P114
S. Manly

NAME Soln Key Slay

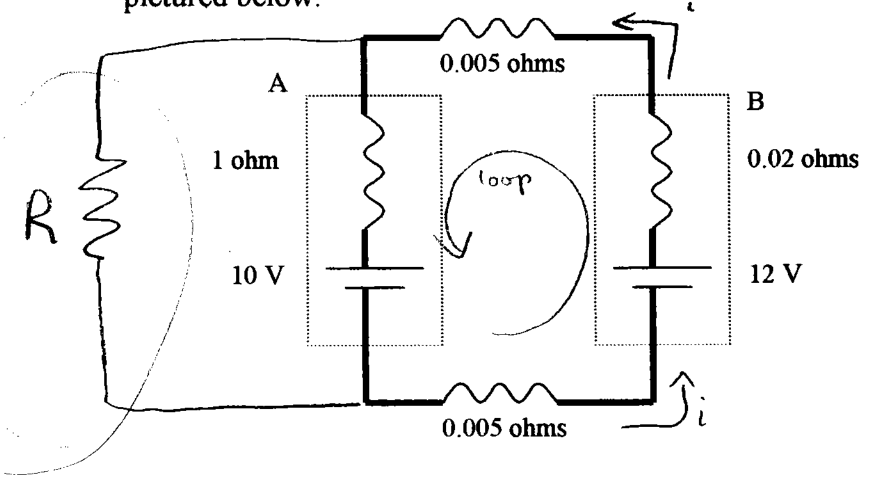
University of Rochester
Spring 2000

Exam 2 (March 2, 2000)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless noted otherwise. Try to be neat. TA's are known to be less generous with partial credit if they have to work hard to decipher the paper!

Problem 1 (25 pts) :

A car needs a jump-start. The weak battery in car (A) is connected by copper jumper cables to a good car battery in car (B). Assume the good battery has an emf of 12 V and an internal resistance of 0.02 ohms and the weak battery has an emf of 10 V and an internal resistance of 1 ohm. Assume the resistance of each jumper cable is 0.005 ohms. For this part, neglect the fact that the batteries in car A and B are also connected to circuit elements (such as starter motors) in their respective cars. The appropriate circuit is pictured below.



(a) (15 pts) - What is the current in the circuit in the figure? Indicate the direction of the current clearly on the figure.

$\Sigma V_{loop} = 0$ $\quad +12 - i(.02) - i(.005) - i(1) - 10 - i(.005) = 0$

$2 - i(1.03) = 0 \quad \boxed{i = 1.94\ A}$

(b) (10 pts) - Actually, when you jump a car, you are not "charging" the weak battery. Rather you are using the good battery (in the other car) to start the car. Once the car is started, there is an internal charger that recharges the battery. Assume the engine of the car being started can be represented by a single resistor R at the moment the ignition is turned on. Modify the diagram above to include this resistance.

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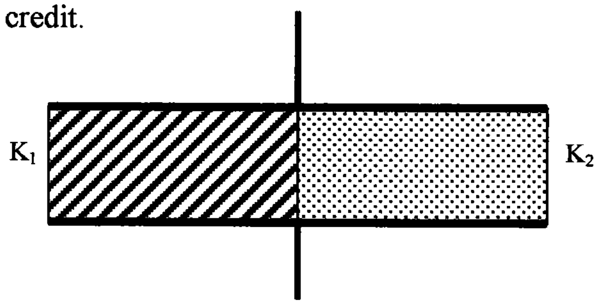
Scores	
1.	___/25
2.	___/25
3.	___/20
4.	___/25
5.	___/15
EC	___/3
Total	___/113

Problem 2 (25 pts):

A parallel-plate capacitor has the space between the plates filled with two slabs of dielectric, one with constant K_1 and one with constant K_2. The thickness of each slab is the same as the plate separation d, and each slab fills half of the volume between the plates. Find the correct expression for the capacitance. You must show your work to receive credit.



(a) $C = \left(\frac{2\varepsilon_0 A}{d}\right)\left(\frac{K_1 K_2}{K_1 + K_2}\right)$

(b) $C = \left(\frac{\varepsilon_0 A}{2d}\right)(K_1 + K_2)$

(c) $C = \left(\frac{\varepsilon_0 A}{2d}\right)\left(\frac{K_1 + K_2}{K_1 K_2}\right)$

(d) $C = \left(\frac{\varepsilon_0 A}{2d}\right)\left(\frac{1}{K_1 + K_2}\right)$

(e) $C = \left(\frac{\varepsilon_0 A}{d}\right)(K_1 + K_2)$

← CAN ALSO Accept (e) if A is Taken to be area of capacitor covering k_1 and k_2 separately

$$C = \frac{Q}{V} = \frac{\sigma A}{V} = \frac{\sigma A \epsilon_0}{\sigma d} = \frac{A\epsilon_0}{d} \text{ in vacuum}$$

$$V = Ed = \frac{\sigma}{\epsilon_0} d$$

$$C = \frac{AK\epsilon_0}{d} \text{ in dielectric}$$

Situation is like two capacitors in parallel

$$\therefore \; C_{eq} = C_1 + C_2 = \frac{\frac{1}{2}A\epsilon_0 K_1}{d} + \frac{\frac{1}{2}A\epsilon_0 K_2}{d} = \frac{A\epsilon_0}{2d}(K_1 + K_2)$$

Answer is (b)

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Problem 3 (20 pts):

A length of wire has a resistance of 120 ohms. The wire is cut into N identical pieces that are then connected in parallel. The resistance of the parallel arrangement is 1.875 ohms. Find N.

Series → $Nr = 120\ \Omega \Rightarrow r = \frac{120}{N}$

Parallel → $\frac{N}{r} = 1.875\ \Omega$

$\frac{N}{120/N} = 1.875$

$N^2 = (120)(1.875)$

$N = 15$

Problem 4 (25 pts, 5 pts for each part):

For spring break you go to England for tea and crumpets with the royal family. You want to look your best, so you take your hairdryer with you. The hairdryer is designed to output 1500 Watts at the standard US wall outlet voltage of 120 Volts

(a) What is the resistance of the heating element?

$P = V^2/R \qquad R = \frac{V^2}{P} = \frac{(120)^2}{1500} = 9.6\ \Omega$

(b) What is the power produced by the hairdryer in England, where the wall voltage is 240 V?

$P = \frac{V^2}{R} = \frac{(240)^2}{9.6\ \Omega} = 6000$ Watts

(c) What is the current through the hairdryer in the US?

$V = iR \qquad i = \frac{120}{9.6} = 12.5$ Amps

(d) What is the current through the hairdryer in England?

$i = \frac{240}{9.6} = 25$ Amps

(e) Why should you care that the hairdryer's power and current are different in England than the US?

The additional current may throw a breaker or blow a fuse. If that does not happen the hairdryer will likely burn up ... perhaps causing a fire. It is designed for considerably less current/power.

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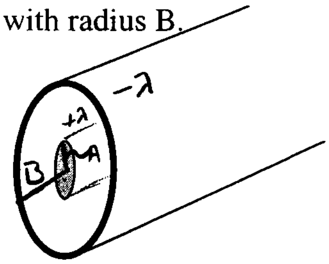
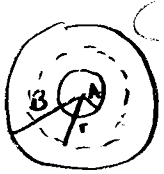
Problem 5 (15 pts):

One possible model for the axon of a nerve cell is a coaxial cable with a charge per unit length of $+\lambda$ C/m on the central conductor of radius A and a charge per unit length of $-\lambda$ C/m on the outer conductor with radius B.



a) (7) pts - Use Gauss's Law to determine the electric field in the regions A<r<B and r>B. You must show your work to receive credit.



(A < r < B)

$$\int \vec{E}\cdot d\vec{A} = |E|\,2\pi r \ell = \frac{+\lambda \ell}{\epsilon_0} \qquad |E| = \frac{\lambda}{2\pi r \epsilon_0}$$

r > B

$$|E| = 0$$

b) (8) pts - Use your answer to part (a) to find the total energy per unit length stored in this configuration. You must show your work to receive credit. *Hint: one can use* dv=2πLrdr *for an integral over a cylindrical volume of length L.*

$$\text{Energy density} = u = \frac{\epsilon_0}{2}E^2 = \frac{\lambda^2}{8\pi^2 r^2 \epsilon_0}$$

In a length L

$$U = \int u\,dv = 2\pi L \int_A^B \frac{\lambda^2}{8\pi^2 r^2 \epsilon_0}\, r\,dr = \frac{2\pi L \lambda^2}{8\pi^2 \epsilon_0}\int_A^B \frac{1}{r}\,dr$$

$$U = \frac{L\lambda^2}{4\pi\epsilon_0}\ln\frac{B}{A}$$

$$\frac{U}{L} = \text{energy per unit length} = \frac{\lambda^2}{4\pi\epsilon_0}\ln\frac{B}{A}$$