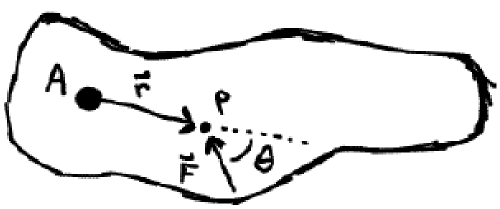
Exam 3 (November 21, 2006)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless specified otherwise.

<u>Problem 1 (18 pts):</u>

The object below can rotate frictionlessly in the plane of the paper about axis A. Initially the object is at rest. A force F is applied to the object at the point P shown below at time t=0.

a) (7 pts) Referring to the drawing below, if F has a magnitude of 5 Newtons, the mass of the object is 2 kg, the magnitude of r is 0.2 meters, and the angle theta is 30 degrees, what is the magnitude of the torque on the object?



b) (4 pts) What is the direction of the torque on the object (referring to the sketch, your answer might be one of the following: up, down, left, right, into paper, out of paper)?

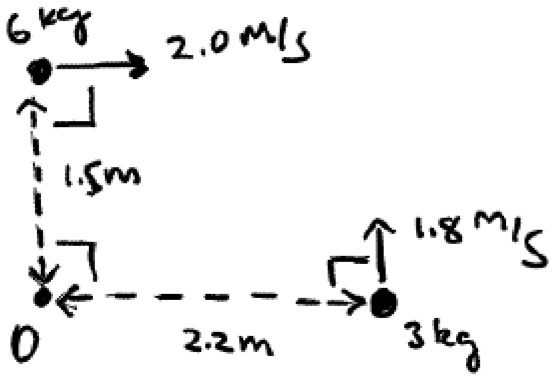
c) (7 pts) If we suppose that the torque you specify in parts (a) and (b) is applied continuously to the object starting at time t=0, what is the angular velocity (magnitude and direction) of the object at t=3 seconds? (Assume the moment of inertia of the object is 2 kgm^2.)

<u>Problem 2 (12 pts):</u>

A cylinder (I=(1/2)MR^2) with mass M=1 kg and radius R=0.2 m rolls without slipping along a level floor with speed v=0.3 m/s. Determine the work required to stop the cylinder from rolling.

Problem 3 (10 pts):

Two objects are moving as shown in the figure below. What is the total angular momentum of the system about point O?



1)	/18
2)	/12
3)	/10
4)	/10
5)	/20
6)	/15
7)	/15
tot	/100

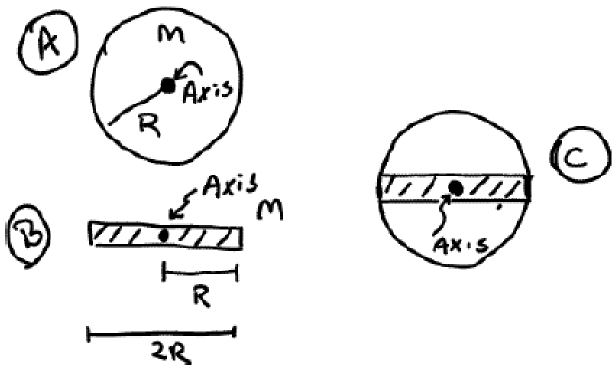
Problem 4 (10 pts):

A few years from now (that will pass faster than you can imagine) you find yourself helping your eight-year-old son make a pinewood derby car to race in the Cub Scout pinewood derby competition. In the pinewood derby, each kid carves a block of wood into the shape of a car and attaches four plastic wheels that are free to rotate about axes formed from nails. The cars race in pairs down an inclined plane "track". A timer is set up to record the race times for each car. The winning car in each heat is the one that reaches the timer at the bottom first. The cars all must have the same mass and length. In order to maintain the mass after the child removes wood to shape the car, little steel masses are glued on the car Where along the length of the car do you advise your son to place the masses in order to maximize the car's speed at the bottom of the incline? Why? ***(Please assume the air resistance and wheel friction is independent of where the mass is placed.)*** **Briefly explain your equations as needed.**

<u>Problem 5 (20 pts):</u>

Consider two objects pictured below. One is a uniform, thin rod of total length 2R and mass M (sketch B). The other is a uniform thin disk of radius R and mass M (sketch A). Each of the objects rotates (without translating) in the plane of the paper about an axis through their geometric center with angular velocity ω.

(10 pts) What is the ratio of the kinetic energy of the rod to the disk?



(10 pts) Derive and circle the correct expression for the kinetic energy of rotation for the object formed by attaching the rod to the disk as shown in sketch C. Assume the object rotates with angular velocity ω. *(Reminder: you must show work to receive credit.)*

a) $\frac{1}{12}MR^2\omega^2$ **b)** $\frac{1}{2}MR^2\omega^2$ **c)** $\frac{1}{6}MR^2\omega^2$ **d)** $\frac{5}{6}MR^2\omega^2$ **e)** $\frac{5}{12}MR^2\omega^2$

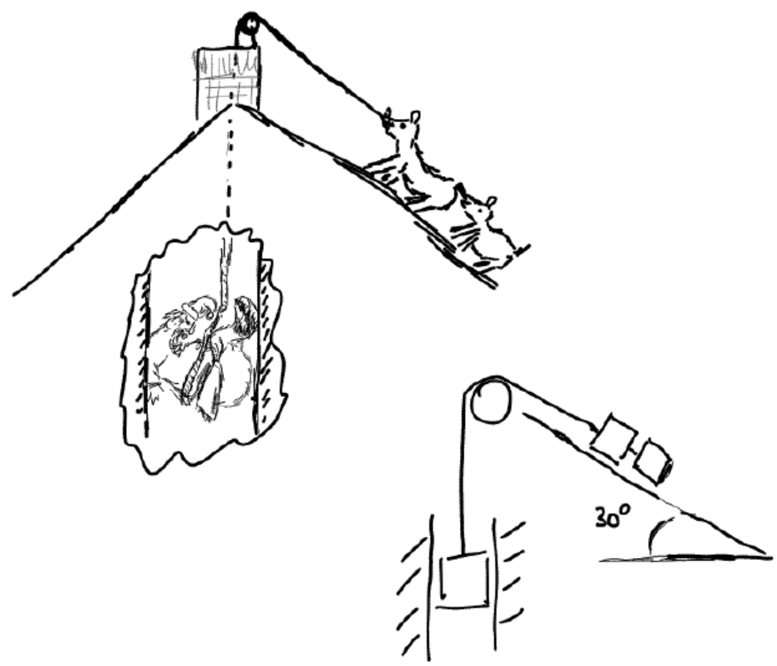
Problem 6 (15 pts):

Polly Puritan sees her true love, Alex, across the ice at the winter festival. Alex sees Polly at the same time. They each race to the middle of the ice to embrace. Polly heads due south with a speed of 5 m/s. Alex rushes northeast to meet Polly at a speed of 6 m/s. They meet in the middle of the ice and embrace passionately. In what direction and at what speed do Alex and Polly move while they embrace? Assume the couple aims their skates in a direction such that they move without friction across the ice once they meet. For your calculational pleasure, Alex has a mass of 81 kg and Polly has a mass of 47 kg.

Problem 7 (15 pts):

Santa and his reindeer are training in the off-season by participating in maneuvers at the NPTCATG (North Pole Tactical Chimney Assault Training Ground), which is sited a few hundred yards from Santa's off-season crib. In the tight quarters drill, sketched below, Santa's reindeer pull Santa up a frictionless chimney with a (massless) rope. Santa has a mass M_s. The rope passes over a massive pulley with a radius R and a mass M. Assume the mass of the pulley is distributed in the form of a uniform disk. The rope is attached to two reindeer with combined mass M_r who slide down a frictionless roof which makes an angle of 30 degrees with the horizontal. Determine three independent equations that would be sufficient to solve for Santa's acceleration up the chimney and the tension everywhere in the rope in terms of the masses and the angle of the roof.

****** In the interest of time, please do not bother solving these equations.*******



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$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$v = v_o + at$$

$$x = x_o + v_o t + \frac{1}{2}at^2$$

$$x = x_o + \left(\frac{v_o + v}{2}\right)t$$

$$v^2 = v_o^2 + 2a(x - x_o)$$

$$x - x_o = \int_{t_o}^{t} v\,dt$$

$$v - v_o = \int_{t_o}^{t} a\,dt$$

$$\sum \vec{F} = m\vec{a}$$

$$F_{friction} = \mu_k N$$

$$F_{friction} = \mu_s N$$

$$F_{centripetal} = \frac{mv^2}{r}$$

$$\vec{F}_{spring} = -k(\vec{x} - \vec{x}_o)$$

$$work = \int F \cdot ds$$

$$power = \frac{dw}{dt}$$

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$s = r\theta$$

$$v = r\omega$$

$$a = r\alpha$$

$$\omega = \omega_o + \alpha t$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$\theta = \theta_o + \left(\frac{\omega + \omega_o}{2}\right)t$$

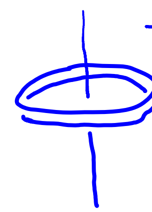
$$\omega = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

$$KE_{translation} = \frac{1}{2}MV^2$$

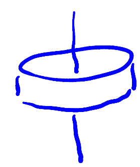
$$KE_{rotation} = \frac{1}{2}I\omega^2$$

$$I = \sum m_i r_i^2 = \int r^2 dm$$

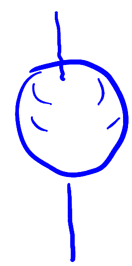
$$X_{cm} = \frac{\sum x_i m_i}{M} = \frac{\int x\,dm}{M}$$



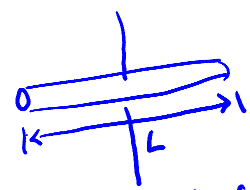
Thin hoop
radius R
$I = MR^2$



Solid cylinder
Radius R
$I = \frac{1}{2}MR^2$



uniform sphere
Radius R
$I = \frac{2}{5}MR^2$



uniform rod of length L
$I = \frac{1}{12}ML^2$

$$\frac{d(x^n)}{dx} = nx^{n-1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$I = I_{cm} + mh^2$$

$$\vec{\tau} = \vec{r} \times \vec{F} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

circumference of circle $= 2\pi r$

area of circle $= \pi r^2$

$$\vec{F}_{grav} = -\frac{Gm_1 m_2}{r^2}\hat{r}$$

quadratic equation $= \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$