P114 University of Rochester NAME Soln key - SEM
S. Manly Spring 2010

Final Exam (May 5, 2010)

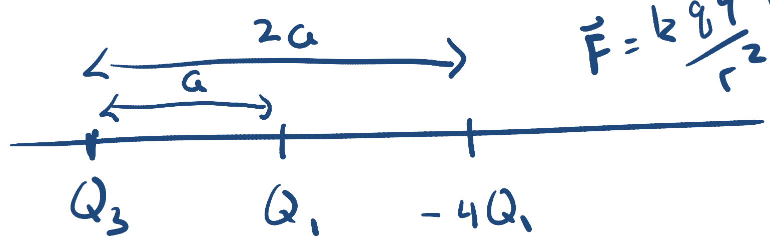
Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show your work where indicated.

Problem 1 (3 pts):

A charge Q_1 is positioned on the x-axis at x = a. Where should a charge $Q_2 = -4Q_1$ be placed to produce a net electrostatic force of zero on a third charge, $Q_3 = Q_1$, located at the origin?

$\vec{F} = k\frac{qq'}{r^2}\hat{r}$

a) At the origin.
b) At x = 2a.
1 – c) At x = -2a.
d) At x = -a.

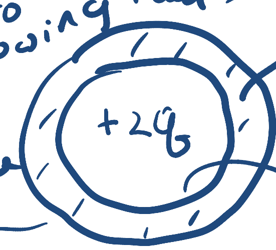


Problem 2 (3 pts):

A charge of +2q is placed at the center of an uncharged conducting shell. What will be the charge on the inner and outer surfaces of the shell, respectively

Can use Gauss' law to prove knowing that → $\vec{E} = 0$ inside shell

3 – a) -2q, +2q
b) –q, +q
1 – c) -2q, -2q
1 – d) -2q, +4q



outer surface +2q

-2q inner surface

Problem 3 (3 pts):

A particle with charge q is at rest when a uniform magnetic field is suddenly turned on. The field points in the z-direction. What is the direction fo the net force acting on the charged particle?

a) In the z-direction.
b) There is no force on the particle. ~ $\vec{F}_B = q\vec{v} \times \vec{B}$ No Force if $\vec{v} = 0$
c) In the negative z –direction
d) The force will be in the xy plane, causing the particle to move in a circle inducing a magnetic field that is in the –z direction.
e) The force will be in the xy plane, causing the particle to move in a circle inducing a magnetic field along the +z direction.

Problem 4 (3 pts):

Which of the following statements concerning plane wave electromagnetic fields are incorrect? (There might be more than one …)

a) Electromagnetic waves in a vacuum travel at the speed of light (c).
b) The magnitudes of the electric field and the magnetic field are equal.
c) Only the electric field vector is perpendicular to the direction of the wave's propagation.
d) Both the electic field vector and the magnetic field vector are perpendicular to the direction of propagation.
e) An electromagnetic wave only carries energy when E=B.

P114 University of Rochester NAME Soln key - SlM
S. Manly Spring 2010

Problem 5 (3 pts):

α : $Z = 2$, $A = 4$

The alpha decay of an atom of $^{238}_{92}U$ yields the atom

+2 - a) $^{234}_{90}U$
b) $^{238}_{90}Th$
c) $^{238}_{93}Np$
d) $^{234}_{91}Pa$
+3 e) $^{234}_{90}Th$

$238 \rightarrow 234$
$92 \rightarrow 90$

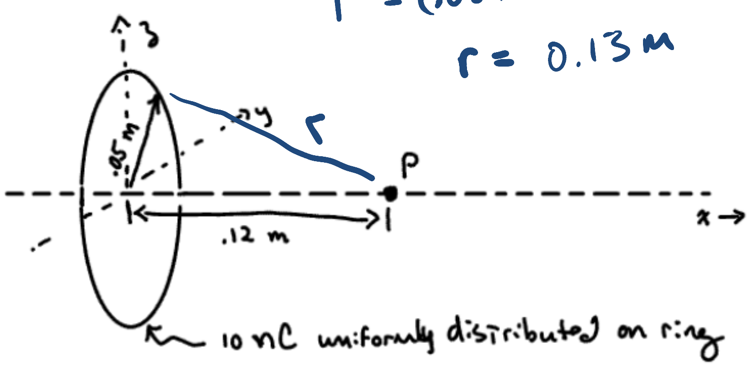
but since Z changes it cannot be U so must be Th

Problem 6 (6 pts, show work):

A ring of radius 0.05 m is in the yz plane with its center at the orgin. The ring carries a uniform charge of 10 nC. The electric potential at x=0.12 m is approximately

a) 217 V
b) 543 V
c) 692 V
d) 809V
e) 963 V

$r^2 = (.05)^2 + (.12)^2$
$r = 0.13\,m$



$V = \dfrac{kQ}{r}$ does not depend on direction + all charge at same distance

$$V = \frac{9\times10^{9}\,(10\times10^{-9})}{.13} = 692\ V$$

Problem 7 (4 pts, show work):

A charge +q moves with speed v in a plane perpendicular to a uniform magnetic field B. The charge has mass m.

a) What is the radius of the circle in which the charge moves (in terms of the variables of the problem)?

$\dfrac{mv^2}{R} = qvB$ $\boxed{R = \dfrac{mv}{qB}}$

b) How much work is done by the magnetic field on the charge as it moves around the circle once?

None $W = \int \vec{F}\cdot d\vec{s}$

$\vec{F}$ is always at right angles to $d\vec{s}$

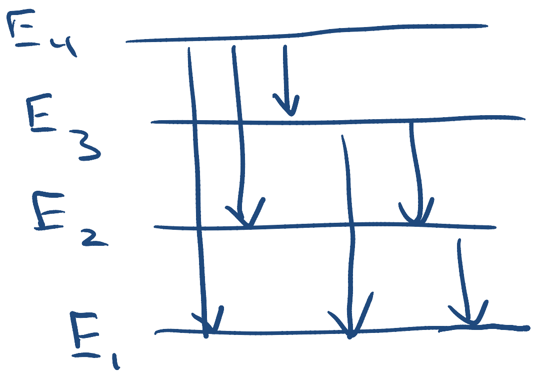
+ So $W = 0$

P114 University of Rochester NAME Solnkey-SLM
S. Manly Spring 2010

1)	/3
2)	/3
3)	/3
4)	/3
5)	/3
6)	/6
7)	/4
8)	/4
9)	/6
10)	/5
11)	/5
12)	/6
13)	/5
14)	/5
15)	/7
16)	/7
17)	/6
18)	/8
19)	/12
tot	/100

Problem 8 (4 pts):

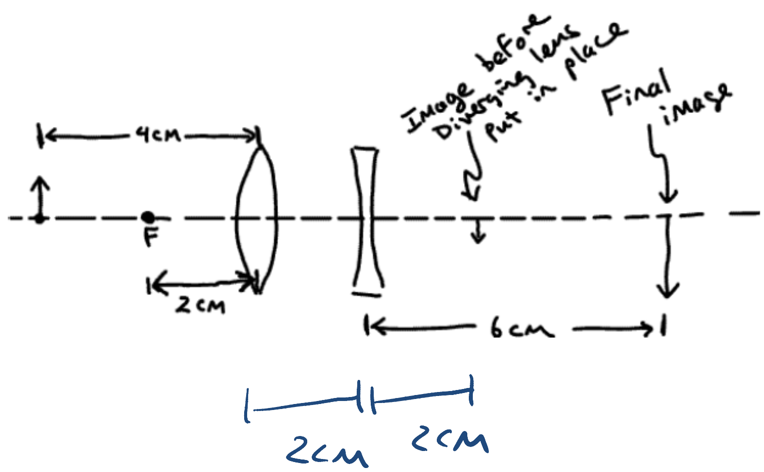
You pass white light through an atomic gas and then a prism. When you look at the pattern of light coming out of the prism, you notice that six specific colors (or frequencies) of light have been absorbed by the gas. According to the Bohr model, this data tells you that the electron in the atom has at least how many energy levels?



6 transitions requires at least 4 energy levels

Problem 9 (6 pts, show work):

An object placed 4 cm to the left of a converging lens of focal length 2 cm produces an image 4 cm to the right of the lens. A diverging lens placed at the focal point of the converging lens as shown in the sketch produces a final image 6 cm to the right of the diverging lens. Determine the focal length of the diverging lens.



$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$$

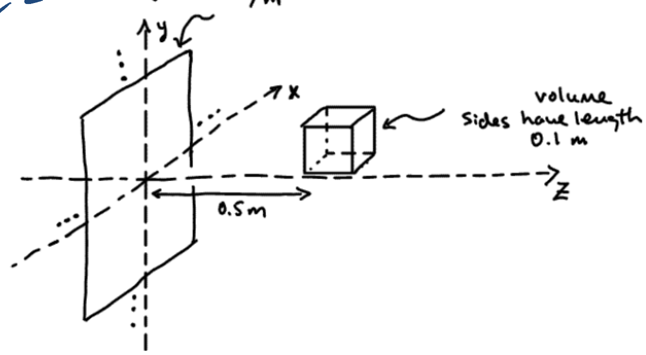
$$\frac{1}{6} + \frac{1}{-2} = \frac{1}{f}$$

$$\frac{1}{f} = -.33$$

$$f = -3\text{cm}$$

Problem 10 (5 pts, show work):

Consider a cubical volume situated with sides parallel to the x-y, x-z, and y-z planes as shown in the sketch. The cube has sides of length 0.1m and is located 0.5 m away from an infinite plane with a uniform electric charge density of ~~+2~~ 2×10^{-4} Coul/m^2. How much energy is stored in the cubical volume in the form of the electric field?



$$u_E = \frac{\epsilon_0}{2} E^2$$

$$E_{\substack{\infty // \\ \text{Plate}}} = \frac{\sigma}{2\epsilon_0}$$

Energy Stored in Volume

$$= u_E(\text{Vol})$$

$$= \frac{\epsilon_0}{2}\frac{\sigma^2}{4\epsilon_0^2}(0.1)^3 = \frac{(2\times10^{-4})^2(.1)^3}{8(8.85\times10^{-12})}$$

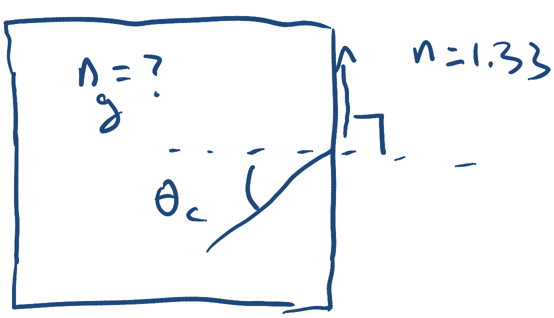
$$= 0.57 \text{ Joules}$$

Problem 11 (5 points, show work):

A ray of light is traveling in a glass cube that is totally immersed in water. You find that if the ray is incident on the glass-water interface at an angle to the normal greater that 48.7 degrees, no light passes from the glass to the water (no light is refracted into the water in physics-speak). What is the refractive index of the class glass? (Assume the refractive index of water is 1.33.)

$$n_{glass} \sin\theta_c = n_w \sin 90$$

48.7°



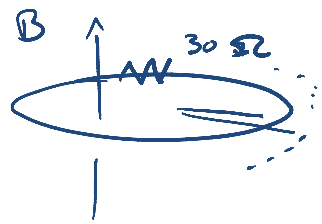
$$n_g = \frac{n_w}{\sin\theta_c} = \frac{1.33}{\sin(48.7)}$$

$$n_g = 1.77$$

P114 University of Rochester NAME Soln Key - SLM
S. Manly Spring 2010

Problem 12 (6 pts, show work):

A respiration monitor has a flexible loop of copper wire, which wraps about the chest. As the wearer breathes, the radius of the loop of wire increases and decreases. When a person in the Earth's magnetic field inhales, what is the average current in the loop, assuming that it has a resistance of 30 ohms and increases in radius from 20 cm to 25 cm over 1 s? For simplicity assume the magnetic field of the Earth to be perpendicular to the plane of the loop and of magnitude 50x10^{-6} Telsa.



when $r = 20\,cm$ $A = \pi(.2)^2 = 0.13\ m^2$

$r = 25\,cm$ $A = \pi(.25)^2 = 0.19\ m^2$

$$\mathcal{E} = \frac{d\Phi_M}{dt} = \frac{B(A_{.25} - A_{.2})}{1\,s} = \frac{(50\times10^{-6})(.19-.13)}{1}$$

$\mathcal{E} = 3\times10^{-6}\,V$ $\mathcal{E} = iR$ $i = \frac{3\times10^{-6}}{30}$

$0.1\ \mu A = i$

Problem 13 (5 pts, show work):

Suppose we find a rock and geologists tell us that when that rock was formed in a volcano it would have contained an equal amount of Iridium-192 (Z=77, symbol = Ir) and Bismuth-209 (Z=83, symbol = Bi). Suppose that Iridium-192 and Bismuth-209 are both naturally radioactive. Also, suppose that Iridium-192 decays with a half-life of 500,000 years and Bismuth-209 decays with a half-life of 1 million years. If the rock is two million years old and you measure the amount of Iridium-192 and Bismuth-209 in the rock, what ratio for the amount of Iridium-192 to Bismuth-209 would you expect to measure?

2 myrs = 4 $t_{1/2}$ for Iridium-192

= 2 $t_{1/2}$ for Bismuth-209

\# $t_{1/2}$: 1, 2, 3, 4

1/2 1/4 1/8 1/16

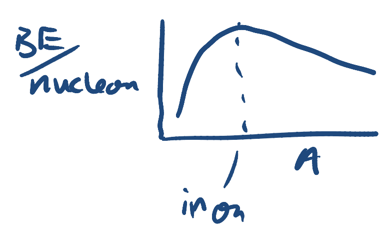
$$\frac{\text{Iridium-192}}{\text{Bismuth-209}} = \frac{1/16}{1/4} = \frac{1}{4}$$

Problem 14 (5 points):

Because of your amazing physics expertise you become a consultant on nuclear terrorism to the U.S. Department of Missles and Urban Development (MUD) after graduation. One day the Grand Pubah Ubersecretary of MUD, Samuel Thudpucker III, calls you to his office to ask for your advice on a national security matter. Samuel sits you down and says, "We have just apprehended a nasty, scumbag terrorist type and interrogated him. He didn't give up much information at first, but after we threatened to make him watch CNN's Nancy Grace endlessly, he broke. The scumbag told us that he and his nasty friends recently acquired a special nuclear bomb that uses iron as the active bomb material. Is this credible? Should we be worried?"

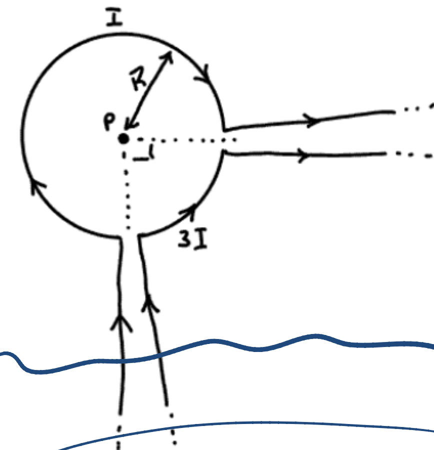
Please give here a brief and appropriate response to Ubersecretary Thudpucker's questions using what you have learned in this course.

Iron is the most stable nucleus
It has the highest Binding Energy/nucleon.
Thus it is not possible to use iron as the source of energy for a bomb using either fission or fusion



Problem 15 (7 points, show work):

Consider the configuration of currents shown below. The currents are confined to the plane of the paper. The "circle of current" is centered at point P and has a radius R. The currents approach and leave the "circle of current" along radial lines. Assume the gaps in the arc of the circle are negligibly small and are expanded in the sketch for your viewing pleasure. The clockwise moving current covers three-quarters of the full arclength of the circle and the counter-clockwise moving current coverse one-quarter of the full arclength of the circle. Prove that the magnetic field at point P – the center of the circle – is zero.



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{l}\times\hat{r}}{r^2}$$

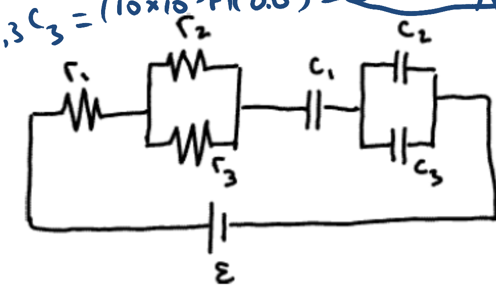
$$\vec{B}_P = \frac{\mu_0}{4\pi}\left[\int_0^{\frac{3}{4}2\pi R}\frac{I}{R}\,ds\ \hat{in} + \int_0^{\frac{2\pi R}{4}}\frac{I}{R}\,ds\ \hat{out}\right]$$

$$\vec{B}_P = \frac{\mu_0}{4\pi}\left[I\,\frac{2\pi R}{R}\,\frac{3}{4}\,\hat{in} + 3I\,\frac{2\pi R}{R}\,\frac{1}{4}\,\hat{out}\right]$$

$$\vec{B}_P = 0$$

Problem 16 (7 points, show work):

Consider the circuit to the right. If r_1=5 ohms, r_2=6 ohms, r_3=10 ohms, C_1=1 mF, C_2=5 mF, C_3=10 mF and the EMF is 10 Volts. The capacitors are uncharged at t=0.



a) After a long time elapses, how much charge will be deposited on each of the capacitors?

$Q = CV$

$Q = C\mathcal{E}$ / C_{eq}

$C_{2,3} = C_2 + C_3 = 15\text{ mF}$

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{23}} = \frac{1}{1} + \frac{1}{15}$

$C_{eq} = 0.94\text{ mF}$

$Q = (0.94\times10^{-3})(10)$

$\boxed{Q_{Tot} = 9.4\times10^{-3}\text{ coul.}}$

$Q_1 = Q = 9.4\times10^{-3}\text{ Coul}$

$Q_{2,3} = 9.4\times10^{-3}\text{ Coul}$

$Q_{2,3} = C_{2,3}V_{2,3}$

$9.4\times10^{-3} = (15\text{ mF})V_{23}$

$V_{2,3} = 0.6\text{ volts}$

$Q_2 = V_{2,3}C_2 = (5\times10^{-3}\text{F})(0.6) = 3\times10^{-3}\text{ coul} = Q_2$

$Q_3 = V_{2,3}C_3 = (10\times10^{-3}\text{F})(0.6) = 6\times10^{-3}\text{ coul} = Q_3$

b) What is the time constant of this circuit?

$$\tau = R_{eq}C_{eq} = (8.75)(.94\times10^{-3}) = \boxed{8.2\times10^{-3}\text{ seconds}}$$

$\frac{1}{r_{2,3}} = \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{6} + \frac{1}{10}$ $\quad r_{2,3} = 3.75\ \Omega$ $\quad r_{eq} = r_1 + r_{2,3} = 5 + 3.75 = 8.75\ \Omega$

Problem 17 (6 points, show your work):

A spacecraft travels along a straight line from Earth to the Moon, a distance of 3.84x108 m. Its speed measured on Earth is 0.50c. $(c = 3\times10^8\ m/s)$

a) How long does the trip take, according to a clock on Earth?

$3\times10^8\ m/s\left(\frac{1}{2}\right) = \frac{3.84\times10^8\ m/s}{T}$ $\boxed{T = 2.56\ s}$

$\gamma = \frac{1}{\sqrt{1-.5^2}} = 1.15$

b) How long does the trip take, according to a clock on the spacecraft?

Clock on space craft measures proper time

$T_{earth} = T_{spacecraft}\,\gamma$ $\boxed{T_{spacecraft} = \frac{T_{Earth}}{1.15} = 2.22\ s}$

c) Determine the distance between the Earth and the moon if it were measured by a person on the spacecraft?

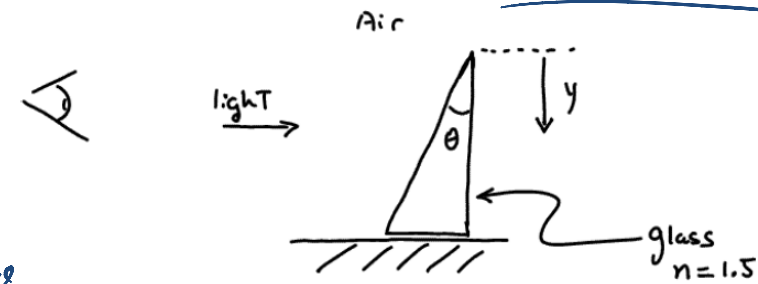
Earth measured distance is proper frame

$D_{Earth} = D_{spacecraft}\,\gamma$ $D_{sp} = \frac{3.84\times10^8\ m}{1.15}$

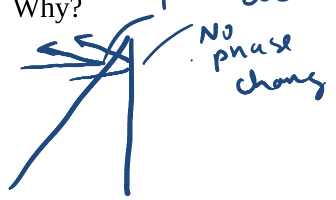
$\boxed{D_{sp} = 3.33\times10^8\ m}$

Problem 18 (8 pts, show work):

A triangular piece of glass is illuminated from the front as shown in the sketch. The light has a wavelength of 500×10^{-9} m. The index of refraction of the glass is 1.5.

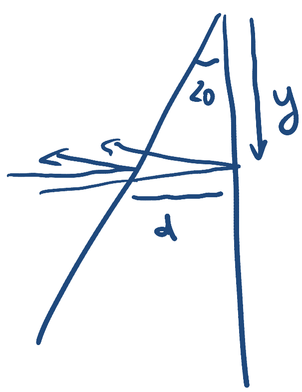


a) At the top of the glass (y=0), is there a dark fringe or a bright fringe? Why?



one of the interfering rays has a phase change ... the other does not. Difference in length traveled at top is negligible $\longrightarrow$ Dark fringe at top

b) If the angle theta (as shown in the graph) is 20 degrees, what is the distance between dark fringes along the direction y as seen by the observer in the sketch?



$\tan 20 = d/y$

$d = y\tan 20$

$c = \lambda\nu$

$v = \lambda_n\nu$

$\frac{c}{v} = n = \frac{\lambda}{\lambda_n}$

$\lambda_n = \lambda/n$

$2d = m\lambda_n$ for dark fringes

$m = 1, 2, \ldots$

$d = y\tan 20$

$2y\tan 20 = m\lambda_n$

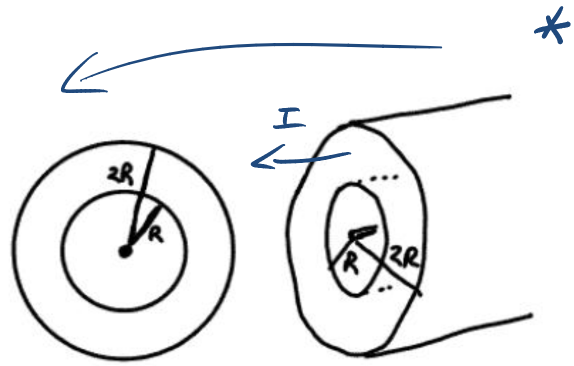
$y = \frac{m\lambda_n}{2\tan 20} = \frac{m\ 500\times10^{-9}}{2\tan 20\ (1.5)}$

$\Delta y = \frac{500\times10^{-9}}{2\tan 20(1.5)}$

$\Delta y = \frac{.7\ \mu m}{1.5}$

$\boxed{\Delta y = 0.47\ \mu m}$

Problem 19 (12 pts, show work):



Consider a ~~thin current-carrying wire surrounded by a~~ cylindrical, current-carrying shell. Referring to the sketch to the right, the ~~wire (of negligible radius) carries a current I into the paper and the outer~~ shell carries a current I out of the paper spread over its surface area according to a current density j(r)=K/r, where K is a constant. The shell has inner radius R and outer radius 2R.

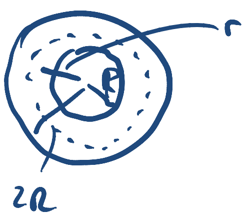
a) (3 pts) Determine the magnetic field in the region r<R in terms of K, I, and R.

$\int \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$ $\qquad I_{encl} = 0$

$|\vec{B}| 2\pi r = \mu_0 I$ $\qquad B = 0$

for $r<R$

b) (3 pts) Determine the magnetic field in the region R<r<2R in terms of K, I, and R.



$|B| 2\pi r = \mu_0 I_{enc}$

$I_{enc} = \int_R^r \frac{K}{r} 2\pi r\, dr = K 2\pi (r-R)$

$|B| = \mu_0 K \frac{(r-R)}{r}$

Direction is counterclockwise

c) (2 pts) Determine the magnetic field in the region r>2R in terms of K, I, and R.

$|B| 2\pi r = \mu_0 I$

$|B| = \frac{\mu_0 I}{2\pi r}$ for $r > 2R$

Direction is counterclockwise

d) (4 pts) Show that K = I/2πR

$I = \int_R^{2R} j\, da = \int_R^{2R} K/r\; 2\pi r\, dr = K 2\pi (2R - R) = K 2\pi R$

$\boxed{K = I/2\pi R}$

Add u_E —

Final Exam Formulas

$\vec{F} = q\vec{E}$

$\vec{F} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{12}^2}\hat{r}_{12}$

$\phi_E = \oint \vec{E}\cdot d\vec{A}$

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$

$\vec{E} = \int_{vol} \frac{k\,dQ}{r^2}\hat{r}$

V = work/charge

$V_{\text{point charge}} = \frac{kQ}{r}$

$V = \int_{vol} \frac{k\,dQ}{r}$

$E_s = -dV/ds$

$V = IR$

$Q = CV$

$U = \frac{1}{2}CV^2$

$P = IV = I^2R = \frac{V^2}{R}$

$|e| = 1.6\times10^{-19}$ coulombs

$k = 8.99\times10^9 \frac{Nm^2}{C^2}$

$\epsilon_0 = 8.85\times10^{-12}\ C^2/Nm^2$

$\mu_0 = 4\pi\times10^{-7}\ \frac{Tm}{A}$

$E = E_0/\kappa$

$C = \kappa C_0$

$Q(t) = C\mathcal{E}(1-e^{-t/RC})$

$Q(t) = Q_0e^{-t/RC}$

$R_{eq} = \Sigma r_i$
$1/C_{eq} = \Sigma 1/C_i$] For series geometry

$\vec{F} = q\vec{v}\times\vec{B} = l\vec{i}\times\vec{B}$

$\vec{\mu} = nIA$

$\vec{\tau} = \vec{\mu}\times\vec{B}$

$B_{solenoid} = \mu_0 nI$

$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{i\,d\vec{l}\times\hat{r}}{r^2}$

$\int_{curve} \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$

$\mathcal{E} = -d\Phi_B/dt$

$\Phi_B = \oint\vec{B}\cdot d\vec{a}$

$\Phi_B = Li$

$\mathcal{E} = -L\,di/dt$

$u_B = B^2/2\mu_0$

$B = E/c$ For plane wave

$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$

$\langle S\rangle = \frac{EB}{2\mu_0}$

$P_{radiation} = S/c$ (Absorbed)

$c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$

$v = \lambda\nu$

$\vec{B} = \mu_0(1+\chi_M)\vec{B}_{ext}$

$\mu_0\chi_M = \mu$

$v = \lambda \nu$

$E = h\nu$

$\lambda = h/p$

$$E_n = -\frac{Z^2 \, 13.6\,\text{eV}}{n^2}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi = E\psi$$

$(A-Z)\,m_n c^2 + Z m_p c^2 - M_x c^2 = \text{TOT BE}$

$$\frac{dN}{dt} = -\lambda N$$

$N = N_0 e^{-\lambda t}$

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$\Delta t' = \gamma \Delta t$

$\Delta x' = \gamma \Delta x$

$n = c/v$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$$

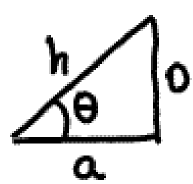
$m = -i/o$

$$P = \frac{1}{f}$$

$\theta \sim 1.22\, \lambda/D$

$\lambda/a = \sin\theta$

$d\sin\theta = m\lambda$



$$\sin\theta = \frac{o}{h} \quad \cos\theta = \frac{a}{h}$$

$\tan\theta = o/a$

Sphere: $A = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

cylinder: $A = 2\pi r L + 2\pi r^2$

$V = \pi r^2 L$

$$\int u^n du = \frac{u^{n+1}}{n+1}$$

$$\int \frac{du}{u} = \ln|u|$$

$$\int e^u du = e^u$$

$$\int \frac{x\,dx}{(x^2+a^2)^{1/2}} = \sqrt{x^2+a^2}$$

$V = V_0 + at$

$x = x_0 + V_0 t + \frac{1}{2}at^2$

$V^2 = V_0^2 + 2a(x-x_0)$

$x = x_0 + \frac{1}{2}(V+V_0)t$

$S = r\theta$

$KE = \frac{1}{2}mv^2$

$PE_{spring} = \frac{1}{2}kx^2$

$$a_c = \frac{\cancel{m}v^2}{r}$$