Physics 142 - November 20, 2007

Happy Thanksgiving!

- Presentations
  - Dec 4
  - Dec 6
  - Dec 11
  - Dec 13

1st come 1st served

2 per day

20 min + disc./quest.

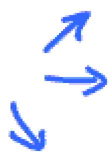
Lasers
Part. Detectors 12/13
Plan. Magn. fields
Elect. Musical Instr.
EM in chem/Medicine 12/13
Wireless comm 12/11
Transistor
Superconductivity 12/11

Buckle your seatbelts . . .

Divergence of vector field $\vec{V}$ (cartesian coordinates)

$$\vec{\nabla}\cdot\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

Measures divergence or convergence of field



+ Divergence

− Divergence

0 Divergence

Can prove to yourself

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{\nabla}\cdot\vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(V_x\hat{i} + V_y\hat{j} + V_z\hat{k}\right)$$

use $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0$

$$\int_S \vec{E}\cdot d\vec{A} = q/\epsilon_0$$

$$q = \int_V \rho \, dv$$

$$\int_V (\vec{\nabla}\cdot\vec{E})\, dv = \int_S \vec{E}\cdot d\vec{A}$$

$$\boxed{\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0}$$

$$\int_S \vec{B}\cdot d\vec{A} = 0$$

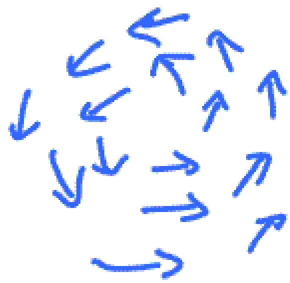
$$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

No Magnetic monopole

Curl of a vector field

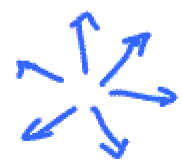
$$\equiv \vec{\nabla} \times \vec{V}$$

Amount of Twist or circulation of vector field



large curl

No curl



No curl

Cartesian coordinates

$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) - \hat{j}\left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right) + \hat{k}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

Stoke's Theorem

$$\oint_S (\vec{\nabla}\times\vec{V})\cdot d\vec{A} = \oint_C \vec{V}\cdot d\vec{l}$$

So

$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d\phi_M}{dt} = -\frac{d}{dt}\oint_S \vec{B}\cdot d\vec{A}$$

use Stoke's thm

$$\oint_S (\vec{\nabla}\times\vec{E})\cdot d\vec{A} = -\frac{d}{dt}\oint_S \vec{B}\cdot d\vec{A}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}}$$

Faraday's law in differential form

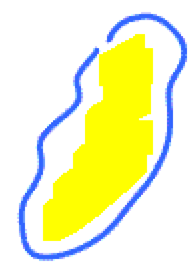
$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 i + \mu_0\epsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \oint_S \vec{J}\cdot d\vec{A} + \mu_0\epsilon_0 \frac{\partial}{\partial t}\int_S \vec{E}\cdot d\vec{A}$$

Stoke's Thm

$$\oint_S (\vec{\nabla}\times\vec{B})\cdot d\vec{A} = \mu_0\oint_S \vec{J}\cdot d\vec{A} + \mu_0\epsilon_0\frac{\partial}{\partial t}\oint_S \vec{E}\cdot d\vec{A}$$

recall $\vec{J}\equiv$ current density

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}}$$



$$\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$

$$\oint_S \vec{B}\cdot d\vec{A} = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} + \mu_0\epsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_M}{dt} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

Integral form

Differential form

Maxwell's Equations

Laplacian of scalar

$$\nabla^2 T = \vec{\nabla}\cdot(\vec{\nabla} T) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}\right)$$

$$\boxed{\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}}$$

Laplacian of a vector field

$$\nabla^2 \vec{V} = (\nabla^2 V_x)\hat{i} + (\nabla^2 V_y)\hat{j} + (\nabla^2 V_z)\hat{k}$$

region w/ No currents

$$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

Take curl of Both sides

$$\vec{\nabla}\times(\vec{\nabla}\times B) = \mu_0\epsilon_0\frac{\partial(\vec{\nabla}\times\vec{E})}{\partial t} = -\mu_0\epsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\,\cancelto{0}{(\vec{\nabla}\cdot\vec{B})} - \underbrace{\vec{\nabla}\cdot(\vec{\nabla}\vec{B})}_{\text{Known as Laplacian}} \qquad \nabla^2\vec{B}$$

in region w/ no currents . . .

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

mult by $\vec{\nabla} \times$ . . . , vector calc identities . . .

$$\nabla^2 \vec{B} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

3 scalar wave equations

$$\left[\begin{array}{l} \nabla^2 B_x = -\mu_0 \epsilon_0 \dfrac{\partial^2 B_x}{\partial t^2} \\ \nabla^2 B_y = -\mu_0 \epsilon_0 \dfrac{\partial^2 B_y}{\partial t^2} \\ \nabla^2 B_z = -\mu_0 \epsilon_0 \dfrac{\partial^2 B_z}{\partial t^2} \end{array}\right]$$

For example

$$\left[\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} + \frac{\partial^2 B_z}{\partial z^2} = -\mu_0 \epsilon_0 \frac{\partial^2 B_x}{\partial t^2}\right]$$

$$\nabla^2 \vec{B} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

similarly

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

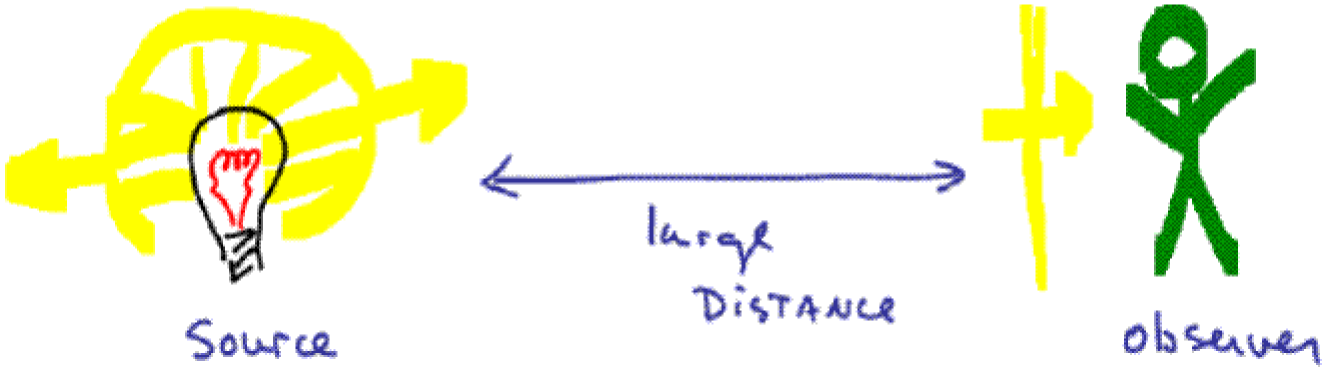
$$\Downarrow$$

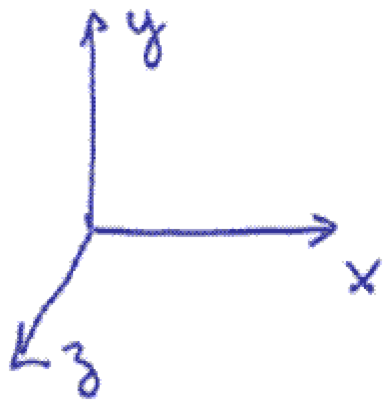
$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t}$$

Wave equations for $\vec{E}, \vec{B}$ w/ propagation velocity $\sqrt{\frac{1}{\mu_0 \epsilon_0}} = c$

Maxwell's eqns are coupled differential equations

Solve for particular Boundary Conditions





$\vec{E} = \vec{E}(x,t)$

Boundary Conditions

$\vec{\nabla}\cdot\vec{E}=0$

$$\frac{\partial E_x}{\partial x}+\underbrace{\cancel{\frac{\partial E_y}{\partial y}}}_{0}+\underbrace{\cancel{\frac{\partial E_z}{\partial z}}}_{0}=0$$

E_x const for all x $\rightarrow$ $E_x = 0$

E_y E_z might not be zero

E is transverse to direction of propagation

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Impose

$\vec{E}$ is along y direction

$\vec{E} = E(x,t)\,\hat{j}$

"polarization"

$$\cancelto{0}{\frac{\partial E_z}{\partial y}} - \cancelto{0}{\frac{\partial E_y}{\partial z}} = -\frac{\partial B_x}{\partial t} \Rightarrow B_x \text{ const}$$

$$-\left(\cancelto{0}{\frac{\partial E_z}{\partial x}} - \cancelto{0}{\frac{\partial E_x}{\partial z}}\right) = -\frac{\partial B_y}{\partial t} \Rightarrow B_y \text{ const}$$

$$\frac{\partial E_y}{\partial x} - \cancelto{0}{\frac{\partial E_x}{\partial y}} = -\frac{\partial B_z}{\partial t}$$

If E_y Δ'es w/ x
B_z Δ'es

} Time dependent B field $\perp$ to E field

$\vec{E}, \vec{B}$ are transverse, mutually $\perp$

$$E_y(x,t) = E_{oy}\cos(kx - \omega t + \varphi)$$

$\frac{2\pi}{\lambda}$ $\qquad$ $\frac{2\pi}{T}$ $\qquad$ phase (initial condition)

$$B_z = -\int \frac{\partial E_y}{\partial x}\,dt$$

Assume zero for now

$\phi = 0$

$$B_z = \int k E_{oy} \sin(kx - \omega t)\,dt$$

$$B_z = \frac{k}{\omega} E_{oy} \cos(kx - \omega t)$$

$$\boxed{B_z = \frac{1}{c} E_y}$$

$$\frac{k}{\omega} = \frac{\frac{2\pi}{\lambda}}{2\pi/T} = \frac{T}{\lambda} = \frac{1}{c}$$

$\boxed{\frac{\lambda}{T} = \text{wave speed}}$

E, B coupled and time dependent

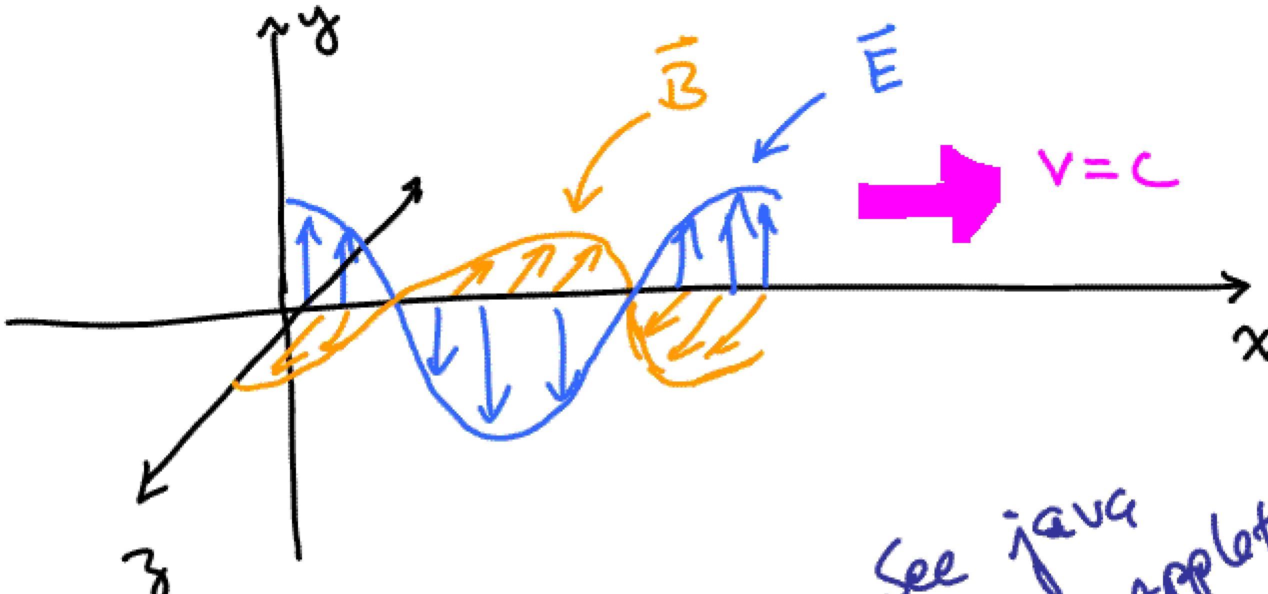
E, B $\perp$ to each other

$\vec{E}, \vec{B}$ in phase

$$E_y(x,t) = E_{0y} \cos(kx - \omega t + \varphi)$$

$$B_z(x,t) = \frac{k}{\omega} E_{0y} \cos(kx - \omega t + \varphi) = \frac{1}{c} E_y$$

$$|\vec{E}| = c|\vec{B}|$$



See java applet !!