P121 University of Rochester NAME Soln key - SM
S. Manly Spring 2003

Exam 3 (April 22, 2003)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given.

Problem 1 (5 parts, no partial credit on each part, 4 pts for each part):

4 pts **(a) A disk is free to rotate about an axis. A force applied at a distance d from the axis causes an angular acceleration α. What angular acceleration is produced if the same force is applied a distance 2d from the axis?**

i) α
ii) 2α
iii) $\alpha/2$
iv) 4α
v) $\alpha/4$

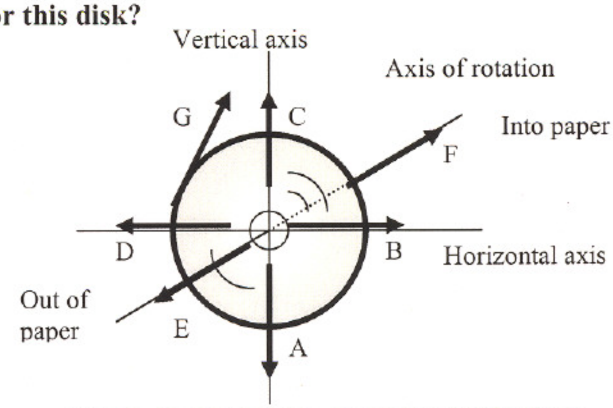
$\tau = I\alpha$ I is fixed

$\tau = d \times F \sim dF$

$dF = I\alpha$ $\therefore (2d)F \rightarrow I(2\alpha)$

4 pts **(b) The disk pictured below rotates clockwise as you are looking at it. Which vector would best represent the angular velocity vector for this disk?**

vector F



4 pts **(c) The disk is slowing down. Which vector would best represent the angular acceleration of the disk?**

Vector E — opposite Direction of Angular velocity

4 pts **(d) Now, forget about the disk and just regard the vectors drawn in the sketch. Which vector is in the direction of $\vec{A} \times \vec{E}$?**

vector D

4 pts **(e) Which vector is in the direction of $\vec{G} \times \vec{D}$?**

vector E

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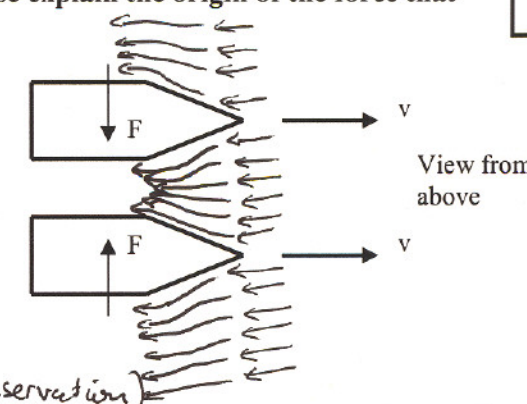
1)	/20
2)	/20
3)	/20
4)	/20
5)	/20
tot	/100

Problem 2 (20 pts):

Two small motor boats are traveling side-by-side as shown in the sketch below. They are traveling at a slow to moderate rate of speed. The people steering the boats notice there is a tendency for the two boats to move toward each other. They must continually correct for this. Using text and equations and/or sketches, please explain the origin of the force that causes the two boats to drift toward one another.

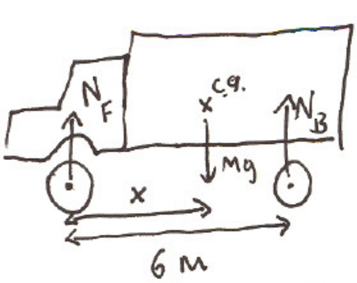


Consider the passage of the water around the two boats as sketched ⇒

The water between the boats is forced into a channel with less cross sectional area and its velocity relative to the boat is increased according to $A_1 V_1 = A_2 V_2$ (mass conservation) because water is an incompressible fluid. On the side away from the other boat the effect is less pronounced because the water is being squeezed from only one side. So the velocity of the water flow relative to the boat surface is larger on the side toward the other boat. By Bernoulli's principle $(P + \frac{1}{2}\rho v^2 + \rho g h = \text{const})$ we know this means the ~~water~~ pressure against the side of the boat is largest on the side away from the other boat. This pressure asymmetry leads to the forces shown.

Problem 3 (20 pts):

Upon graduation you get a job as an engineer with Jimmy Joe-Bob's Trucking Emporium. Big Jimmy Joe-Bob likes the tires of his trucks to wear evenly. Because Jimmy knows you were a physics star in college, he asks you to help him out. You know that for the tires to wear evenly, the tires must all support the same amount of weight. For a given loaded truck you determine that the two front wheels (together) support 7500 N and the two back wheels (together) support 12500 N. (Assume the driver-side and passenger-side wheels each support the same amount of weight.) The front axle and back axle are separated by 6 meters. *(This truck has only two axles and four wheels.)* What distance and in which direction (toward the front or back of the truck) do you need to shift the center-of-gravity of the truck in order for each wheel to support the same amount of weight?



$$Mg = N_F + N_B$$

$\Sigma \tau$ abt front axle

$$\Sigma \tau = 0 = Mg\,x - N_B(6) = 0$$

current location of center of grav. $\rightarrow x = \dfrac{N_B\, 6}{N_F + N_B} = \dfrac{(12500)\,6}{(12500)+(7500)} = 3.75$ M

For tires to wear evenly $x = 3$, $N_F = N_B = 10000$

∴ Must shift center of gravity 0.75 M forward.

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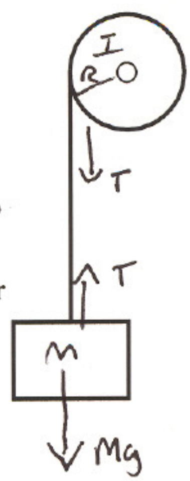
Problem 4 (20 pts):

A mass of 5 kg is tied attached to a massless string that is wound about a cylinder of mass 2 kg and radius 10 cm. The cylinder rotates without friction about an axle that is attached to a support. The mass is allowed to fall. The string does not slip on the surface of the cylinder.

4 PTS (a) What is the acceleration of the falling mass?



For MASS

$Ma = Mg - T$

$F = MA$

For cylinder

$\tau = I\alpha$

$I\alpha = I\frac{a}{R} = TR$

$I = \frac{1}{2}MR^2 = \frac{1}{2}(2)(.1)^2 = .01 \text{ kg m}^2$

$I\frac{a}{R} = M(g-a)R$

$a\left(\frac{I}{R} + MR\right) = MgR$

$a = \frac{MgR}{\left(\frac{I}{R}+MR\right)} = \frac{(5)(9.8)(.1)}{\left[\frac{.01}{.1} + 5(.1)\right]}$

$\boxed{a = 8.16 \text{ m/s}^2}$

4 PTS (b) What is the magnitude of the angular acceleration of the cylinder assuming your answer to (a)?

$\frac{a}{R} = \alpha \qquad \frac{8.16}{.1} = 81.6 \frac{\text{rad}}{\text{s}^2}$

6 PTS (c) What is the angular momentum of the system about the axis of the cylinder 2 seconds after the mass begins to fall (assuming the mass starts with zero velocity and assuming your answers to part (a) and (b) are correct?

$V = V_0 + at$

$V = 0 + \cancel{9.8}\,8.16(2) = \cancel{19.6}\,16.3 \text{ m/s}$

$\frac{V}{R} = \omega = \cancel{196}\,163.2 \frac{\text{rad}}{\text{s}}$

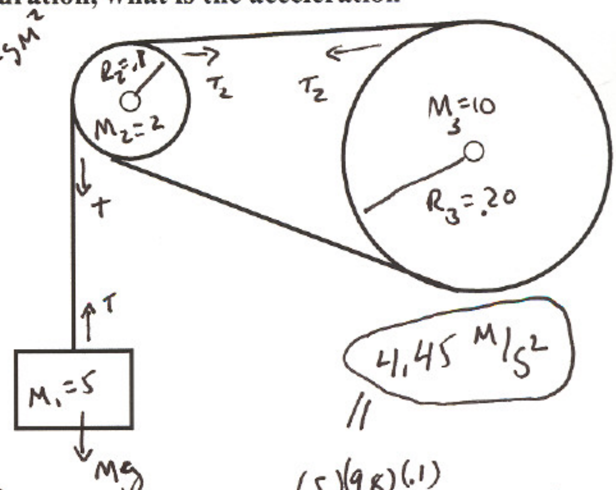
$|\vec{L}| = L_{MASS} + L_{cylinder}$

$\vec{L}(t=2) = MVR + I\omega$

$|\vec{L}| = 5(16.3)(.1) + (.01)(163.2) = 9.78 \frac{\text{kg m}^2}{\text{s}}$

– direction is out of paper.

6 PTS (d) Now suppose the system is attached by a massless belt to another cylinder of mass 10 kg and radius 20 cm that can rotate without friction about an axle that is attached to a support. The system is sketched below. In this configuration, what is the acceleration of the falling mass?



HANGing Mass : $Ma = Mg - T$ I

Cyl. 2 : $\Sigma\tau_2 = I_2\alpha_2 = I_2\frac{a}{R_2} = TR_2 - T_2R_2$ II

Cyl. 3 : $\Sigma\tau_3 = I_3\alpha_3 = I_3\frac{a}{R_3} = T_2R_3$ III

$I_3 = \frac{1}{2}M_3R_3^2 = .2 \text{ kg m}^2$

Have used linear Accel $= a$ at all 3 points → Mass falling + Edge of each cylinder

$I_2\frac{a}{R_2} = (Mg - Ma)R_2 - I_3\frac{a}{R_3^2}R_2$

subst for T, T_2 in eqn II

$a\left(\frac{I_2}{R_2} + MR_2 + I_3\frac{R_2}{R_3^2}\right) = MgR_2$

units okay

$a = \frac{MgR_2}{\left(\frac{I_2}{R_2} + MR_2 + I_3\frac{R_2}{R_3^2}\right)} = \frac{(5)(9.8)(.1)}{\left[\left(\frac{.01}{.1}\right) + (5)(.1) + (.2)\frac{.1}{(.2)^2}\right]}$

$= 4.45 \text{ m/s}^2$

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Problem 5 (20 pts):

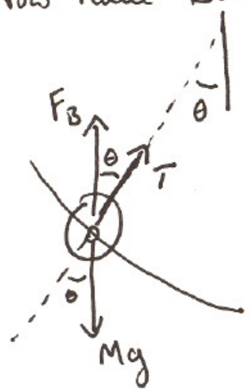
A simple pendulum is made of a spherical solid glass marble 1 cm in radius hanging from a length of fishing line. ***(Please neglect the obvious complication of attaching a spherical marble to fishing line.)*** **The density of the glass in the marble is 2.5×10^3 kg/m^3. For comparison, the density of water is 1×10^3 kg/m^3.**

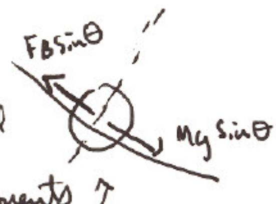
(a) The pendulum is made to swing in air with small oscillations and the period is measured to be 2 seconds. What is the distance from the point of support of the fishing line to the center of the marble, i.e., what is the length of the pendulum? ***(Please ignore air resistance.)***

For simple pendulum $T = 2\pi\sqrt{\frac{L}{g}}$ $\quad 2 = 2\pi\sqrt{\frac{L}{9.8}}$ $\quad L = 99.3$ cm

(b) The very same pendulum is now immersed in water. What is the new period of the pendulum? ***(Assume the water is totally nonviscous, i.e., ignore water resistance even though that is probably a poor assumption.)***

Now have Bouyant force up in addition to gravitational force down.





Assume small angle $\Rightarrow \sin\theta \approx \theta$

Look at motion along Arclength coordinate s

$$M\frac{d^2s}{dt^2} = F_B\theta - Mg\theta$$

$s = L\theta$
$\theta = \frac{s}{L}$

$$M\frac{d^2s}{dt^2} = F_B\frac{s}{L} - Mg\frac{s}{L}$$

$$\boxed{\frac{d^2s}{dt^2} + \left(\frac{g}{L} - \frac{F_B}{ML}\right)s = 0}$$

SHM with $\omega^2 = \frac{1}{L}\left(g - \frac{F_B}{M}\right)$

$$\omega = 2\pi\nu = 2\pi\frac{1}{T} = \sqrt{\frac{1}{L}\left(g - \frac{F_B}{M}\right)} \qquad \boxed{T = 2\pi\sqrt{\frac{L}{\left(g - \frac{F_B}{M}\right)}}} = 2\pi\sqrt{\frac{.993}{9.8 - \frac{4.1\times10^{-2}}{.01}}}$$

$$= \boxed{2.62\,s}$$

F_B = wt of Displ $H_2O = \left[\frac{4}{3}\pi(.01)^3\right]\underbrace{1000}_{\rho_w}(9.8) = 4\times10^{-3}\,g = \cancel{3.4\times10^{-2}N}\ 4.1\times10^{-2}N$

$M = \left[\frac{4}{3}\pi(.01)^3\right]\underbrace{2500}_{\rho_{marble}} = 0.01$ kg