Physics 142 - November 6, 2007

- Exam 2 on Thursday
  - Q+A Session B+L 106
    Wednesday 3:30-5:00
  - checking about running it from 4-5:30 instead
- Presentation groups
  - Elect Spokesperson
  - Each group to meet briefly w/ me
    before Thanksgiving break.

Induction

Last Time

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_M}{dt}$$

Induced EMF

$$\phi_M \equiv \int_{loop} \vec{B} \cdot d\vec{A}$$

Faraday's Law

Lenz's Law – An induced current in a closed conducting loop will appear in such a way as to oppose the change that created it.

Gives direction of induced effect

$$\phi_m = L\,i$$

$L \equiv$ constant of self-inductance

$$\mathcal{E} = -\frac{d\phi_m}{dt} = -L\frac{di}{dt}$$

$$\boxed{\mathcal{E} = -L\frac{di}{dt}}$$

unless problem is specifically about mutual inductance ... usually treat inductance in a circuit as self-inductance

$$\phi_{m_{(2)}} = M\,i_{(1)}$$

$$\phi_{m_{(1)}} = M\,i_{(2)}$$

SAME "M"
cross-talk dictated by geometry

$M \equiv$ constant of mutual inductance

$$\mathcal{E}_{(2)\text{ by }(1)} = -M\frac{di_{(1)}}{dt}$$

$$\mathcal{E}_{(1)\text{ by }(2)} = -M\frac{di_{(2)}}{dt}$$

Energy density in the fields:

$$\boxed{u_E = \frac{\epsilon_0 E^2}{2}} \quad \boxed{u_B = \frac{B^2}{2\mu_0}}$$

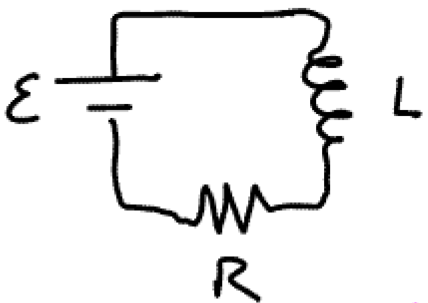
general — Says nothing about circuits or sources or boundary conditions!

Energy in Inductor

$$U = \frac{1}{2} L I^2$$

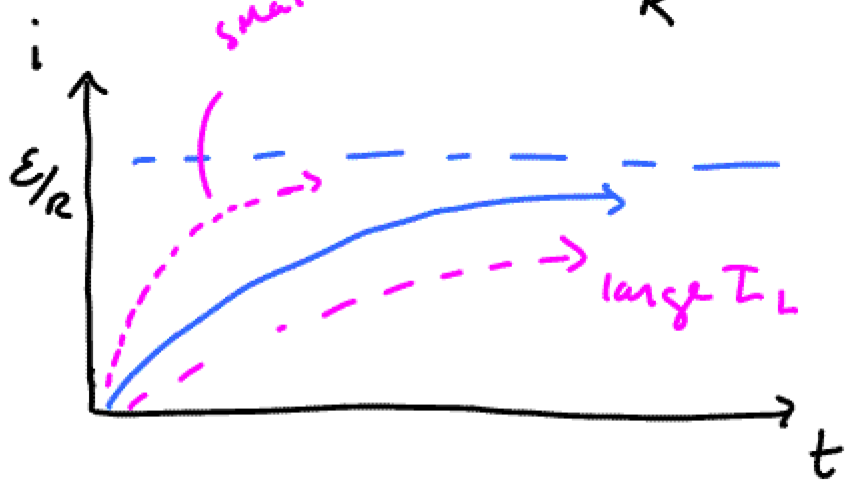
similar to $U_{capacitor} = \frac{1}{2} C V^2$

LR circuit



$$\mathcal{E} - L\frac{di}{dt} - iR = 0$$

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-tR/L}\right) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)$$

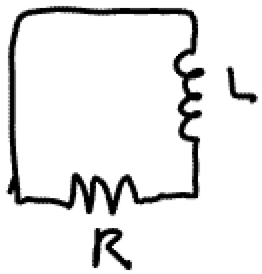


Inductive time constant $\equiv \frac{L}{R} = \tau_L$

$V = iR$ for V across R

$$V = \mathcal{E}\left(1 - e^{-tR/L}\right) = \mathcal{E}\left(1 - e^{-t/\tau_L}\right)$$

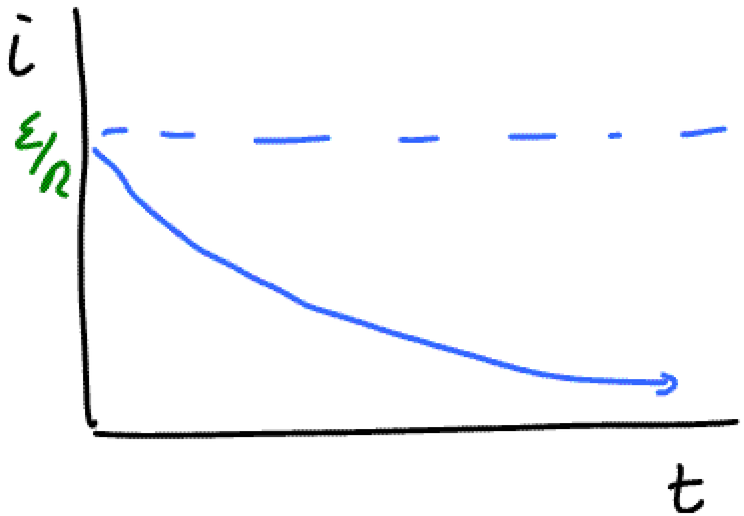
Remove EMF, Short L across R



$$0 = iR + L\frac{di}{dt}$$

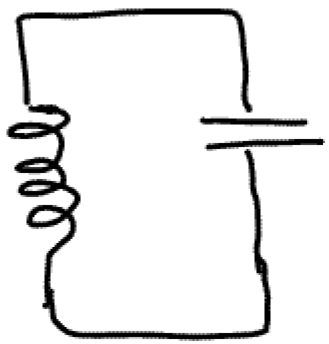
After i Reaches value of $\mathcal{E}/R$

$$i = \frac{\mathcal{E}}{R} e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$$



LC circuit

$$U = U_B + U_E = \frac{1}{2}Li^2 + \frac{q^2}{2C}$$



R in circuit $\longrightarrow 0$

SHM

$$m\frac{d^2x}{dt^2} = -kx$$

same form

$$\frac{dU}{dt} = 0 = \frac{1}{2}L\,2i\frac{di}{dt} + \frac{2q}{2C}\frac{dq}{dt}$$

$$i = \frac{dq}{dt}$$

$$0 = L\frac{di}{dt} + \frac{q}{C}$$

$$\underbrace{0 = L\frac{d^2q}{dt^2} + \frac{q}{C}}_{\text{diff eqn}}$$

$$q(t) = Q\cos(\omega t + \varphi)$$

$$\omega = \frac{1}{\sqrt{LC}}$$

HARMONIC

Energy Flow

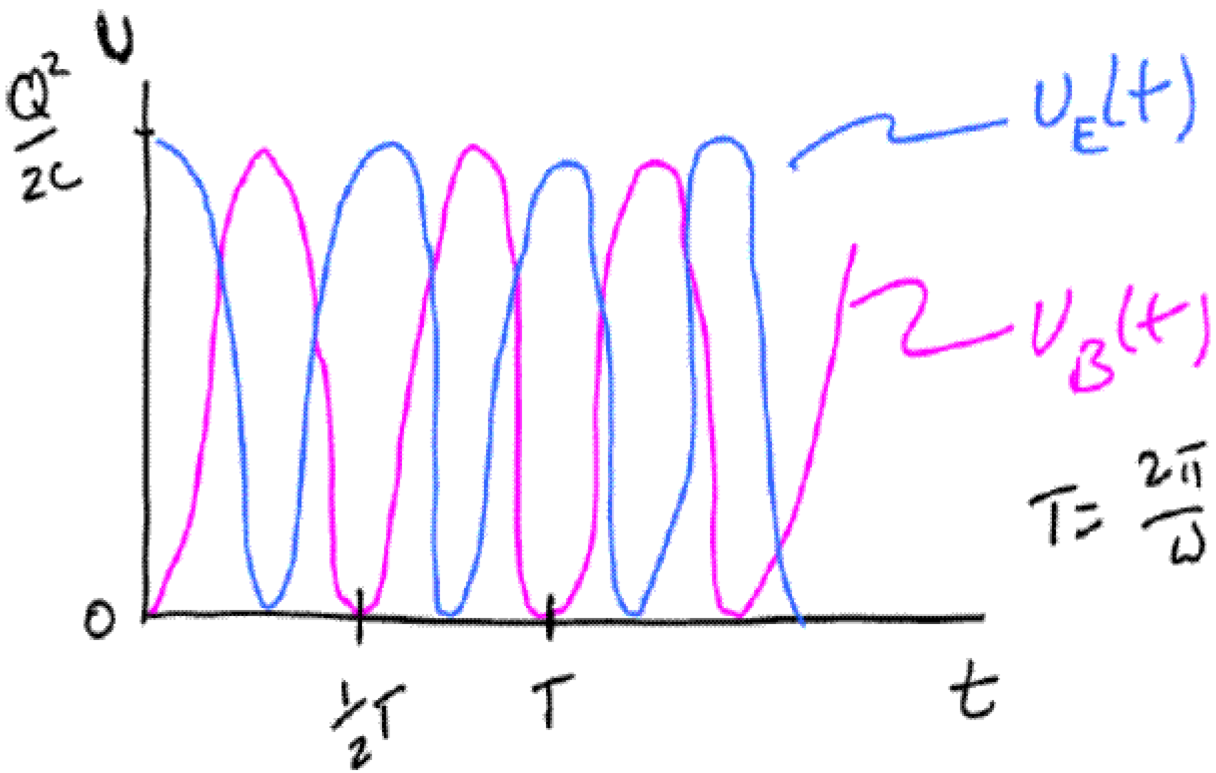
$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \varphi)$$

Energy in capacitor or E field

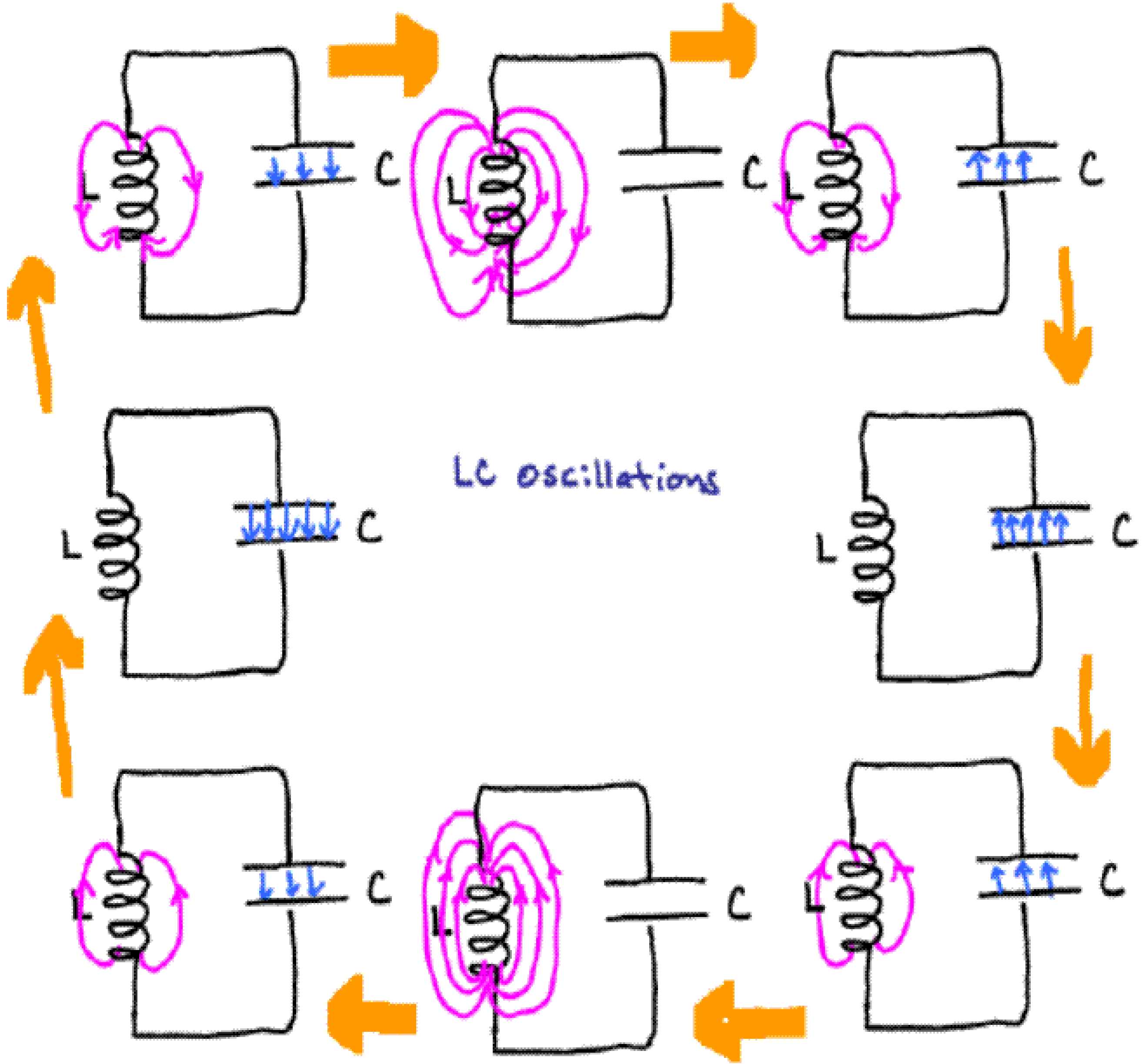
$$i(t) = \frac{dq(t)}{dt} = -Q\omega \sin(\omega t + \varphi)$$

$$U_B = \frac{1}{2}Li^2 = \frac{L}{2}Q^2\omega^2\sin^2(\omega t + \varphi)$$

LC Oscillations

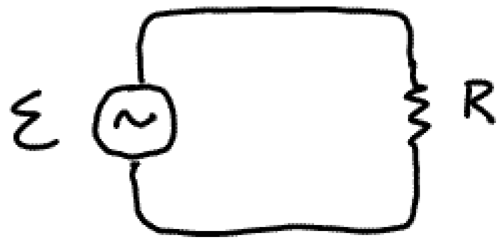


$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$$


LC oscillations
L
C

AC Circuits

$$\mathcal{E} = \mathcal{E}_{max} \sin \omega t$$



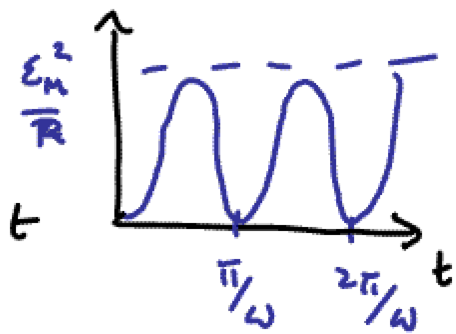
Kirchoff $\quad \mathcal{E} - IR = 0$

$$I = \frac{\mathcal{E}}{R} = \frac{\mathcal{E}_{max} \sin \omega t}{R}$$

I is inphase with $\mathcal{E}$

Power (instantaneous)

$$P = IV = I\mathcal{E} = \frac{\mathcal{E}_{max}^2}{R} \sin^2 \omega t$$



Average Power

$$\langle \sin^2 \rangle \sim \frac{1}{2}$$

$$\bar{P} = \frac{\mathcal{E}_{max}^2}{2R}$$