P121 University of Rochester NAME Solutions Key
S. Manly Spring 2001

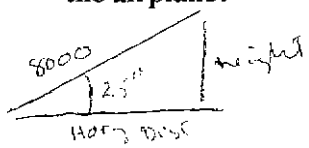
Exam 1 (February 8, 2001)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (20 pts):

A radar facility at ground level near an airport detects an airplane approaching directly from the north. The first observation shows the range to the plane is 8000 ft in a direction that is 25 degrees above the horizon. The radar tracks the airplane for another 85 degrees in the vertical north-south plane as shown in the drawing. The range at final contact is 3000 ft. The two observations and the radar are all in the vertical north-south plane.

a) At the instant of the first observation, what is the horizontal distance from the radar to the airplane?



Horizontal Dist = $8000 \cos 25^\circ = 7250$ ft

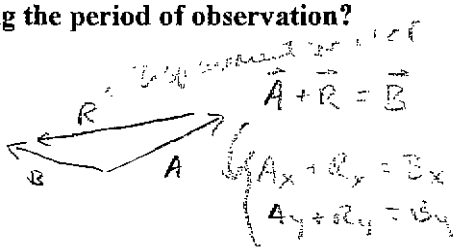
b) At the instant of first observation, what is the height of the airplane? (Assume the ground is perfectly flat and level and that the radar is effectively at ground level.)

height = $8000 \sin 25^\circ = 3381$ ft

c) What is the displacement of the airplane during the period of observation?

Initial vector = (3381, 7250)

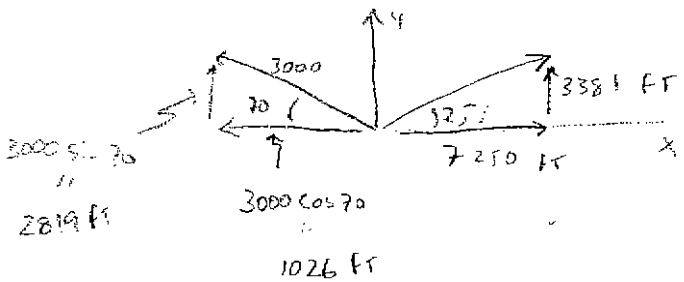
Final vector = (−1026, 2819)



$\vec{A} + \vec{R} = \vec{B}$

$A_x + R_x = B_x$

$A_y + R_y = B_y$



x comp:

$7250 + R_x = -1026$

$R_x = -8276$ ft

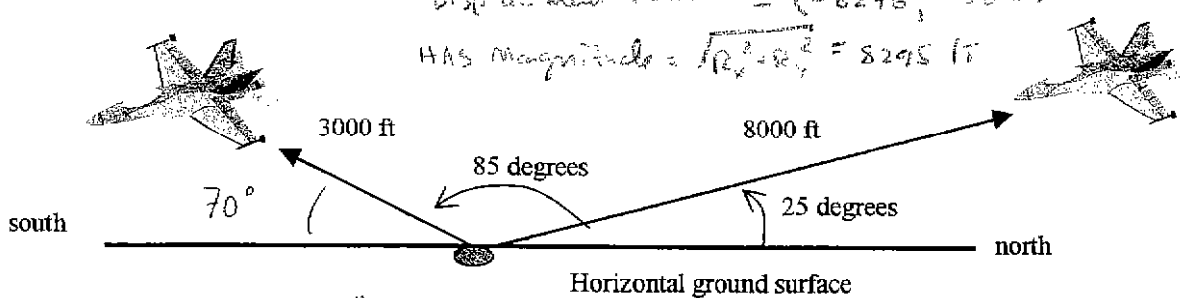
y comp:

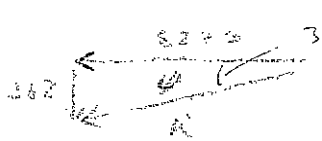
$3381 + R_y = 2819$

$R_y = -562$ ft

Displacement vector = (−8276, −562)

Has Magnitude = $\sqrt{R_x^2 + R_y^2} = 8295$ ft





$\tan \varphi = \frac{562}{8276} \Rightarrow \varphi = 3.9^\circ$

Displacement vector is 8295 ft at 3.9° down from horizontal

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Problem 2 (20 pts):

The motion of a butterfly is plotted on the position vs. time graph below. At which labeled point or points are the following things true:

a) **The butterfly is at the greatest distance from the starting point?**

B

b) **The butterfly's speed is minimum?**

B, D, F

c) **The butterfly's acceleration is zero?**

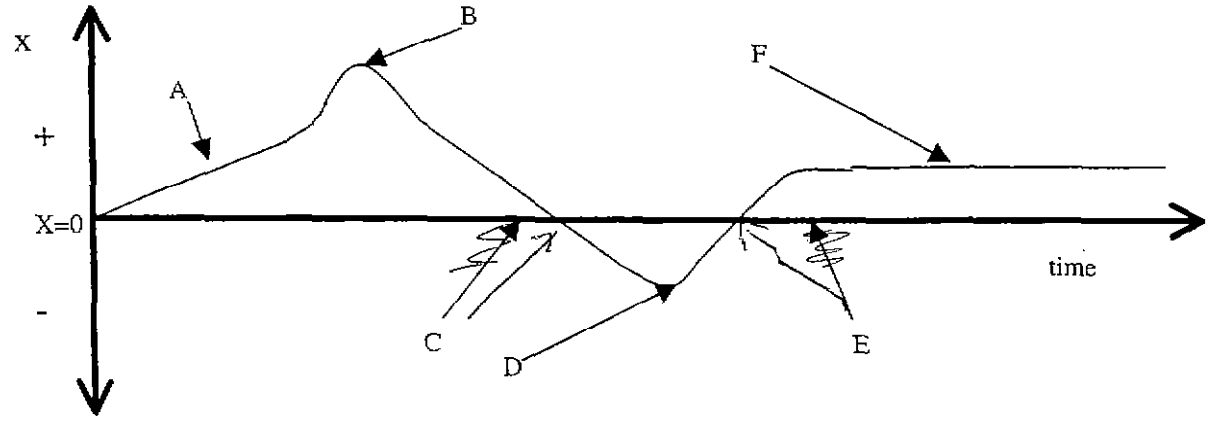
A, C, E, F

d) **The butterfly's velocity in the x direction is constant?**

A, C, E, F

e) **The butterfly's acceleration is at its most negative value?**

B



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1)	/20
2)	/20
3)	/20
4)	/20
5)	/20
tot	/100

Problem 3 (20 pts):

A basketball player jumps straight upward 76 cm.

a) What is the velocity of the player just as his feet leave the ground?

Const a = −9.8 m/s

.76 m v=0

0=y v_{oy}=?

$$v_y^2 = v_{oy}^2 + 2a(y - y_0)$$

$$0 = v_{oy}^2 - 2(9.8)(0.76)$$

$$v_{oy}^2 = 14.9\ (m/s)^2$$

$$\boxed{v_{oy} = 3.86\ m/s\ \text{up}}$$

b) What is the total time the player is in the air?

$$v_y = v_{oy} + a_y t$$

$$-3.86 = 3.86 - (9.8)t$$

$$t = 0.79\ s$$

→ uses symmetry, v at start going up = v at end going down. Other ways to solve do not need this.

c) How much time does the player spend in the bottom 15 cm of his motion?

76 cm

15 cm a = −9.8

v_{oy} = 3.86

Time from y=0 to y=+15 cm

$$y = y_0 + v_{oy}t + \tfrac{1}{2}at^2$$

$$0.15 = 3.86t - \tfrac{1}{2}9.8t^2$$

−or−

$$0 = 4.9t^2 - 3.86t + 0.15$$

use quadratic eqn

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3.86 \pm \sqrt{3.86^2 - 4(4.9)(0.15)}}{9.8}$$

$$= \frac{3.86 \pm 3.46}{9.8}$$

= 0.75 s −or− 0.041 s

Time spent in bottom 15 cm ~0.04 s on total of (2) 0.04 s = 0.08 s

d) How much time does the player spend in the top 15 cm of his motion?

Similar to (c)

$$0.61 = 3.86t - \tfrac{1}{2}9.8t^2$$

$$0 = 4.9t^2 - 3.86t + 0.61$$

$$t = \frac{3.86 \pm \sqrt{(3.86)^2 - (4)(4.9)(0.61)}}{9.8}$$

$$t = \frac{3.86 \pm 1.71}{9.8}$$

t = 0.57 s (time from y=0 to y=0.61 going down) −or− 0.22 s (y=0 to y=0.61 going up)

TOTAL time in top 15 cm = (0.57 − 0.22) = 0.35 s

e) Why do basketball players seem to hang in the air at the top of a jump?

The speed of the player is much greater at the bottom of the motion than at the top. Therefore it takes much more time for the player to cover a given distance at the top than at the bottom, as shown in parts (c) and (d).

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Problem 4 (20 pts):

A fan turns at 300 revolutions per minute. The tip the fan blades are at a radius of 25 cm from the axis of rotation.

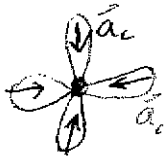
a) What is the instantaneous speed of one of the fan blade tips?

$300\ \text{rev/s} = \frac{300\ (2\pi r)}{60\ s} = \frac{300\ (2\pi(0.25))}{60}\ \text{m/s} = 7.8\ \text{m/s}$

b) What is the magnitude of the centripetal acceleration of the fan blade tips?

$a_c = \frac{v^2}{R} = \frac{(7.8\ \text{m/s})^2}{0.25\ \text{m}} = 246.7\ \text{m/s}^2$

c) What is the direction of the acceleration vector for each fan blade tip?



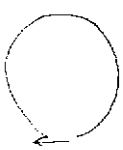
radially inward toward the Axis of rotation

d) What is the average speed of one of the fan blade tips over one revolution?

Motion is constant

∴ INSTANTANEOUS speed = Average speed = 7.8 m/s

e) What is the average velocity of one of the fan blade tips over one revolution?



Zero

Velocity is a vector quantity

initial and final position vectors are identical

∴ Ave velocity = 0 in exactly 1 revolution

f) Suppose you replaced the fan blades with new and longer fan blades. What is the maximum radius of the fan such a that the speed of the blade tips does not exceed the speed of sound? (Assume the speed of sound is 331 m/s.)

$\frac{300\ 2\pi R}{60} = 331\ \text{m/s} \qquad R = 10.5\ \text{m}$

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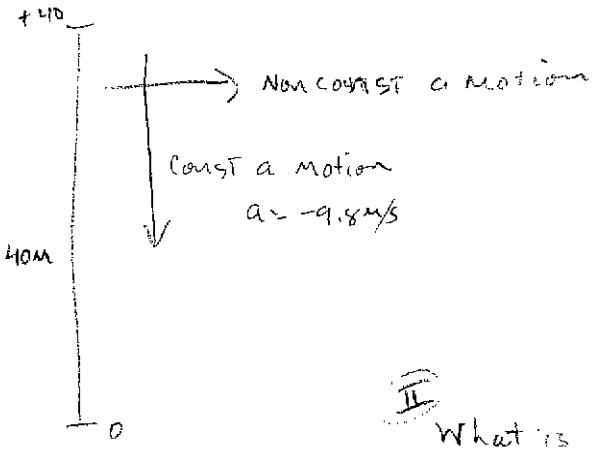
Problem 5 (20 pts):

Slobo the Magnificent made a living with the circus by "diving" from a 40-meter high platform into a small pool of water. At one point, Slobo became upset that he couldn't get a tan because he spent his days in the circus tent diving into pools. So, he convinced his boss to move his act outside. For his first outdoor dive, Slobo slathered on suntan lotion and climbed the platform. He slowly walked off the end of the platform, as usual, hoping to land in his pool, which has a diameter of 4 m centered on a vertical line from the end of his platform. However, a big, horizontal gust of wind occurred that blew Slobo to the side with an acceleration of 2t m/s^2, where t is the time elapsed after Slobo walked off the edge of the platform. By what horizontal distance did Slobo miss the center of the pool? Show all work. The good news is that Slobo had a nice tan to show off at his wake!



I. determine Time to fall 40 m

$$y = y_0 + v_{0y}t + \frac{1}{2}at^2$$

$$0 = +40 + 0t - \frac{1}{2}9.8t^2$$

$$40 = \frac{9.8}{2}t^2 \quad t = 2.8\ s$$

II What is horizontal distance traveled in 2.8 s?

$$\frac{dv_x}{dt} = a_x = 2t$$

$$\int dv_x = \int a_x dt$$

$$v_x - v_{0x} = v_x = \int_0^t 2t\,dt = t^2$$

$$\frac{dx}{dt} = v_x$$

$$x = \int_0^t t^2 dt = \frac{t^3}{3}$$

t = time in air

distance traveled horizontally $= \frac{(2.8)^3}{3} = 7.3\ m$