P114
S. Manly

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University of Rochester
Spring 2000

Final Exam (May 3, 2000)

140 SAT for big exam

h 6.62×10^{-34} J·s

c

ϵ_0 8.85×10^{-12} $C^2/N\cdot m^2$

μ_0 $4\pi\times10^{-7}$ Wb/A·m

yd to meter
1 yd = .91 m

If you *object* to having your grades posted on the class website under your social security number, check here ☐.
I will post those with no check in the box.
If you *object* to having your graded exam placed in the box outside my door for you to retrieve, check here ☐.
I will make available those with no check in the box.

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless noted otherwise. Try to be neat. TA's are known to be less generous with partial credit if they have to work hard to decipher the paper!

Problem 1 (12 pts) :

You lounge on the beach a week from now. Bored with looking at all the skin on the neighboring towels, you pick up a pair of eyeglasses belonging to a friend and dream of the good ol' days back in physics class ...

(a) 3 pts - Your friend is nearsighted. Are the lenses in the eyeglasses converging or diverging?

Diverging

(b) 3 pts - The absolute value of the focal length of each lens is 10 cm. For an object located 6 cm in front of one of the ~~lens~~ lenses, where is the image located?

$f = -10$ cm

$\frac{1}{f} = \frac{1}{i} + \frac{1}{o}$

$\frac{1}{-10} = \frac{1}{i} + \frac{1}{6}$

$\frac{1}{i} = \frac{-1}{6} + \frac{1}{-10}$

$i = -3.75$ cm

3.75 cm in front of lens
same side as object

(c) 3 pts - Is the image upright or inverted?

upright
because M is (+)

(d) 3 pts - What is the magnification of the lens?

$M = \frac{-i}{o} = -\frac{i}{6} = -\frac{-3.75}{6} = +0.625$

P114
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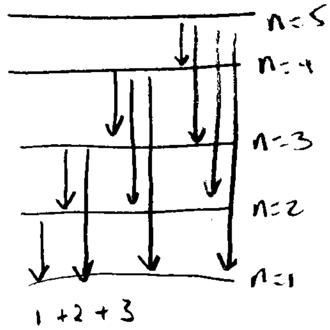
Problem 2 (12 pts)

Atomic and modern physics short answer:

Scores	
1.	___/12
2.	___/12
3.	___/10
4.	___/10
5.	___/10
6.	___/10
7.	___/10
8.	___/10
9.	___/10
10.	___/10
11.	___/5
Total	___/109

(a) 4 pts - A monochromatic (all one frequency) beam of light excites hydrogen atoms in a glass tube. Light of 10 different frequencies is observed to be emitted by the gas in the tube as the hydrogen atoms fall back to the ground state. What is the energy of the photons in the incident monochromatic beam of light?



$E_{incident} = E_{n=5} - E_{n=1}$

$= -\frac{13.6}{5^2} - -13.6$

$E_{inc} = 13.06$ eV

(b) 4 pts - How many quantum states are available to electrons in the n=3 state of a multi-electron atom, according to Schroedinger's theory of the atom?

$n = 3$

$\ell = 0, 1, 2$

$m_\ell = 1 \quad 3 \quad 5 = 9$ STATES

$m_s = \pm \frac{1}{2}$ $\quad \times 2$ $\quad$ 18 STATES

$|\ell| = 2 \quad m_\ell: -2, -1, 0, +1, +2$

$|\ell| = 1 \quad m_\ell = -1, 0, +1$

$|\ell| = 0 \quad m_\ell = 0$

(c) 4 pts - For a given light source, if the photoelectric effect is observed to occur for one metal, can you conclude that the effect will also be observed for a different metal under the same conditions? Why or why not?

NO

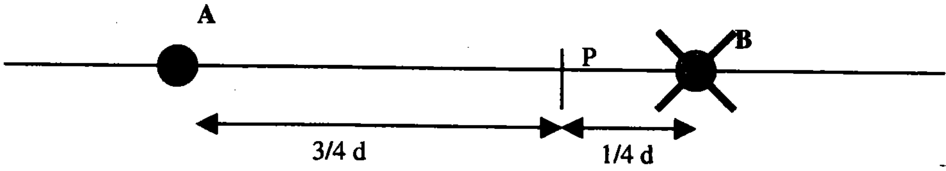
The work fn of the 2ND metal could be higher than that of the 1ST metal. If it is high enough the incident photons won't have enough energy to eject the electron.

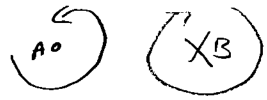
$h\nu_\gamma$ must be > work fn of metal.

P114 NAME Soln Sht University of Rochester
S. Manly Spring 2000

Problem 3 (10 pts):

Consider the two infinite parallel, current-carrying wires shown in the figure below. One (A) carries current I out of the page, one (B) carries current I into the page. The wires are a distance d apart. Determine the magnitude and direction of the magnetic field at a point P along the line joining the two wires that is 3/4 d from wire A and 1/4 d from wire B. (hint: Make use of Ampere's Law.)





They add at pt P

$\vec{B}$ is up at P

$\oint B \cdot dl = \mu_0 i_{encl}$ for each wire

B fields can be superimposed vectorially

$B_A \Rightarrow \quad B 2\pi r = \mu_0 I$

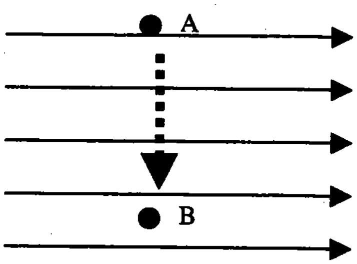
$B_A = \frac{\mu_0 I}{2\pi \frac{3}{4}d}$

$B_B = \frac{\mu_0 I}{2\pi \frac{1}{4}d}$

$\vec{B}_P = \frac{\mu_0 I}{2\pi d}\left[\frac{4}{3} + 4\right] = \frac{16}{3}\frac{\mu_0 I}{2\pi d}\ \widehat{up}$

Problem 4 (10 pts):

A positive charge is moves from point A to point B along the path shown in each of the following scenarios. The distance between the points A and B, d_{AB}, is identical in each of the cases. Rank them in the order of increasing work by the field on the charge … least work first, most work last.

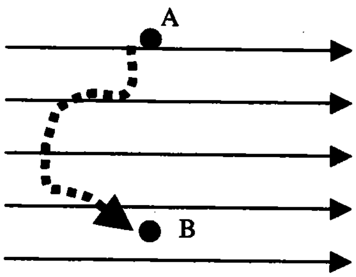


$\vec{F} = q\vec{E}$

$W = \vec{F} \cdot d\vec{s}$

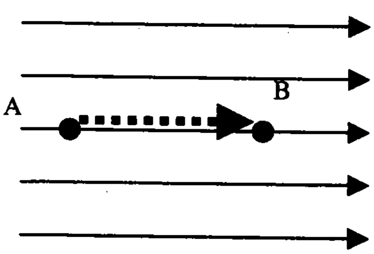
0

(a)



Net work = 0

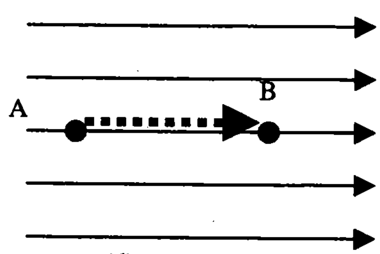
(b)



(c)

Work is positive

qEd



(d)

$F = q\vec{v} \times \vec{B}$

Net work = 0

(a) = (b) = (d) = 0 < (c)

P114
S. Manly

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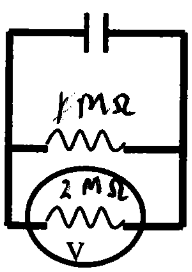
University of Rochester
Spring 2000

Problem 5 (10 pts) :

A capacitor is charged to a potential of 15.0 V and is then connected in parallel to a voltmeter having an internal resistance of 2 million Ohms and a resistor having a resistance of 1 million Ohms. After a time of 5.00 s the voltmeter reads 5.0 V. What is the capacitance?



RC circuit

What is equivalent R_{eq}

$$\frac{1}{R_{eq}} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2} \qquad R_{eq} = \frac{2}{3}\ M\Omega$$

$$Q = CV \qquad V = Q/C$$

$$Q(t) = Q_0 e^{-t/RC_{eq}}$$

$$CV(t) = CV_0 e^{-t/RC_{eq}}$$

$$V(t) = V_0 e^{-t/RC_{eq}}$$

$$5.0 = 15 e^{-5/R_{eq}C}$$

$$\ln\frac{5}{15} = -\frac{5}{R_{eq}C}$$

$$R_{eq}C = \frac{5}{1.098} = \frac{5}{1.1} = 4.54$$

$$C = 4.54 / 0.66\times10^{6}\ \Omega$$

$$\boxed{C = 6.8\mu F}$$

Problem 6 (10 pts) :

The rubidium isotope ^{87}Rb (Z=37) has a half-life of 4.9×10^{10} years. It decays into ^{87}Sr (Z=38).

(a) 4 pts - What type of radiation is emitted by ^{87}Rb when it decays?

One $n \rightarrow p + e^-$ $\qquad \beta^-$

(b) 6 pts - Rubidium is often used to determine the age of rocks and fossils. Rocks containing the fossils of early animals contain a ratio of ^{87}Sr to 87Rb of 0.0100. Assuming that there was no ^{87}Sr present when the rocks were formed, calculate the age of these fossils.

$$t_{1/2} = \frac{0.693}{\lambda} \qquad \lambda = \frac{0.693}{t_{1/2}}$$

$$\lambda = 1.4\times10^{-11}\ yr^{-1}$$

$$N(t) = N_0 e^{-\lambda t}$$

$$.01 = \frac{Sr}{Rb} = \frac{N_0(1-e^{-\lambda t})}{N_0 e^{-\lambda t}} \begin{matrix} \text{\# Rb decayed} \\ \text{\# Rb remaining} \end{matrix}$$

$$(1-e^{-\lambda t}) = e^{-\lambda t}(.01)$$

$$\frac{1}{1.01} = e^{-\lambda t}$$

$$\ln\frac{1}{1.01} = -\lambda t = -1.4\times10^{-11}\, t$$

$$\boxed{t = 7.1\times10^{8} \text{ years}}$$

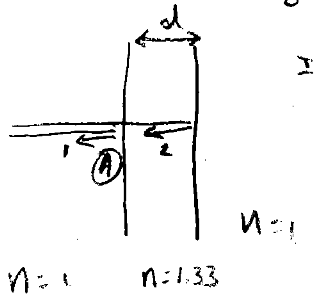
P114
S. Manly

NAME Soln - Sln

University of Rochester
Spring 2000

Problem 7 (10 pts) :

(a) 5 pts - As a soap bubble evaporates, its surface appears black just before it pops. Why does it appear dark rather than bright just before popping? A sketch and a formula or two might be helpful in explaining this.



Interference between waves 1 (180° phase Δ) and 2 (no phase change) at pt A gives

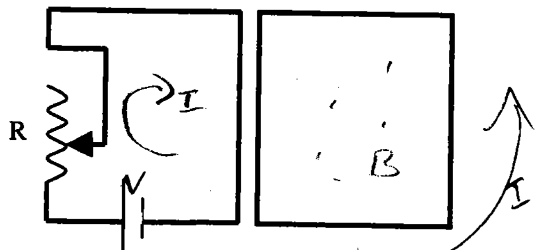
$2d = (m + \tfrac{1}{2})\lambda_n$ constr. interference $m = 0, 1, 2 \cdots$

$2d = m\lambda_n$ destructive interference $m = 1, 2 \cdots$

The smallest d that can give constr. interference is $m=0 \Rightarrow d = \frac{\lambda_n}{4}$

as d becomes smaller than this one approaches the condition for destructive interference. This will be the case as $d \to 0$ just before the bubble pops.

(b) 5 pts - If the variable resistance R in the left-hand circuit of the figure below is increased, what is the direction of the induced current in the right-hand circuit?



induced B thru RT loop is upward. (Caused by left loop)

R is increased
∴
I is decreased because V is fixed and V = IR

Flux is up
I decreased → Flux is decreased
Induced current such as to increase flux
Counterclockwise

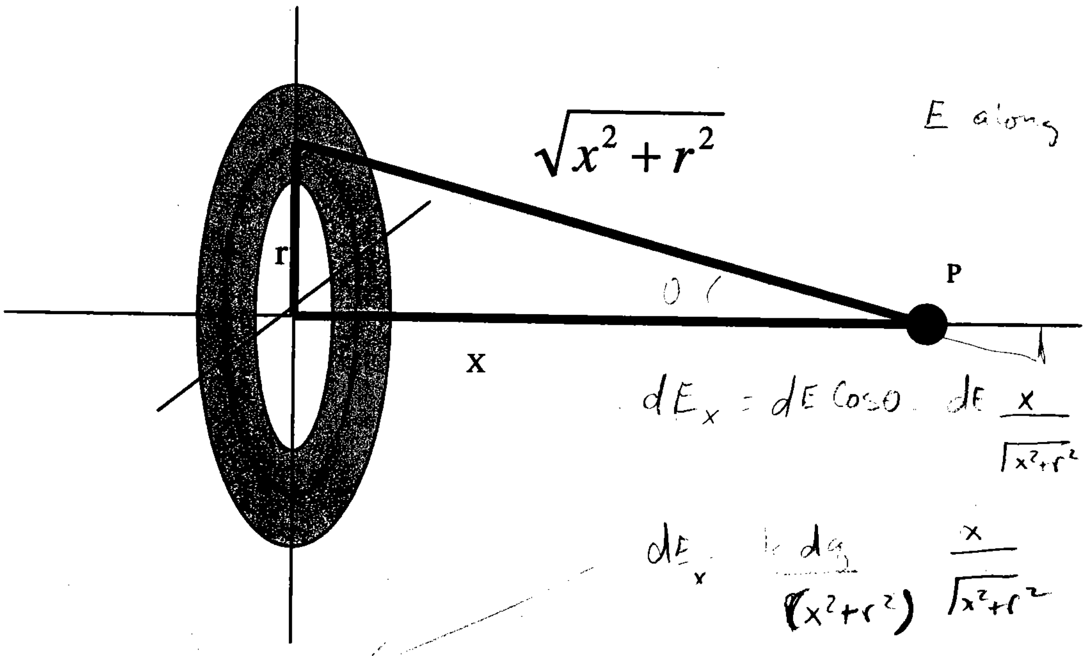
P114
S. Manly

NAME Soln Slm

University of Rochester
Spring 2000

Problem 8 (10 pts) :

Consider a conducting annulus (a thin disk with a hole in the center) with a uniform charge density σ. Let the annulus lie in the yz plane centered on the x-axis, as shown in the drawing below. Determine the electric field (magnitude and direction) at a point P a distance x from the origin along the x-axis. The inner radius of the annulus is A and the outer radius is B. (Hint: $dq=\sigma 2\pi r dr$ for a differential ring of charge with radius r and radial thickness dr.)



E along x by symmetry

$dE_x = dE\cos\theta = dE\,\frac{x}{\sqrt{x^2+r^2}}$

$dE_x = \frac{k\,dq}{(x^2+r^2)}\,\frac{x}{\sqrt{x^2+r^2}}$

$$dE_x = \frac{k x \sigma 2\pi r dr}{(x^2+r^2)^{3/2}} = k x \sigma 2\pi \frac{r dr}{(x^2+r^2)^{3/2}}$$

$$E_x = \int_A^B (k x\sigma 2\pi)\frac{r dr}{(x^2+r^2)^{3/2}} = (k x \sigma 2\pi)\left[-\frac{1}{\sqrt{x^2+r^2}}\right]_A^B$$

$$E_x = k x\sigma 2\pi\left[\frac{1}{\sqrt{x^2+A^2}} - \frac{1}{\sqrt{x^2+B^2}}\right] = \frac{x\sigma}{2\epsilon_0}\left[\frac{1}{\sqrt{x^2+A^2}} - \frac{1}{\sqrt{x^2+B^2}}\right]$$

or

$$E_x = k\sigma 2\pi\left[\frac{x/A}{\sqrt{(\frac{x}{A})^2+1}} - \frac{x/B}{\sqrt{(\frac{x}{B})^2+1}}\right] = \frac{x\sigma}{2\epsilon_0}\left[\frac{\frac{x}{A}}{\sqrt{(x/A)^2+1}} - \frac{x/B}{\sqrt{(\frac{x}{B})^2+1}}\right]$$

P114 NAME Soln - SJM University of Rochester
S. Manly Spring 2000

Problem 9 (10 pts) :

A couple of your colleagues … who apparently finished exams early and started celebrating … told me that George Jetson zipped by the University last night in his little spacecar moving at 0.92c. According to them, he measured the length of the football field in Fauver Stadium as he passed by (at constant velocity). Assuming these students are telling a true story, what length did George measure (in yards)? (hint: The football field was constructed to be 100 yards long.)

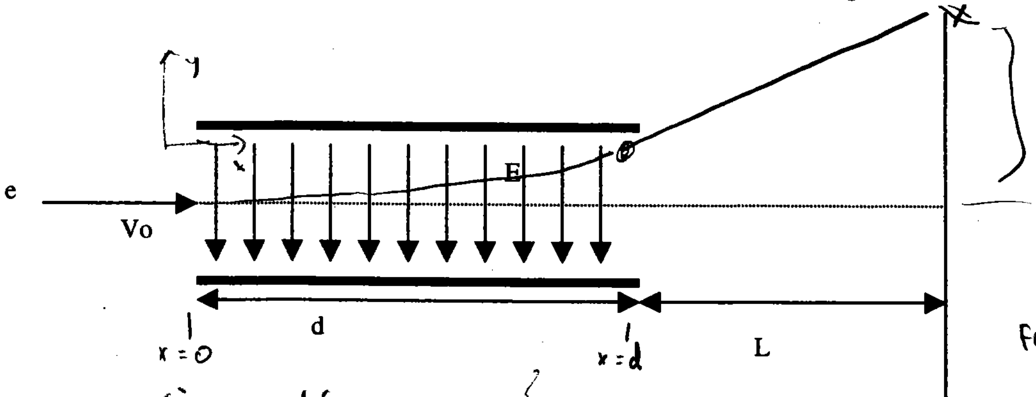
$$\gamma = \frac{1}{\sqrt{1-(.92)^2}} = 2.5$$

$\Delta x' = \gamma \Delta x$ — length observed by George; Football field = 100 yds in proper frame

$$\frac{100\text{ yds}}{2.5} = 40\text{ yds}$$

Problem 10 (10 pts) :

An electron with charge e and mass m is projected horizontally at right angles into a uniform downward electric field of magnitude E and length d. The initial velocity of the electron is v_o. After passing through the region with the electric field, the electron hits a screen a distance L beyond the field region. Where does the electron hit the screen relative to where it would have hit with no electric field? Ignore the effects of gravity.



$$\frac{|e|E}{m_e}\frac{d}{V_o^2}(1+L) = Y \text{ at screen}$$

Const Acceleration problem

$\vec{F} = |e|\vec{E}$ up for $0<x<d$

$\frac{|e|\vec{E}}{m_e} = a_y$ $V_{ox} = V_o$ $V_{oy} = 0$

$a_x = 0$

$a_x = 0$ $V_x(d) = V_o$

$x = x_0 + \frac{1}{2}(V_o + V)t$

$d = 0 + \frac{1}{2}(2V_o)t$

$d = V_o t$

$t = d/V_o$

t to $x=d$ $= d/V_o$

Y at this point is

$Y = Y_0 + V_{oy}t + \frac{1}{2}a_y t^2$

$Y(d) = 0 + 0 + \frac{1}{2}\frac{|e|E}{m_e}\left(\frac{d}{V_o}\right)^2$

$Y(d) = \frac{|e|E}{2m_e}\left(\frac{d}{V_o}\right)^2$

$V_y(d) = a_y t = \frac{|e|E}{m_e}\frac{d}{V_o}$

at $x=d$: $V_y(d)$, $V_x(d)$

From $x=d$ to $x=L$

$a_y = a_x = 0$

$Y(d+L) = Y(d) + V_y(d)t_{d\to L}$

$L = V_x(d)\, t_{d\to L}$

$t_{d\to L} = \frac{L}{V_o}$

$Y(d+L) = Y(d) + \frac{|e|E}{2m_e}\frac{d}{V_o}\frac{L}{V_o}$

$Y(d+L) = \frac{|e|E}{2m_e}\left(\frac{d}{V_o}\right)^2 + \frac{|e|E}{2m_e}\frac{dL}{V_o^2}$

$$\boxed{Y = \frac{|e|E\,d}{2m_e V_o^2}(1+dL)}$$

P114
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NAME Sohn Sin

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Problem 11 (5 pts) :

What advice do you have for students taking P113 in the fall that will help them through the course? Please write it below.

Anything is Acceptable if it is [illegible] write Answer to the question