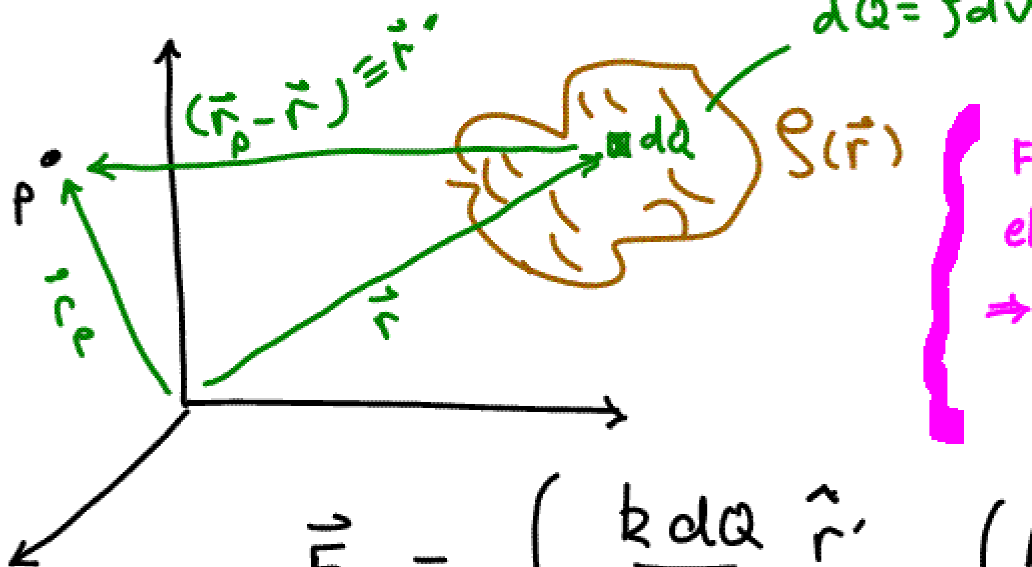
Physics 142 - September 18, 2007

- Lecture 11:05 - 12:20

Last Time



Finding the electric field
→ Brute Force

$$\vec{E}_p = \int_{\substack{\text{charge}\\\text{dist}}} \frac{k\, dQ}{r'^2}\,\hat{r}' = \int_{\substack{\text{volume}\\\text{(of charge dist)}}} \frac{k\,\rho(\vec{r})\,dv}{r'^2}\,\hat{r}'$$

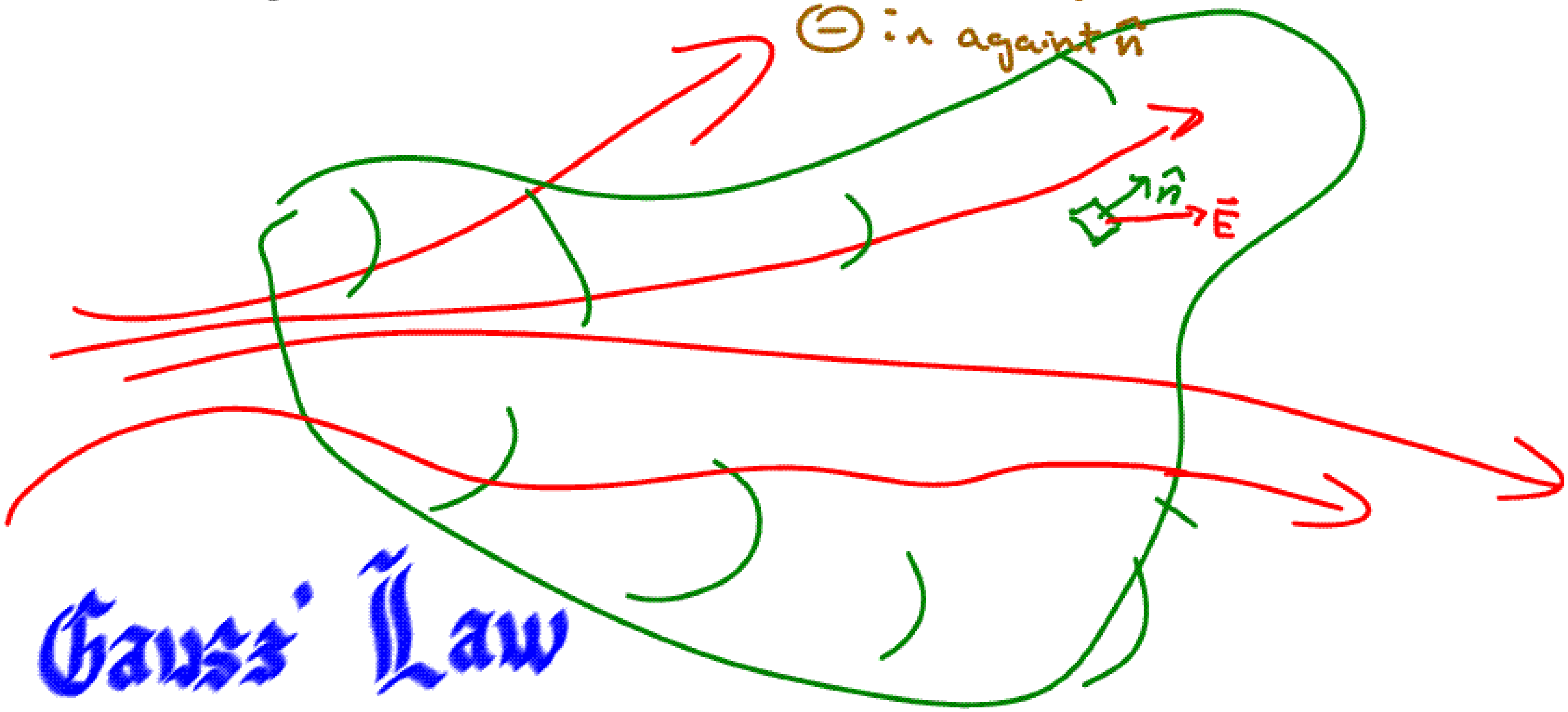
This can be hard ... find easier way to determine $\vec{E}$

$$\phi = \oint \vec{E} \cdot \hat{n} \, dA$$

Sum $\vec{E}_{\parallel}$ to $\hat{n}$ over surface

⊕ if along $\hat{n}$

⊖ if against $\hat{n}$



Gauss' Law

$$\boxed{\phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}}$$

Enclosed by the surface

Gauss' Law true in general ... most useful under selected conditions of symmetry

integral over volume inside gaussian surface (NOT necessarily all charge)

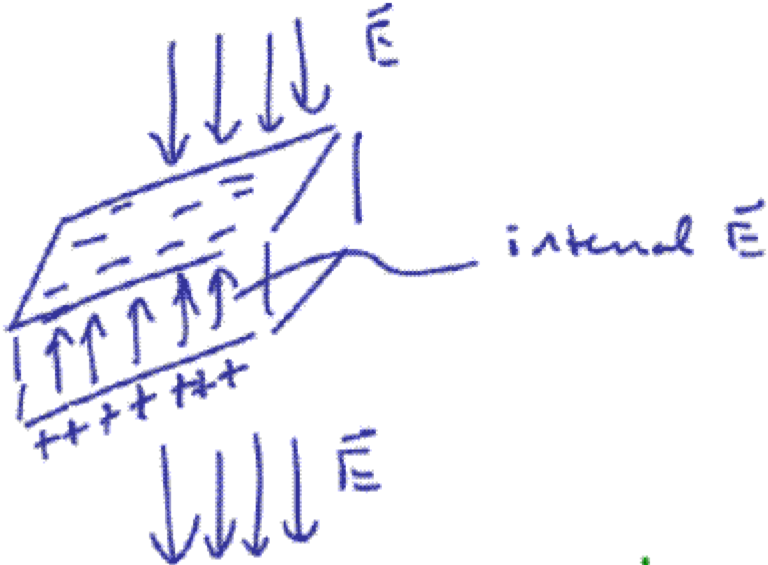
$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\int_V \rho \, dv$$

Easy if $|\vec{E}|$ is constant on surface - can pull out of integral

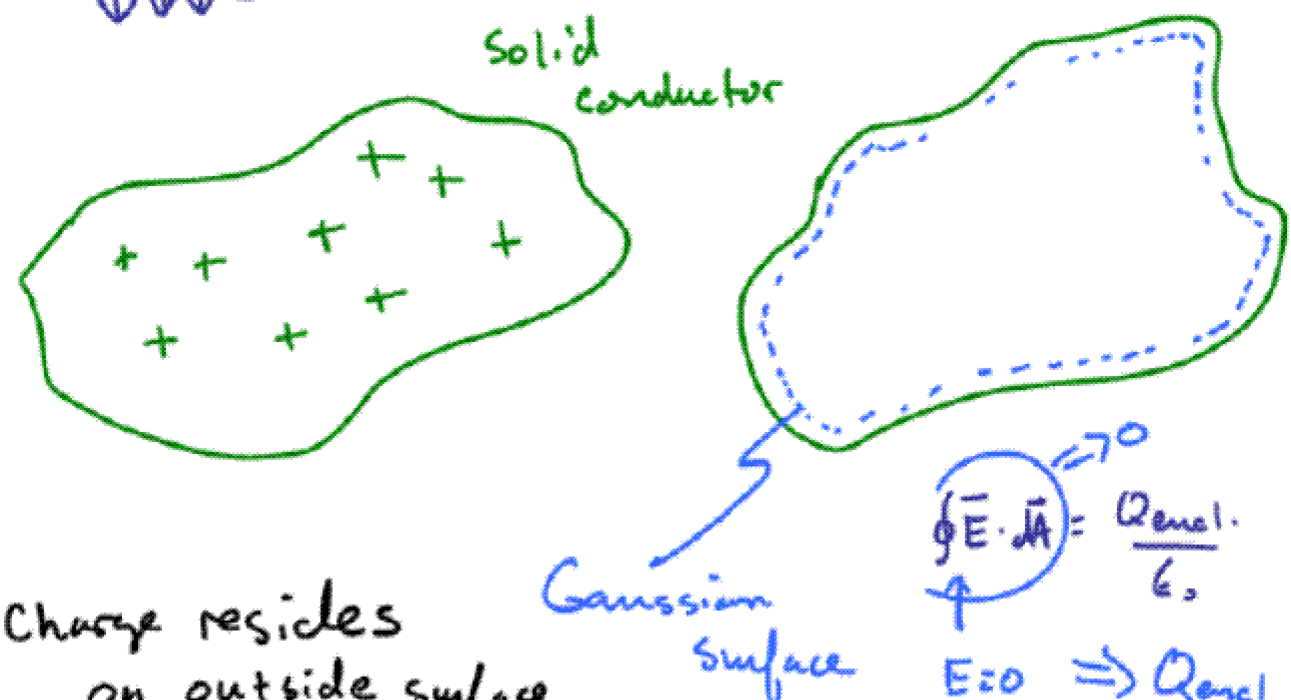
Easy if $\vec{E} \perp d\vec{A}$ or $\vec{E} \parallel d\vec{A}$

integral over Gaussian surface



$\vec{E} = 0$ inside a conductor

Important

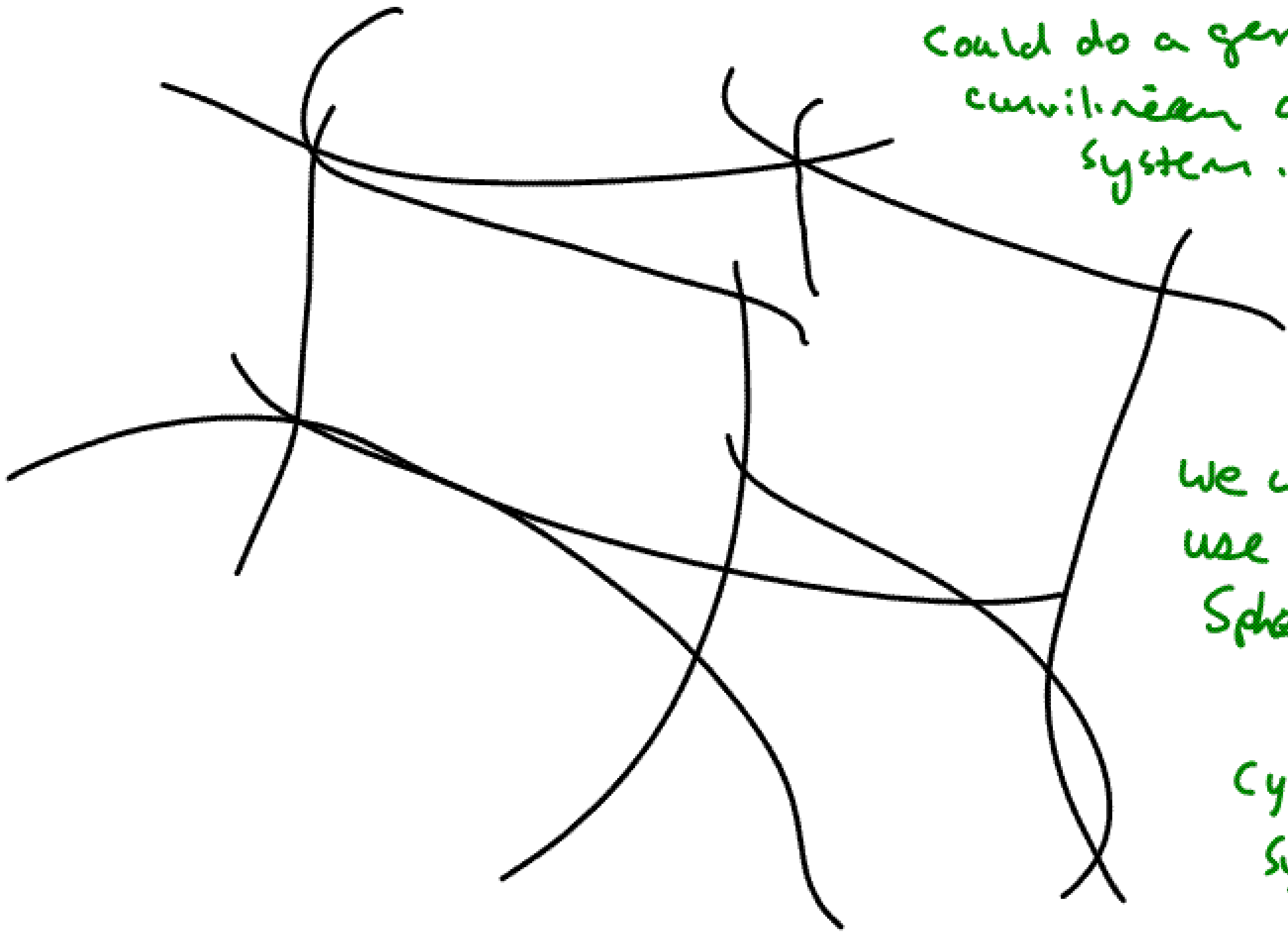


$$\oint \underbrace{\vec{E}}_{E=0} \cdot d\vec{A} = \frac{Q_{encl.}}{\epsilon_0} \Rightarrow 0$$

$E = 0 \Rightarrow Q_{encl} = 0$

Charge resides on outside surface of conductor

Curvilinear Coordinate Systems



Could do a generalized curvilinear coord. system . . . .

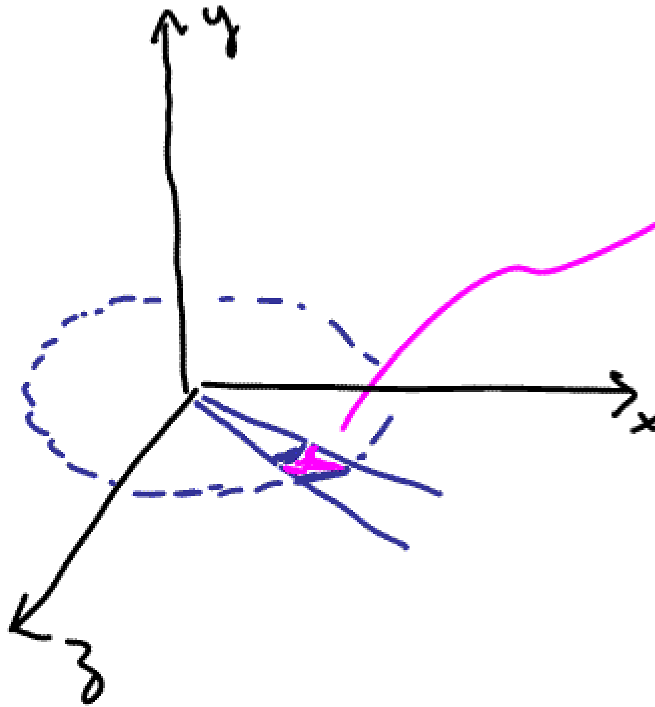
we will only use Spherical Polar + Cylindrical Systems

$s = r\theta$

$ds = r\,d\theta$

Find Area of disk

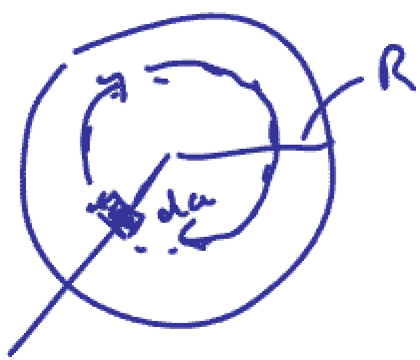
Integrate dA around θ and then along r → double integral.

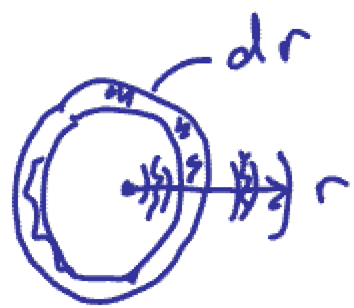


$$dA = r\,d\theta\,dr$$

$$\int_0^R \int_0^{2\pi} dA =$$

$$\int_0^R \int_0^{2\pi} r\,d\theta\,dr =$$

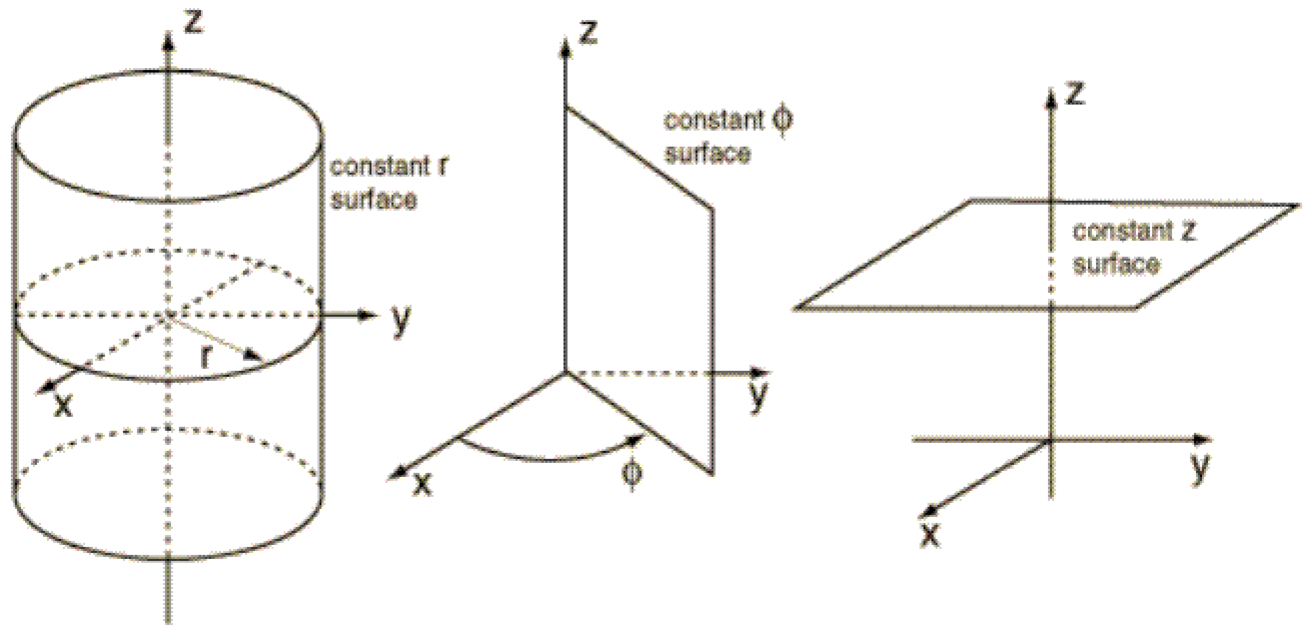




$$\int_0^R r\,2\pi\,dr =$$

$$2\pi \frac{R^2}{2} = \pi R^2$$

cylindrical coordinates

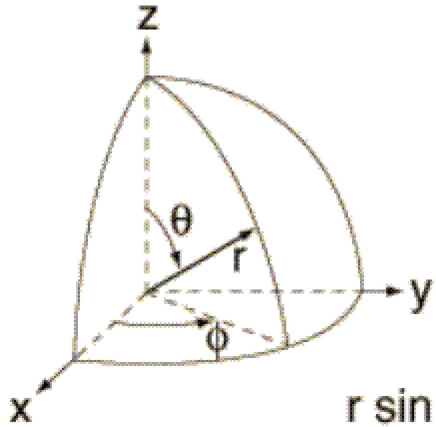


Some figures in this section from:

http://hyperphysics.phy-astr.gsu.edu/hbase/sphc.html

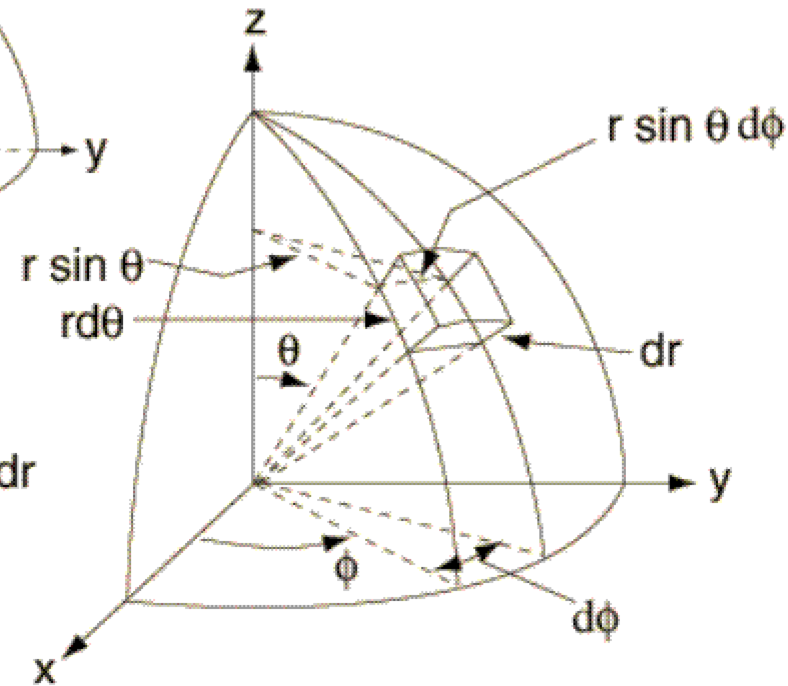
Also Griffiths, Intro to Electromagnetism

Spherical polar coordinates

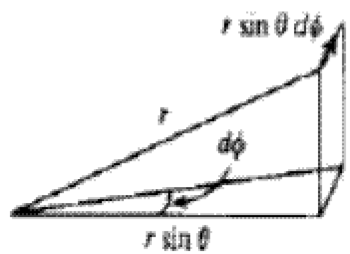


Volume element

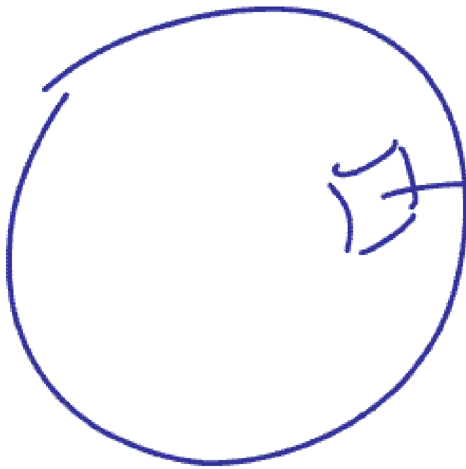
$dV = r^2 \sin\theta \, d\theta \, d\phi \, dr$



$$dl_\phi = r \sin\theta \, d\phi.$$

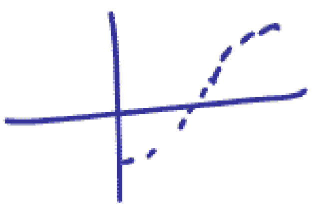


Sphere Area $= \int_0^{\pi}\int_0^{2\pi} dA = \int_0^{\pi}\int_0^{2\pi} r^2 \sin\theta \, d\phi \, d\theta$



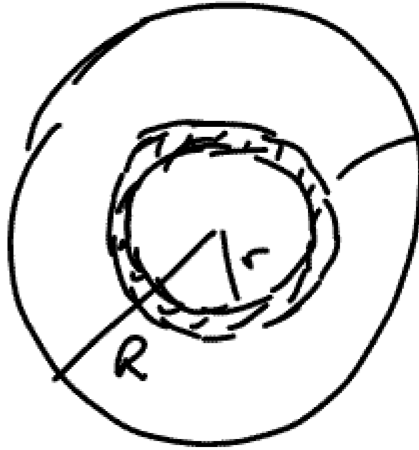
$dA = r^2 \sin\theta \, d\theta \, d\phi$

$$\int_0^{\pi} r^2 \sin\theta \underbrace{\int_0^{2\pi} d\phi}_{2\pi} \, d\theta$$



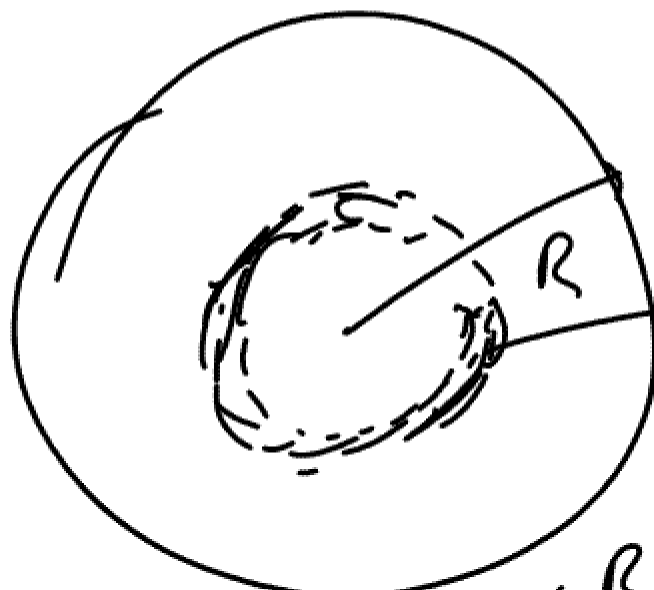
$$2\pi r^2 \int_0^{\pi} \sin\theta \, d\theta = 4\pi r^2$$

Griffiths — Intro. to Electrodynamics



$dA = 2\pi r \, dr$

$$\int dA = \int_0^R 2\pi r \, dr$$
$$= \pi R^2$$



Spherical shell

$dv = 4\pi r^2 dr$

$$\int_0^R dv = \int_0^R 4\pi r^2 dr = 4\pi \frac{R^3}{3}$$

$\rho = kr^3$ for $r < R$

$= 0$ for $r \geq R$

TOTAL charge

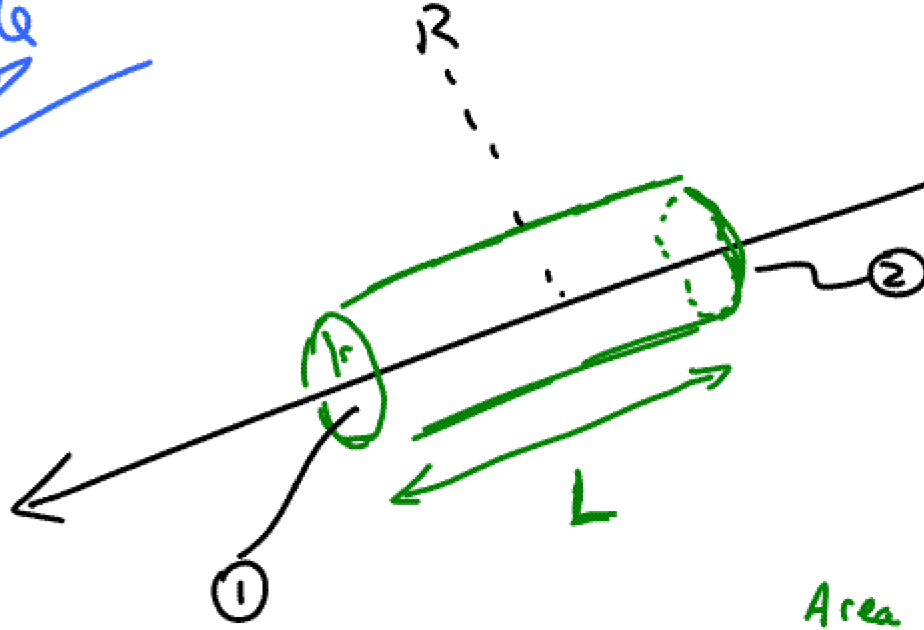
$$\int \rho \, dv = \int_0^R \rho \, 4\pi r^2 \, dr$$

$$= \int_0^R k \, r^3 \, 4\pi r^2 \, dr$$

$$= 4\pi k \frac{R^6}{6}$$

Example

$+\lambda = \text{chg}/\text{length}$



∞ line charge

Find Field at point R

Area of gaussian surface
$= 2\pi r L + (2)\pi r^2$

Cylindrical symmetry
$\vec{E}$ radially out

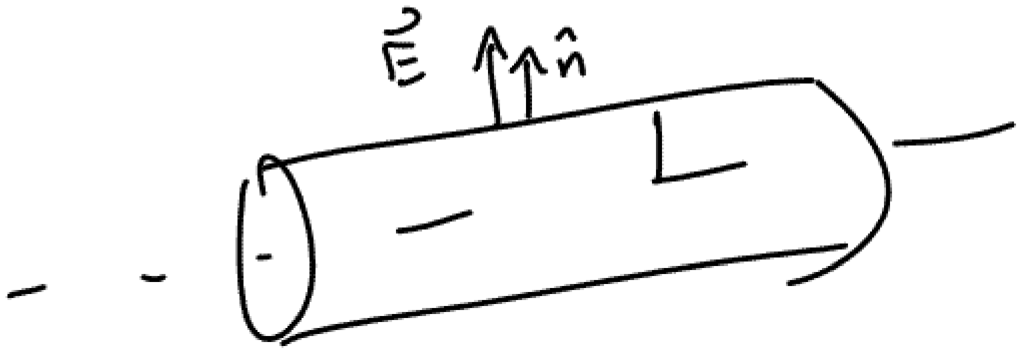
$$\oint \vec{E}\cdot\hat{n}\,dA = \frac{Q_{enc}}{\epsilon_0}$$

$L\lambda$

Endcaps don't contribute

①

$\vec{E}\cdot\hat{n} = 0$



$$\oint \vec{E}\cdot\hat{n}\, dA = |E|\, 2\pi r L = \frac{L\lambda}{\epsilon_0}$$

$$|E| = \frac{\lambda}{\epsilon_0 2\pi r}$$

$\vec{E}$ Field a distance r
from an ∞ line charge