Physics 142 – October 18, 2007

- Presentation Topic preferences
- Will make groups + post on web soon
- Next week — class in your Jammies

Last Time

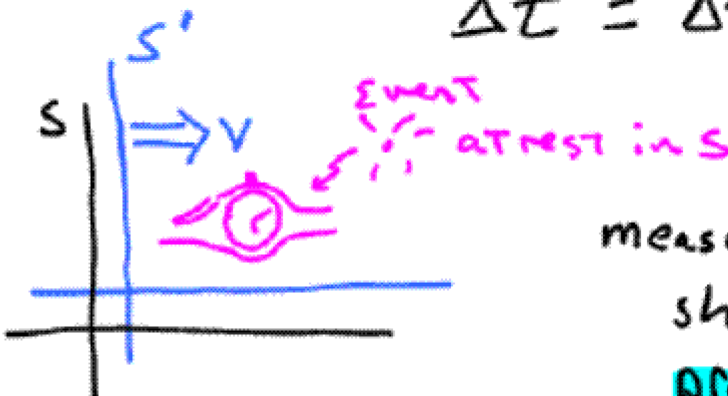
Special Theory of Relativity

only valid for inertial frames of reference

Assume

- Speed of light, c, constant for all observers in all inertial reference frames
- "Physics" invariant

Time dilation



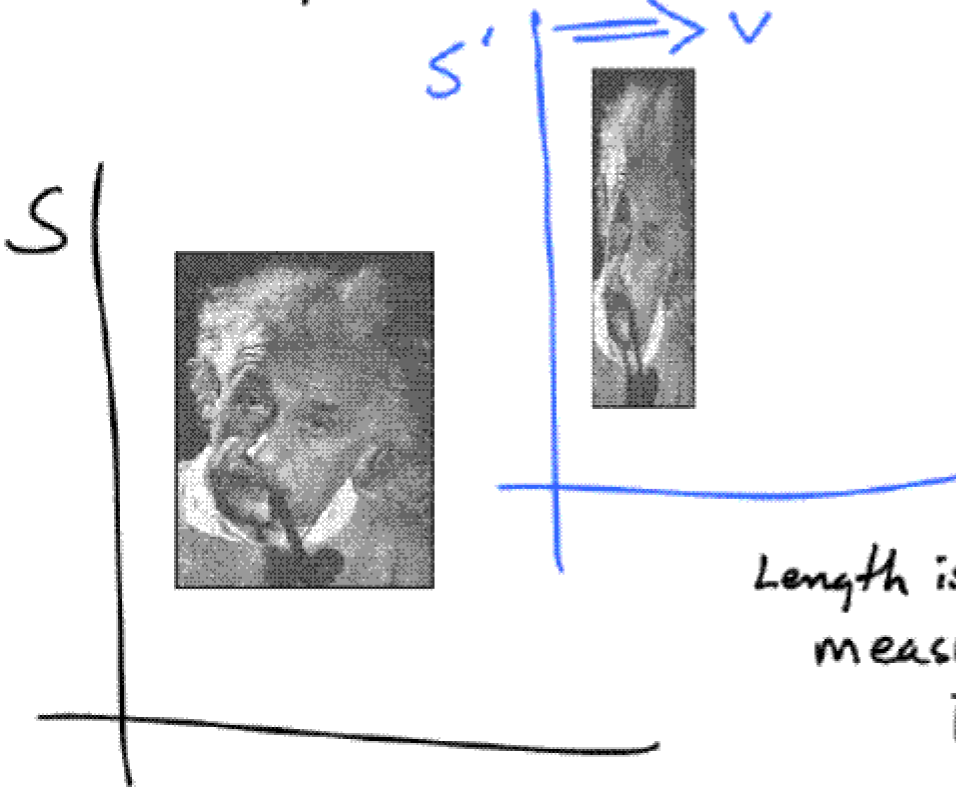
$$\Delta t' = \Delta t \, \gamma$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} > 1$$

measured time is shortest in proper frame of reference

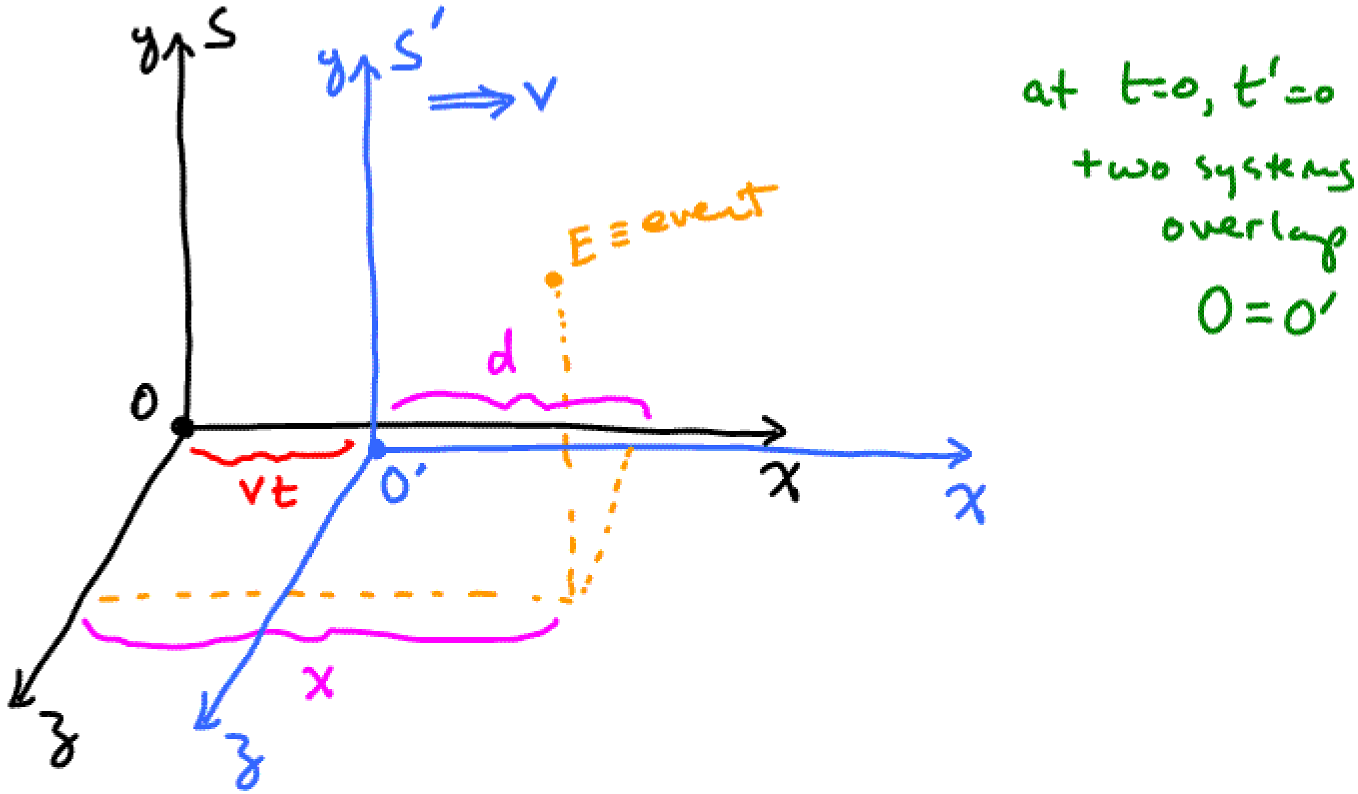
$\boxed{\text{frame where event is at rest}}$

Length Contraction



$$\Delta x' = \frac{\Delta x}{\gamma}$$

Length is greatest when measured in Proper frame of Reference



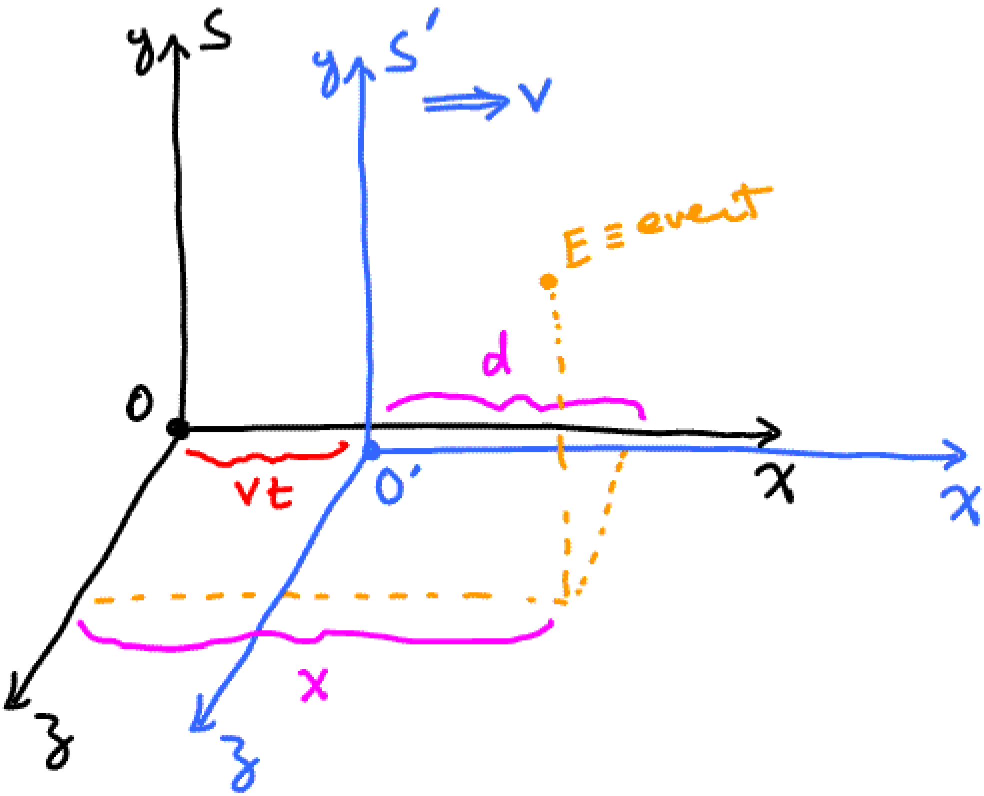
Classical physics (Galilean transformations)

$$x' = d = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$



Relativistic Transf.

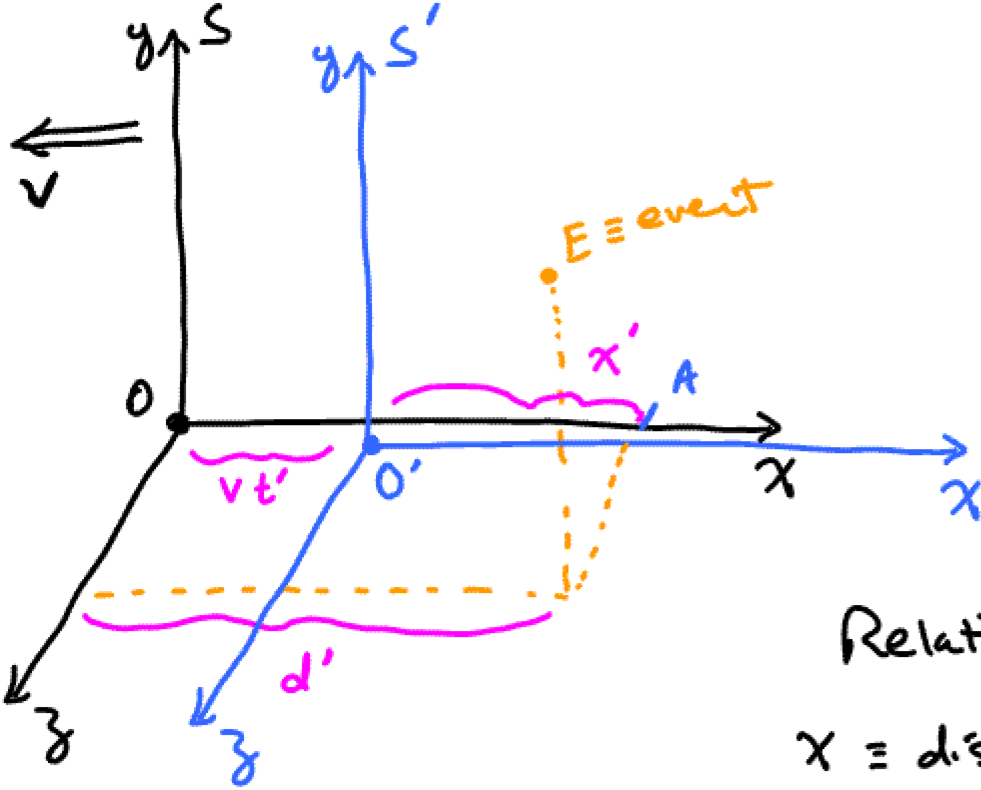
$$d = \frac{x'}{\gamma}$$

$$x' = d\gamma$$

$$d = x - vt$$

$$\frac{x'}{\gamma} = x - vt$$

$$x' = \gamma(x - vt)$$



Galil. trans.

$$x' = d' - vt'$$

$$d' = x \qquad t' = t$$

$$x' = x - vt$$

Relativistically

$x \equiv$ dist from O to A in S

$$d' = \frac{x}{\gamma}$$

$$x' = d' - vt'$$

$$x' = \frac{x}{\gamma} - vt'$$

$$x = \gamma(x' + vt')$$

$$x = \gamma(x' + vt') \qquad x' = \gamma(x - vt)$$

sub in

$$x = \gamma\left[\gamma(x - vt) + vt'\right]$$

bit of Algebra

$$t = \gamma\left(t' - \frac{v}{c^2}x'\right) \qquad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

Lorentz Transformations

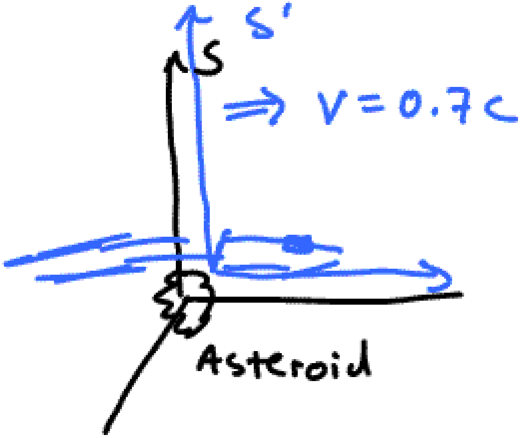
$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$V \to$ small $\Rightarrow \gamma = 1$ $\quad \frac{v}{c} \to 0$ get Gal. Trans.



$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = 1.4$$

Event 1 $\quad t=0,\ t'=0$ origins coincide

Spaceship passes Asteroid

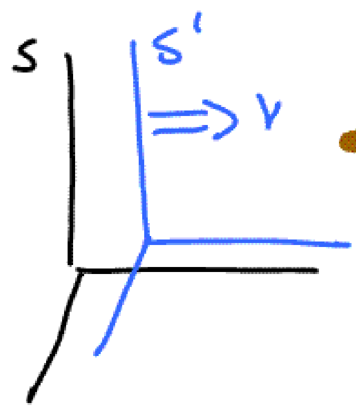
Event 2 $\quad$ Laser flashes at $\underbrace{x_2 = 3\text{ km},\ t_2 = 5\,\mu s}_{\text{in } S}$

what does event 2 look like in frame S'?

$$x_2' = \gamma(x_2 - vt_2) = (1.4)\left[3 - .7\,\overset{\text{km/s}}{(3\times10^{5})}\,5\times10^{-6}\,\text{s}\right] = 2.73\text{ km}$$

$$t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right) = 1.4\left[5\times10^{-6} - (.7)\ 3\right] = -2.8\,\mu s$$

Velocity Transformations



$\vec{u} \equiv$ velocity of object in S

in S $\longrightarrow$ $u_x = \dfrac{dx}{dt}$

$u_y = dy/dt$

$u_z = dz/dt$

in S'

$$u_x' = \frac{dx'}{dt'} = \frac{\gamma(dx - v\,dt)}{\gamma(dt - \frac{v}{c^2}dx)} = \frac{\gamma(\overbrace{\tfrac{dx}{dt}}^{u_x} - v)}{\gamma(1 - \frac{v}{c^2}\underbrace{\tfrac{dx}{dt}}_{u_x})}$$

$$\boxed{u_x' = \frac{u_x - v}{1 - \frac{v}{c^2}u_x}}$$

$$u_y' = \frac{dy'}{dt'} = \frac{dy}{\gamma(dt - \frac{v}{c^2}dx)} = \boxed{\frac{u_y}{\gamma(1 - \frac{u_x v}{c^2})}}$$

$$\boxed{u_z' = \frac{u_z}{\gamma\left(1 - \frac{u_x v}{c^2}\right)}}$$

Suppose flying to LA

Pilot says speed is $5/8\, c$

$$\hookrightarrow \quad \frac{dx}{dt} = v \quad \text{speed on ground}$$

your proper time $\tau = \tau$

$$\eta = \frac{dx}{d\tau}$$

— meas on gnd

← Mes on plane

Proper velocity

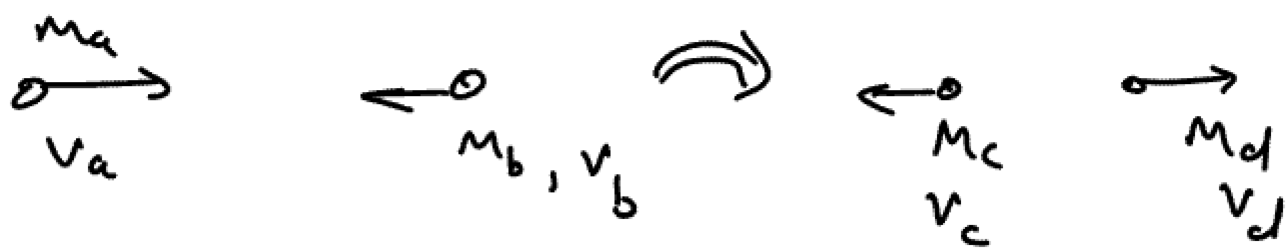
Velocity relevant if you have to decide whether to eat on plane

$$\eta = \frac{dx}{d\tau}$$ transform like dx

$$dt = \gamma \, d\tau$$

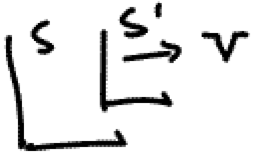
$$\eta = \frac{dx}{d\tau} = \gamma \frac{dx}{dt} = \gamma v$$ proper velocity

Momentum cons. should hold relativistically



Newton $m_a v_a + m_b v_b = m_c v_c + m_d v_d$

$v_a \rightarrow$



$$v_a \longrightarrow \frac{\gamma(v_a - V)}{\gamma\left(1 - \frac{V}{c^2} v_a\right)}$$

use $p \equiv m\eta \longrightarrow$ Momentum cons holds

if

$P_x, P_y, P_z, (?)$ define $4^{\underline{th}}$ entity

x, y, z, t

$$E = \gamma m c^2 = mc^2\eta$$

$\equiv$ Relativistic energy

$$E = \frac{mc^2}{\sqrt{1 + (\frac{v}{c})^2}}$$

$$E = mc^2\left(1 - (\tfrac{v}{c})^2\right)^{-1/2}$$

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 \cdots$$

$$E = mc^2 + \frac{1}{2}mv^2 + \text{h.o.t.}$$

Rel energy

const

KE

particle not moving

$$E = mc^2$$