0 = N\cos\theta - Mg$$

$$N = \frac{Mg}{\cos\theta}$$

$$\therefore F_{radial} = \frac{Mg}{\cos\theta}\sin\theta$$

∴ For circular motion

$$\frac{Mg\sin\theta}{\cos\theta} = \frac{mv^2}{L\cos\theta}$$

or

$$v^2 = Lg\sin\theta$$

$$v = \sqrt{Lg\sin\theta}$$

$$v = \sqrt{(.05)(9.8)\sin 25}$$

$$v = 0.45\ \text{m/s}$$

NAME Solution Set – SM