Physics 142 - November 27, 2007

- Presentation
  - Dec. 4 Transistor
    Planetary Magnetic fields
  - Dec. 6 Elect. Musical Instr.
    Laser
  - Dec. 11 Wireless
    Superconductivity
  - Dec. 13 Particle Detectors
    Electromagnetism in Medicine and/or chem

- If potentially interested in being physics TI next term (+ not already)
  contact Janet Fogg

Last Time

Divergence of vector field $\vec{V}$ (cartesian coordinates)

$$\vec{\nabla}\cdot\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

Measures divergence or convergence of field

+ Divergence

− Divergence

0 Divergence

Can prove to yourself

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{\nabla}\cdot\vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(V_x\hat{i} + V_y\hat{j} + V_z\hat{k}\right)$$

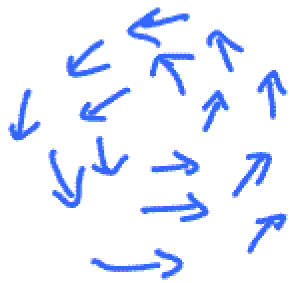
use $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0$

Curl of a vector field

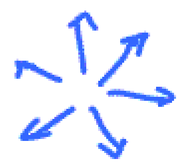
$\equiv \vec{\nabla} \times \vec{V}$

Amount of Twist or circulation of vector field



large curl

No curl



No curl

Cartesian coordinates

$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) - \hat{j}\left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right) + \hat{k}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\oint_S \vec{B}\cdot d\vec{A} = 0$$

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} + \mu_0\epsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_M}{dt} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A}$$

Integral form

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

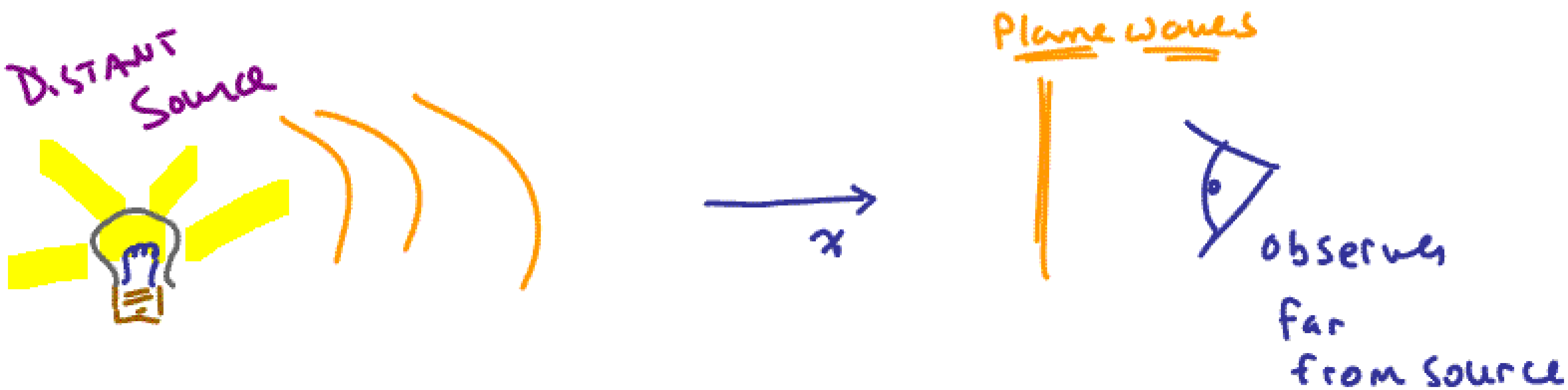
Differential form

Maxwell's Equations

Can show Maxwell's Eqns $\Rightarrow$ Coupled wave equations for $\vec{E}, \vec{B}$

Must specify "Boundary conditions" and Solve

Electromagnetic "plane wave" Solution useful, simple + best guide for intuition



- $\vec{E}, \vec{B}$ are transverse to direction of propagation
- $\vec{E}, \vec{B}$ are perpindicular and in phase

Assume E is oriented along y-axis
"Polarized" along y

- Solns

$$E_y(x,t) = E_{oy} \cos(kx - \omega t + \varphi)$$

$k = \frac{2\pi}{\lambda}$

$\omega = \frac{2\pi}{T} = 2\pi\nu$

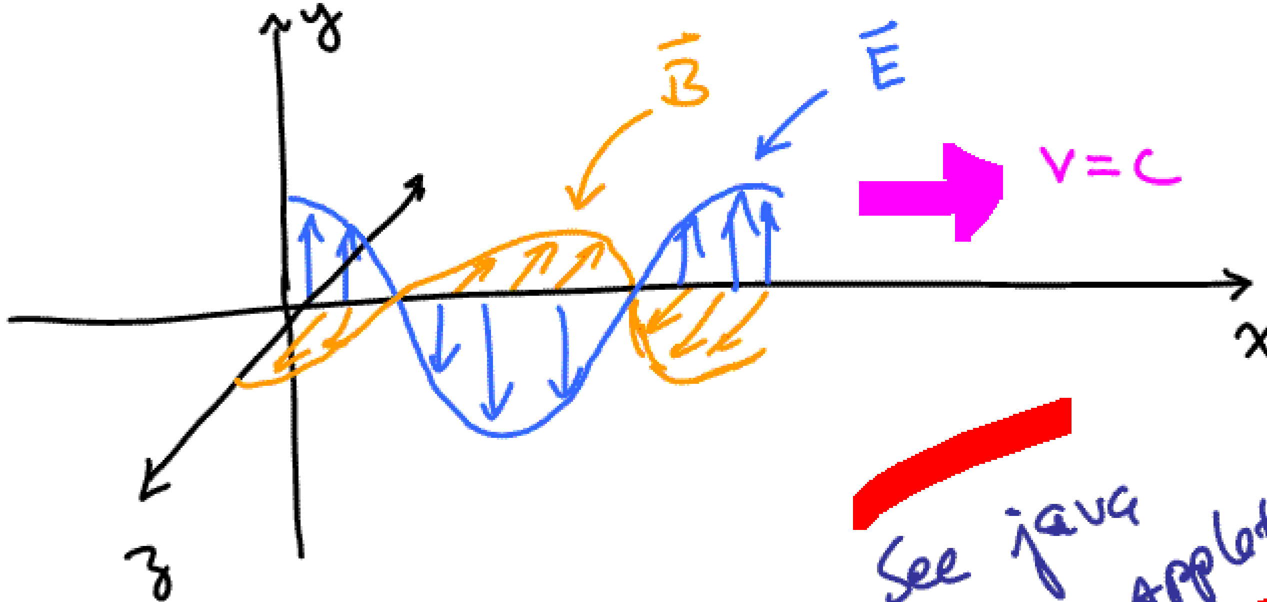
Phase Angle

$$B_z(x,t) = B_{oy} \cos(kx - \omega t + \varphi) = \underbrace{\frac{k}{\omega}}_{\frac{1}{c}} E_{oy}(\cos kx - \omega t + \varphi)$$

- Magnitude of $\vec{E}, \vec{B}$ related

$$\frac{k}{\omega} = \frac{2\pi/\lambda}{2\pi/T} = \frac{T}{\lambda} = \frac{1}{c}$$

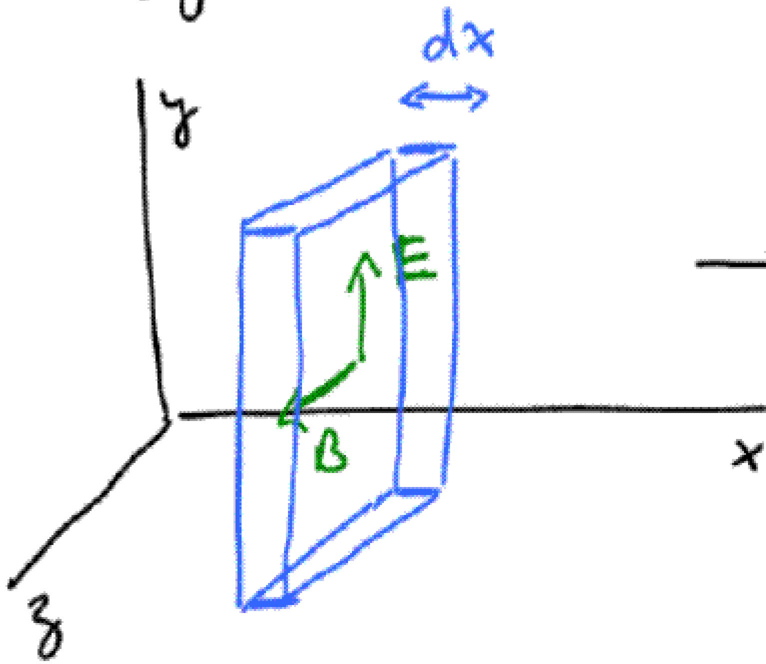
$$|\vec{E}| = c\,|\vec{B}|$$



See java applet !!

Energy Flow in EM Waves

Ohanian



$\longrightarrow$ plane wave

$$u_E = \frac{\epsilon_0}{2} E^2 \qquad u_B = \frac{1}{2\mu_0} B^2$$

$$dU = \text{Energy in Volume} = (u_E + u_B)\,\text{volume}_{\text{Box}}$$

$$dU = \left(\frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2\right)(\text{Area})\, dx$$

$$E = cB \qquad \epsilon_0 = \frac{1}{\mu_0 c^2} \qquad c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$du = \left[\frac{1}{2\mu_0 c^2} EcB + \frac{1}{2\mu_0} B \frac{E}{c}\right](\text{Area})\, dx$$

dU = differential Energy

Moves thru box in time $dt = \frac{dx}{c}$

$$du = \left(\frac{1}{\mu_0 c} EB\right) \text{Area}\, dx$$

$$\frac{dU}{dt} = \frac{EB}{\mu_0} \text{ (Area)}$$

$$\frac{dU}{dt}\frac{1}{\text{Area}} = \frac{EB}{\mu_0} = \frac{\text{Watts}}{m^2} \equiv \text{Intensity}$$

Energy flux

Add direction

$\vec{S} \equiv$ poynting vector $\equiv$ Energy flow

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$\vec{E}, \vec{B}, \vec{S}$ all fluctuate w/ time

Time Average of $S \equiv \bar{S} = \langle S \rangle$

$\langle |\vec{S}| \rangle$

$$E = E_0 \sin \omega t$$

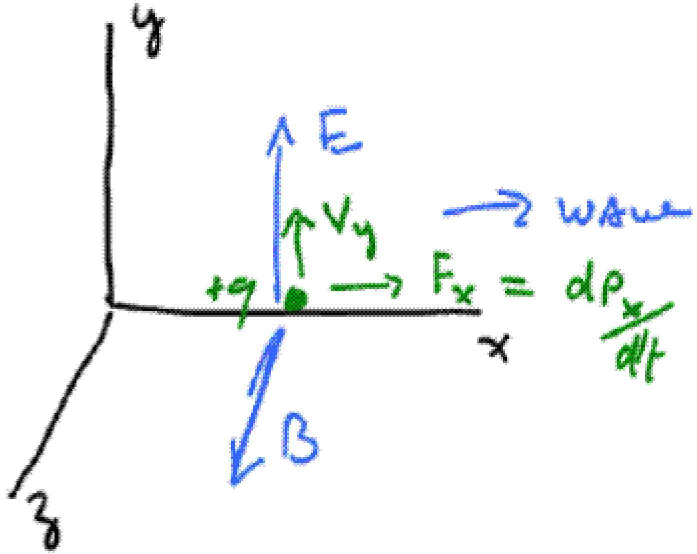
$$B = \frac{E_0}{c} \sin \omega t$$

$$S = \frac{1}{\mu_0 c} E_0^2 \sin^2 \omega t$$

$$\bar{S} = \langle S \rangle = \frac{E_0^2}{2 \mu_0 c}$$

CAN write in terms of B

$$= \frac{c}{2\mu_0} B^2$$



$$\frac{dp_x}{dt} = F_x = q(\vec{v}\times\vec{B})_x = q(v_yB_z - \cancel{v_zB_y})$$

0

$$\frac{dp_x}{dt} = q v_y B_z \qquad B_z = \frac{E_y}{c}$$

$$\frac{dp_x}{dt} = \frac{q}{c} v_y E_y$$

$$\frac{d\,w(\mathrm{ork})}{dt} = q\,\vec{E}\cdot\vec{v} = q\,v_yE_y$$

$w \sim F\cdot d$

$\sim qE\,\frac{d}{t}\,t$

$$\frac{dP_x}{dt} = \frac{1}{c}\frac{dW}{dt}$$

$$dP_x = \frac{1}{c}\,dW$$

$$\boxed{P = \frac{U}{C}}$$

Momentum in EM wave

$$= \frac{1}{c}\left(\text{Energy in EM wave}\right)$$

EM wave Absorbed

$$\vec{F} = \frac{d\vec{P}}{dt}$$

$$F = \frac{dP}{dt} = \frac{dU}{dt}\frac{1}{c} = \frac{1}{c}\,\frac{\frac{E}{\text{Area}}}{dt}$$

Area

$$F = \frac{1}{c} S \,(\text{Area})$$

$$\frac{F}{\text{Area}} = \text{pressure} = \frac{S}{c}$$

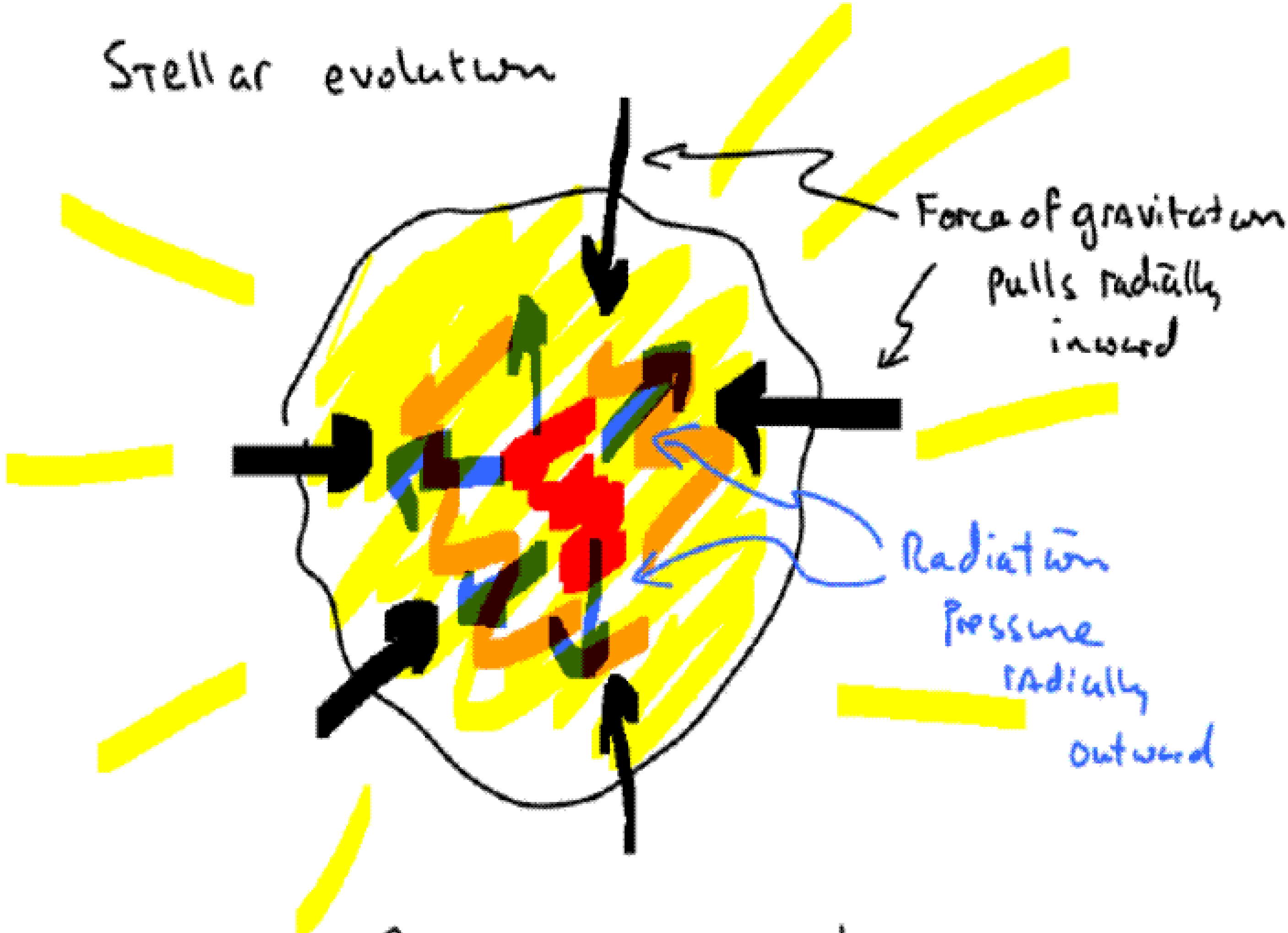
$$\text{Radiation Pressure} = \frac{S}{c}$$

TOTAL Absorption

$\times 2$ if reflection

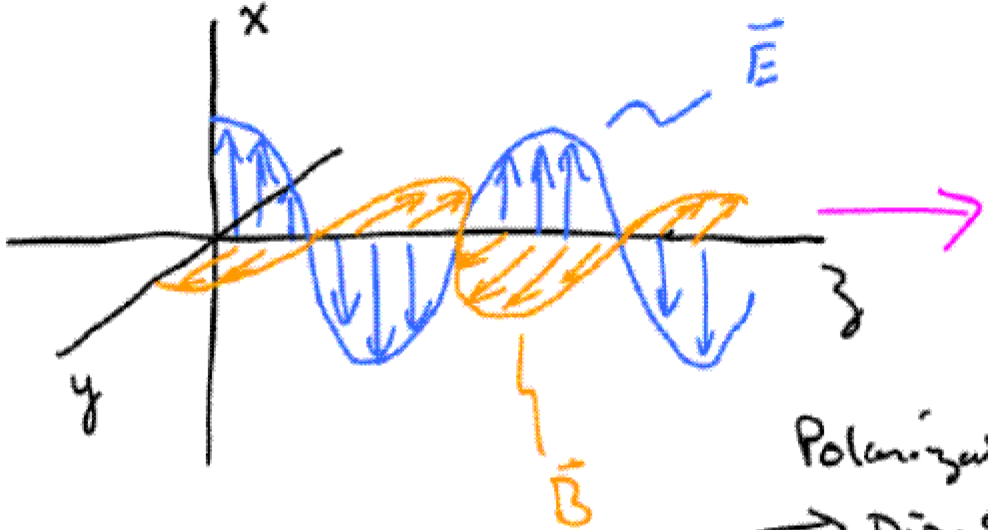
$$\langle P \rangle = \frac{\langle S \rangle}{c}$$

Stellar evolution



Stars are systems where the radiation pressure (from thermonuclear reactions in the core) is in equilibrium with the gravitational attraction of the matter toward the center of the star.

Polarization of Electromagnetic waves



Polarization
→ Dir. of $\vec{E}$ vector

$\vec{E}$ ~~orientd~~ oriented along x axis

wave is polarized along x-axis

gen soln = (A) + (B)

MOST general Soln

Superposition of two orthogonal waves

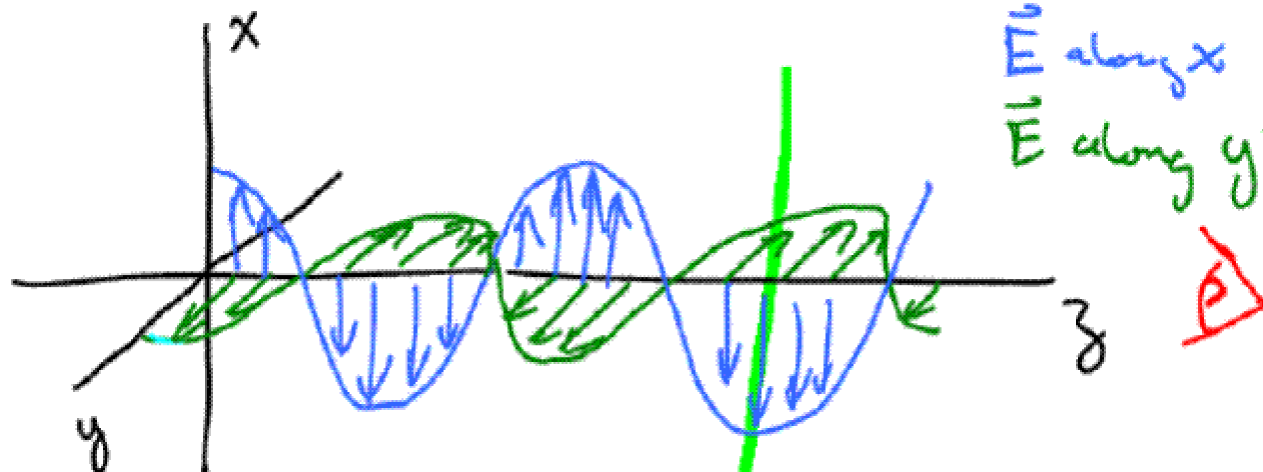
(basis in Mathematics)

1 – plane polarized along x-axis

1 – " " " y-axis

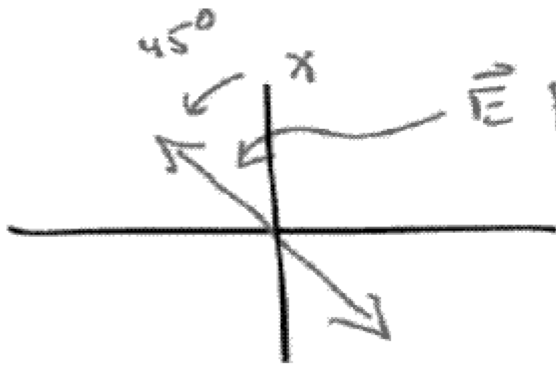
$\vec{E}$ along x

$\vec{E}$ along y



$\vec{E}_x = E_{0x} \cos(kz - \omega t)\,\hat{i}$

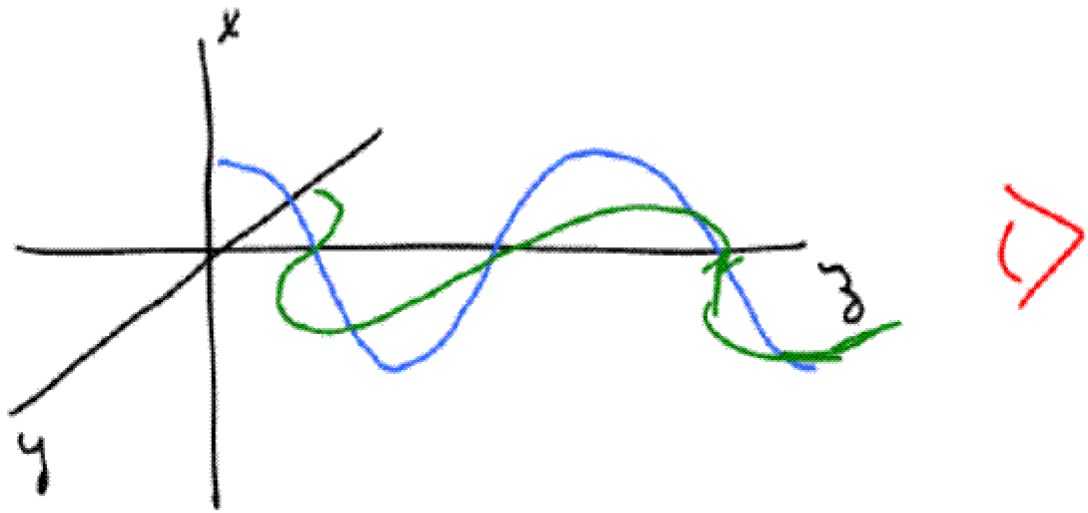
$\vec{E}_y = E_{0y} \cos(kz - \omega t)\,\hat{j}$



Resultant $\vec{E}$ Field vector

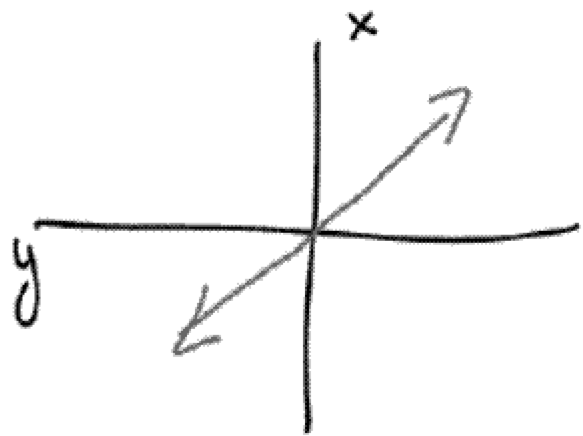
Eye sees

Linear polarization

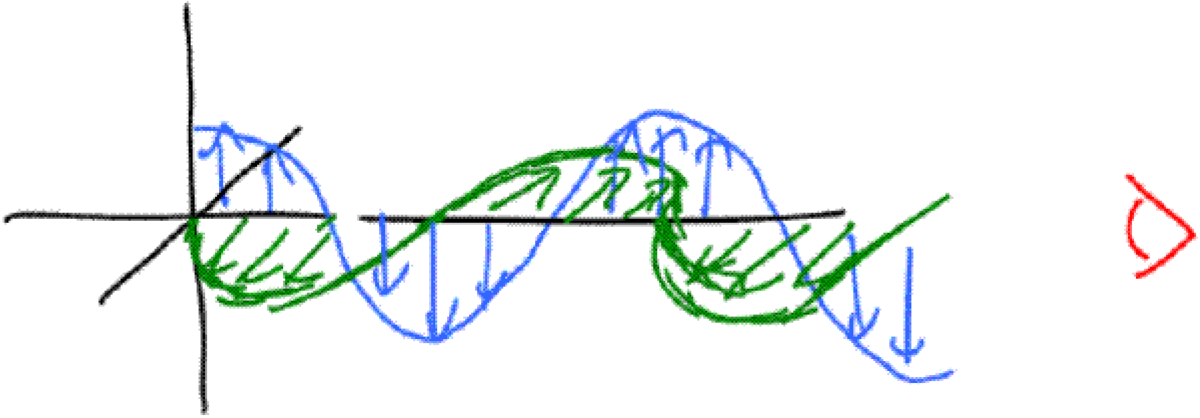


$$E_x = E_{0x}\cos(kz-\omega t)\,\hat{i}$$

$$E_y = E_{0x}\cos(kz-\omega t+\pi)\,\hat{j}$$



Again –
linear Polarization



$$E_x = E_{0x} \cos(kz - \omega t)\ \hat{i}$$

$$E_y = E_{0x} \cos(kz - \omega t + \pi/2)\ \hat{j}$$

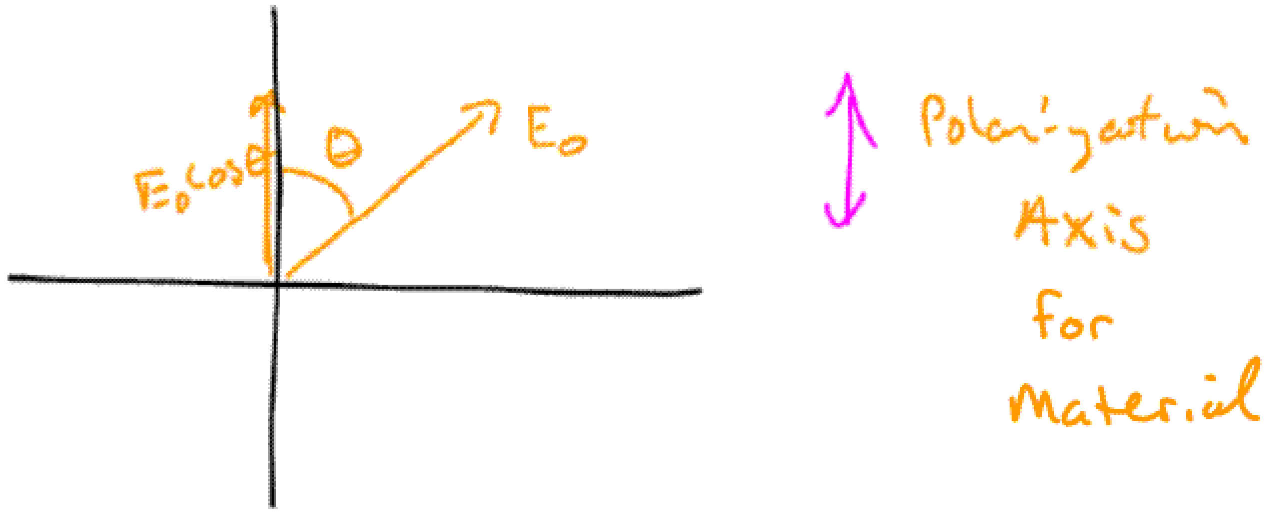
Resultant vector Rotates w/ Time

Clockwise
Right circular
Polarization

Can Δ pol. from linear ↔ circular

Asymmetric index of refraction

$n = c/v$



$$I_{New} \sim E_0^2 \cos^2\theta \sim I_{init} \cos^2\theta$$