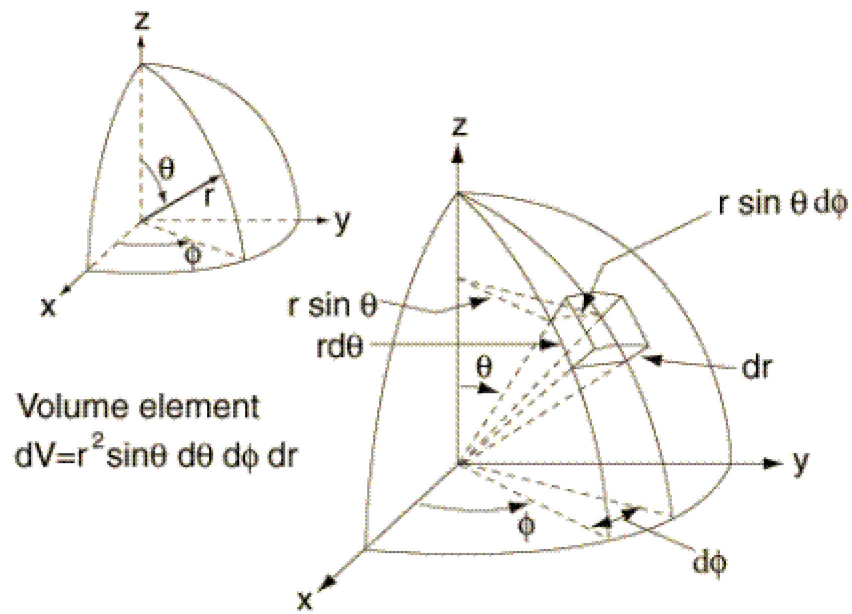
Physics 142 - September 20, 2007

Last Time —

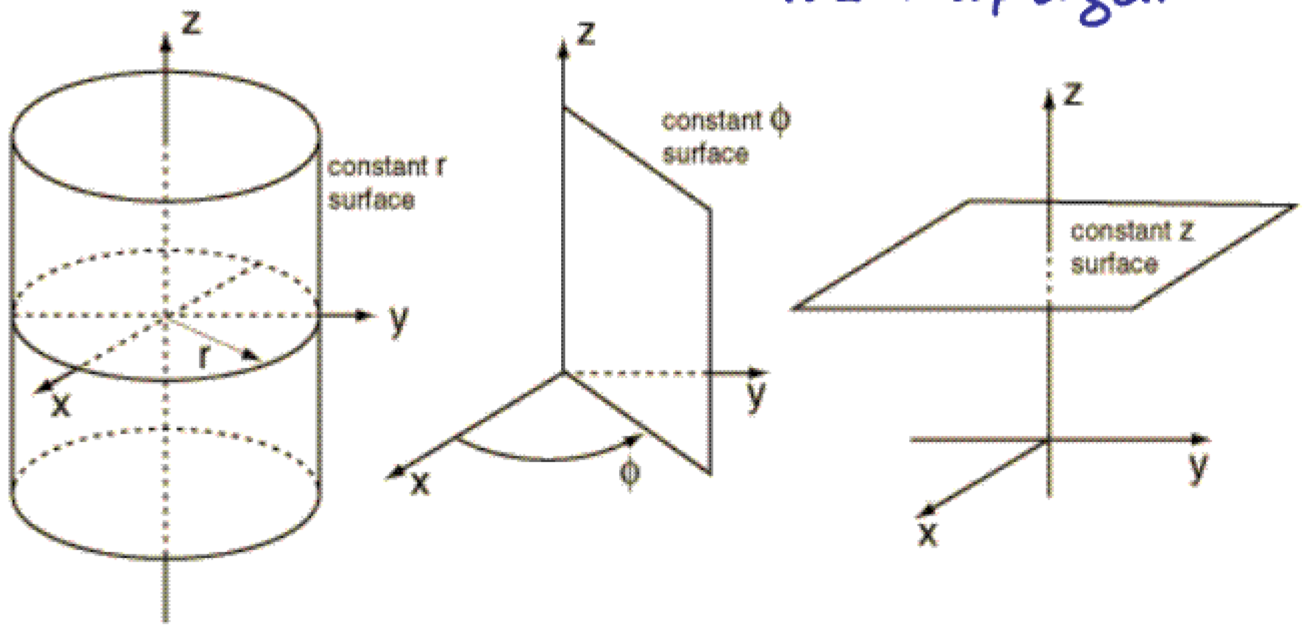
Gauss' Law $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$

Always True ... Most useful under certain conditions of symmetry

Curvilinear Coordinates



$dv = r\,d\phi\,dz\,dr$

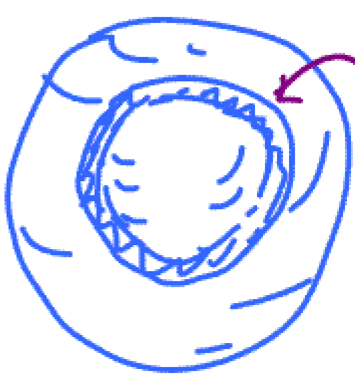


I'll limit us to problems in 1 variable

Usually that means radial dependence

Effectively integrates out Angular dependence

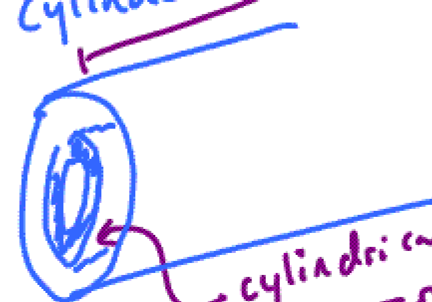
sphere



Shell

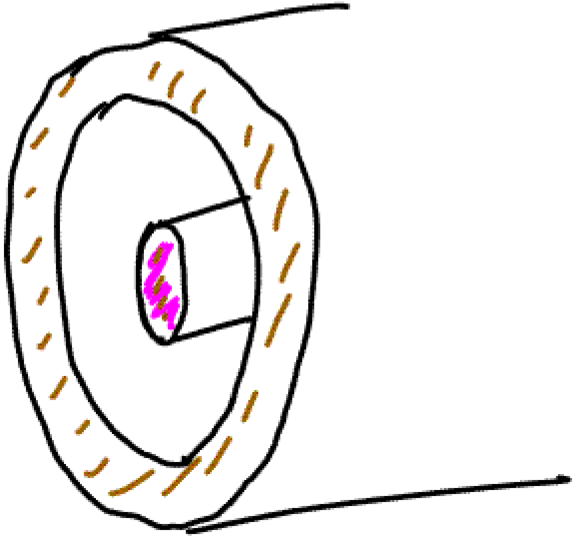
$dv = 4\pi r^2 dr$

cylinder l



cylindrical shell

$dv = 2\pi r l\,dr$



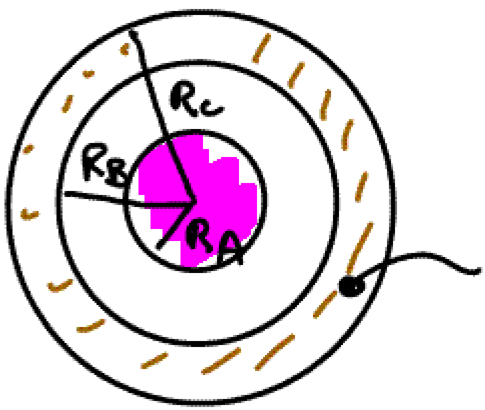
noncondutcting core

radius R_A

has $+\lambda$

distributed

$\rho(r) = ar \quad r < R_A$

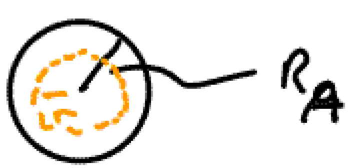


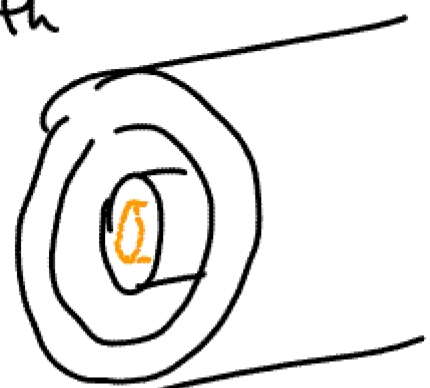
Conductor

Sheath

Find $\vec{E}$ in all space

$r < R_a$





$\vec{E}$ radially outward by symmetry

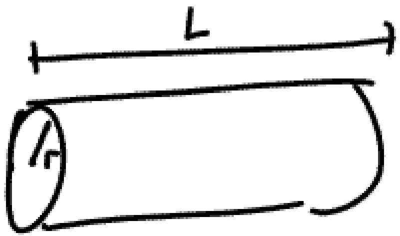
$dv = 2\pi r L\,dr$

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

endcaps do Not contribute $\quad \vec{E} \perp d\vec{A}$

$$|\vec{E}| \int dA = |\vec{E}|\, 2\pi r L$$

Pipe shell



$A = 2\pi r L$

Q_{encl} ?

$$\int \rho\, dv$$

$$= \int_0^r a r\, 2\pi r L\, dr$$

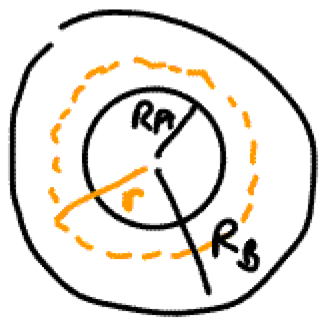
$$= a 2\pi L \int_0^r r^2 dr = \frac{a 2\pi L r^3}{3}$$

$$Q_{encl} = \frac{a 2\pi L r^3}{3}$$

$$|\vec{E}|\, 2\pi r L = \frac{a\, 2\pi L\, r^3}{3\epsilon_0}$$

$$\vec{E} = \frac{a r^2}{3\epsilon_0} \quad \text{radially out} \quad r < R_A$$

$R_A < r < R_B$



$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$|\vec{E}|\, 2\pi r L = \frac{1}{\epsilon_0} \frac{a\, 2\pi L\, R_A^3}{3}$$

$R_A < r < R_B$

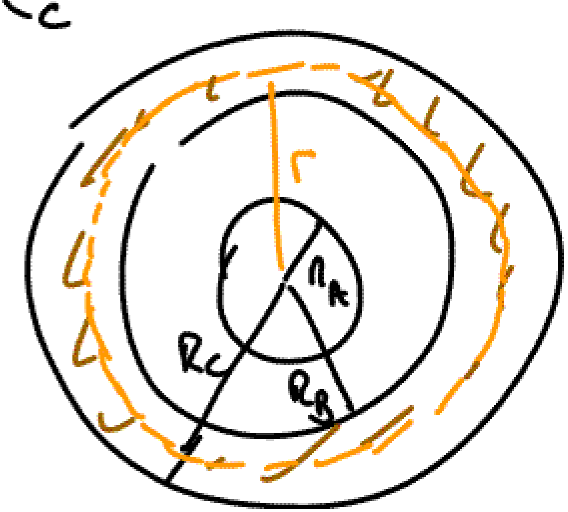
$$\vec{E} = \frac{a}{\epsilon_0 3} \frac{R_A^3}{r}$$

radially outward

$r < R_A$

$$\vec{E} = \frac{a r^2}{3\epsilon_0}$$

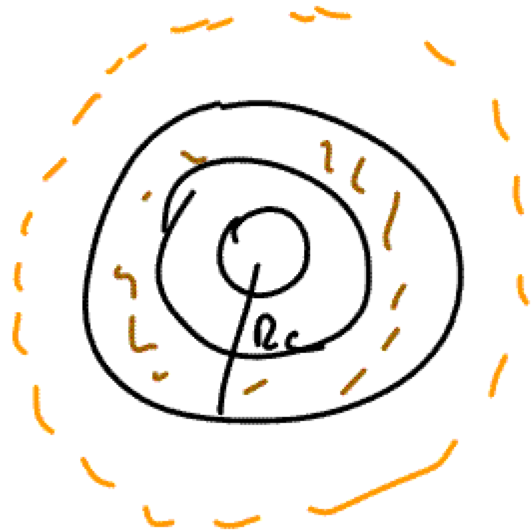
$R_B < r < R_c$



$\vec{E} = 0$

because region inside conductor

$r > R_c$



Ans. is

same as in 2^{ND} region $(R_A < r < R_B)$

because symmetry And Q_{encl}

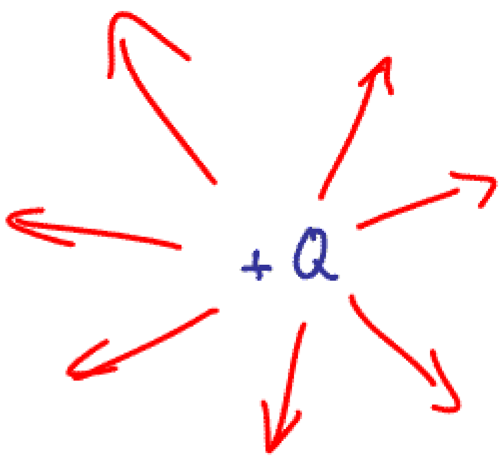
are the same

$r > R_c$

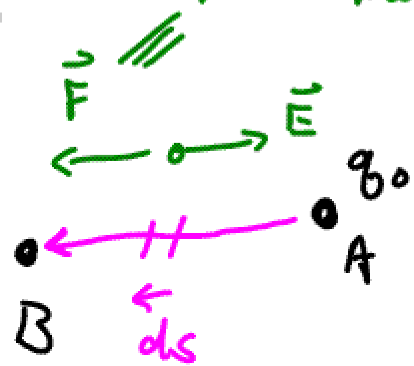
$$\vec{E} = \frac{\alpha}{\epsilon_0 3} \frac{R_A^3}{r} \quad \text{radially outward}$$

Recall how useful Energy considerations are for Mechanics

Electric field + Energy



Force I exert to move q_0 from A to B



How much work do I do to do this?

$$W = \int_A^B \vec{F}\cdot d\vec{s} = \int_A^B F\,ds = \int_A^B q_0 E\,ds = -\int_{R_A}^{R_B} q_0 E\,dr$$

$$= -q_0 \int_{R_A}^{R_B} \frac{kQ}{r^2}\, dr = -q_0 kQ \left[-\frac{1}{r}\right]_{R_A}^{R_B}$$

$$= q_0 kQ \left[\frac{1}{R_B} - \frac{1}{R_A}\right]$$ Net $\oplus$ quantity

$$\Delta V \equiv \frac{W}{q_0} \equiv \text{potential difference} \qquad \frac{\text{work}}{\text{charge}}$$

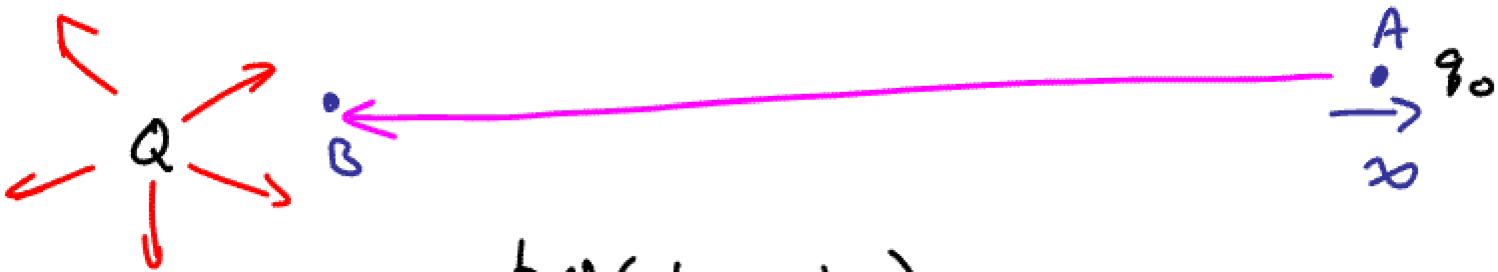
$$\equiv -\frac{\Delta U}{q_0}$$

$U \equiv$ potential energy of system

$$\Delta V \equiv V_B - V_A \equiv V_{AB}$$

$$\text{unit} = \frac{\text{Joules}}{\text{Coulomb}} \equiv \text{Volt}$$

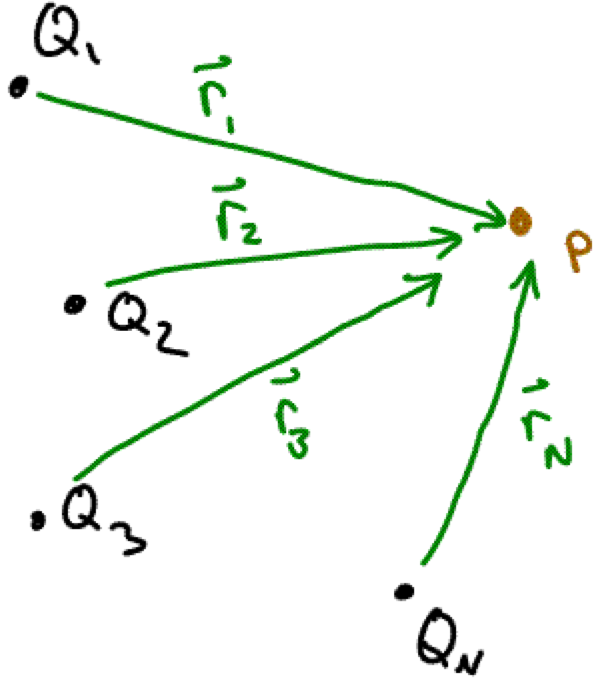
define potential at $\infty \longrightarrow 0$



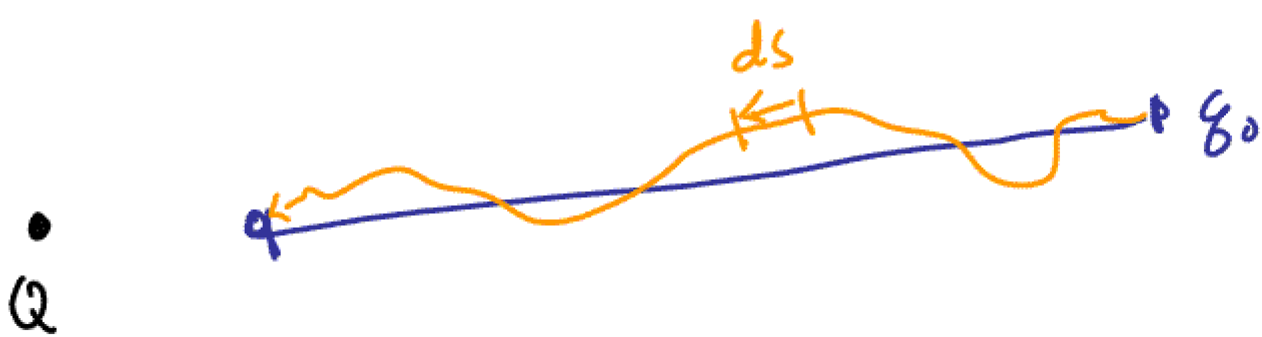
$$\frac{W}{q_0} = kQ\left(\frac{1}{R_B} - \cancel{\frac{1}{R_A}}\right)$$

0 at $R_A \to \infty$

$$\boxed{V(r) = \frac{kQ}{r} \quad \text{potential of a point charge}}$$



$$V_P = \sum_N V_i = \sum_{i=1}^{N} \frac{kQ_i}{r_i}$$



Electrostatics is a conservative force

Potential diffs are Path Independent