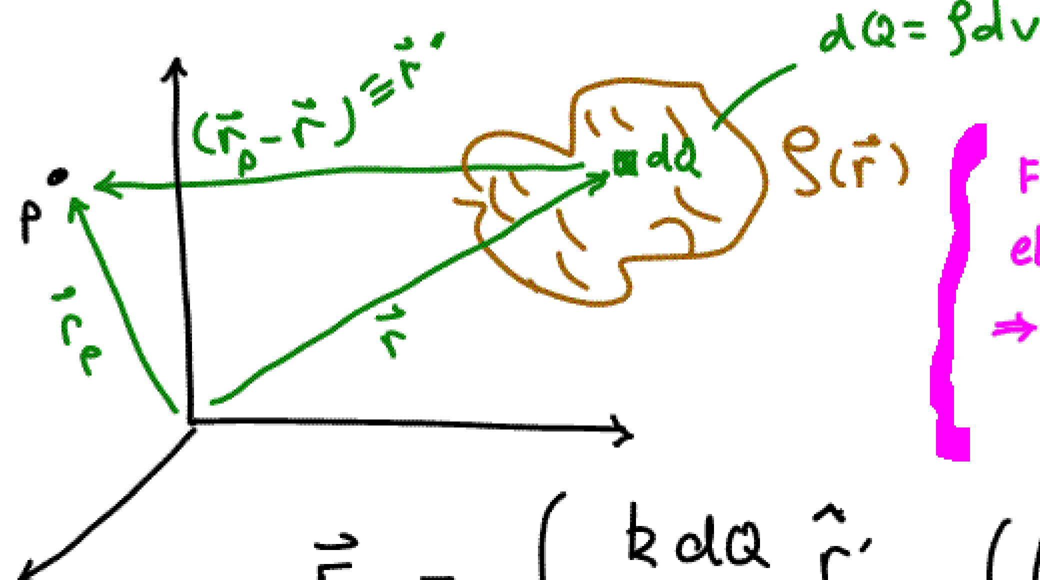
Physics 142 - Sept. 13, 2007

- Problem Set 1 due at end of class
- Solns will be posted on class website today (Hopefully)

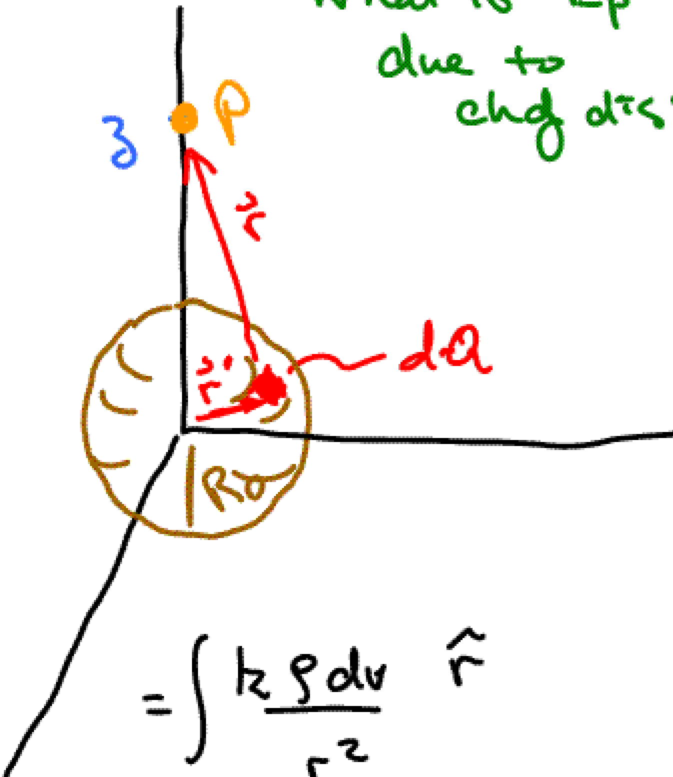
Last Time



Finding the electric field
→ Brute Force

$$\vec{E}_p = \int_{\text{charge dist}} \frac{k\, dQ}{r'^2}\,\hat{r}' = \int_{\substack{\text{volume} \\ \text{(of charge dist)}}} \frac{k\, \rho(\vec{r})\, dv}{r'^2}\,\hat{r}'$$

What is $\vec{E}_P$ due to chg dist.



Spherical uniform charge distribution with Q_{TOT}

$$\rho(\vec{r}') = \frac{Q_{TOT}}{\frac{4}{3}\pi R_0^3} = \text{const}$$

$$\vec{E} = \int_{vol} \frac{k\,dQ}{\mathscr{r}^2}\,\hat{\mathscr{r}}$$

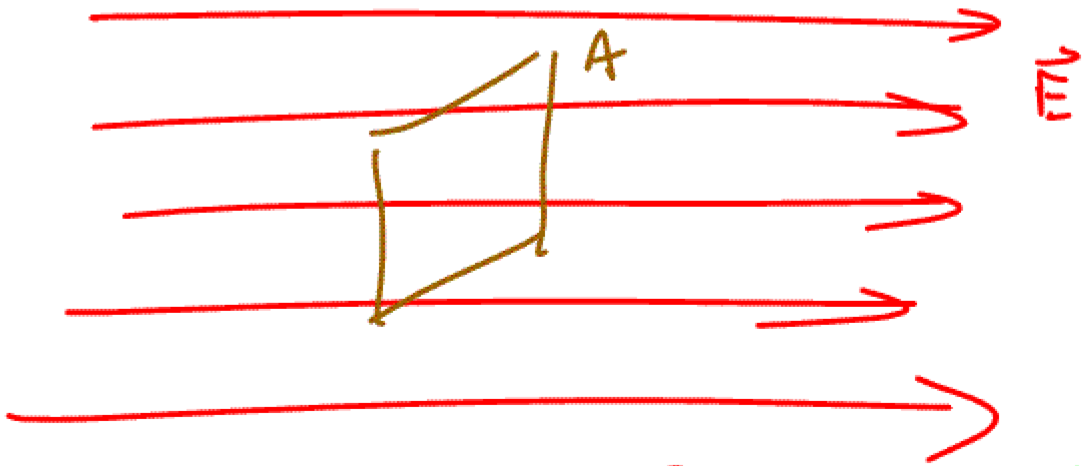
$$= \int_{vol} \frac{k\,\rho\,dv}{\mathscr{r}^2}\,\hat{\mathscr{r}}$$

$$= \int_0^{R_0}\int_0^{\pi}\int_0^{2\pi} \frac{k\,\rho\,r'^2 \sin\theta\, d\theta\, d\phi\, dr}{\mathscr{r}^2}\,\hat{\mathscr{r}}$$

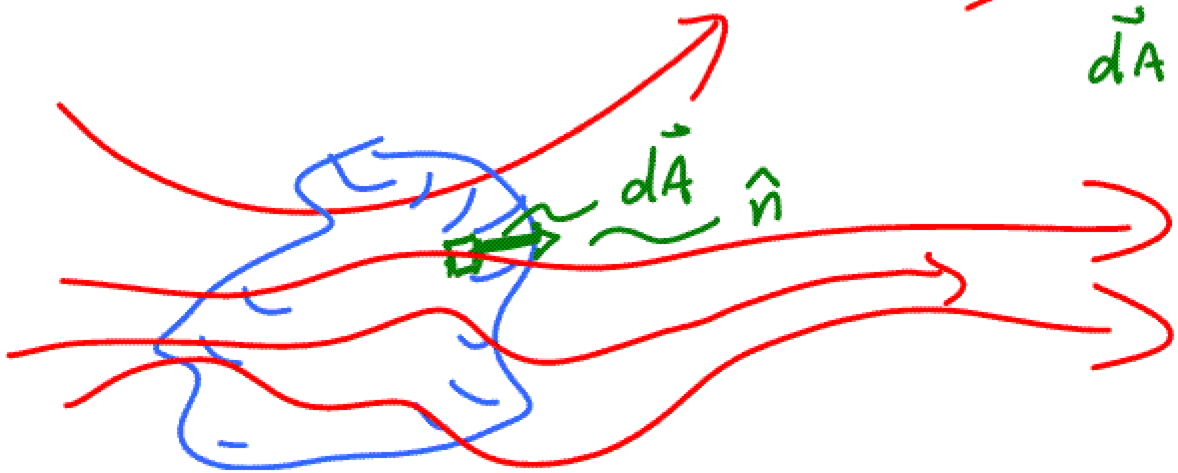
Holy Crap!

Electric Flux

Electric Flux $\equiv \phi \equiv$ Net # of field lines cross a surface



$$\phi = A\,|\vec{E}|$$



$$d\vec{A} = \hat{n}\, dA$$

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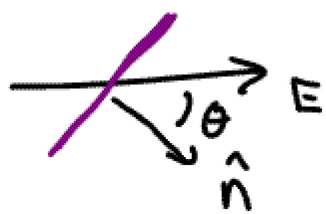
$$\Phi \equiv \int_{\text{surface}} \vec{E} \cdot \hat{n}\, dA = \oint_S \vec{E} \cdot \hat{n}\, dA$$

Look at from front → see

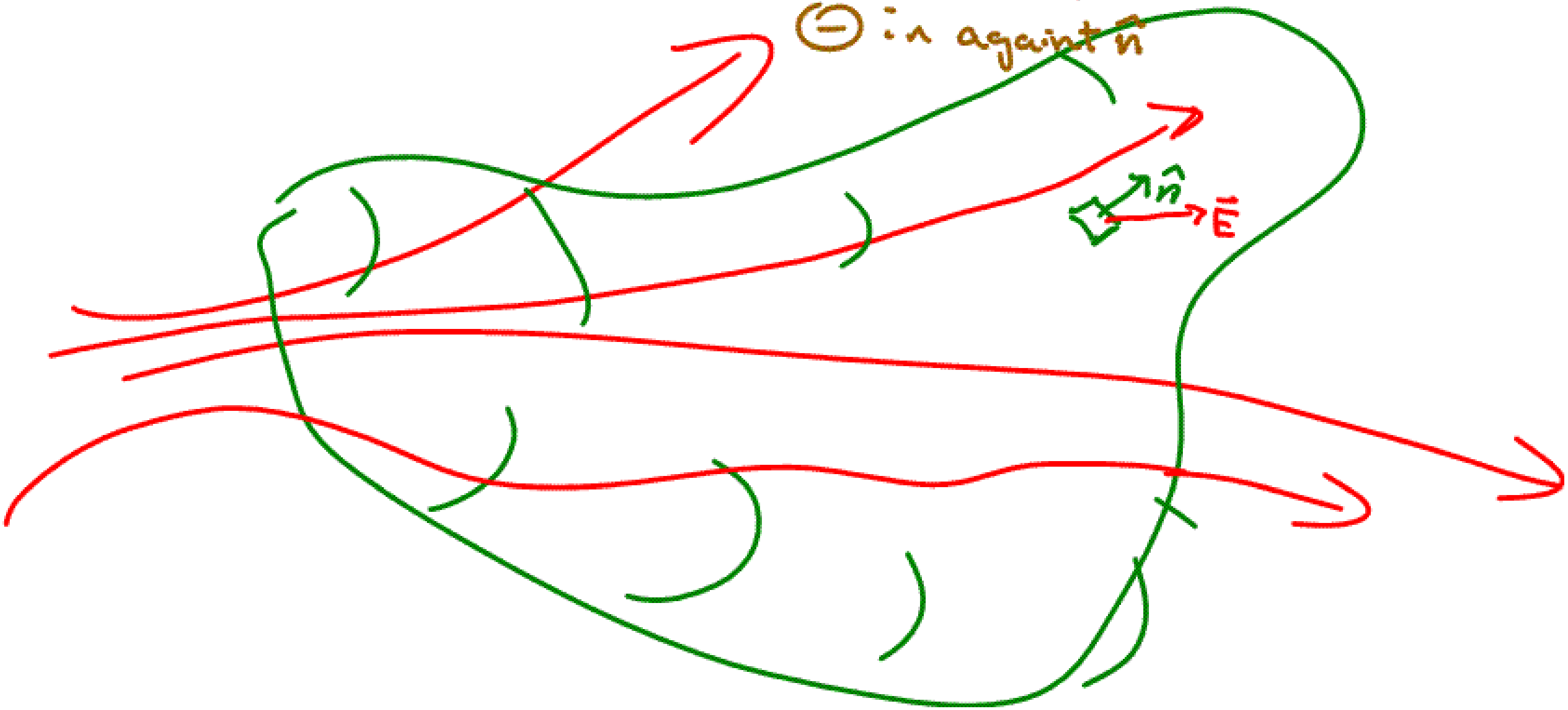


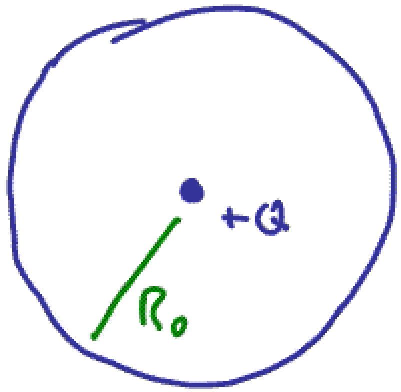
$$dA_2 = dA_1 \cos\theta$$

$$\Phi = \vec{E} \cdot \hat{n}\, dA = |\vec{E}|\, dA \cos\theta$$

$$\phi = \oint \vec{E} \cdot \hat{n} \, dA$$

Sum $\vec{E}_{\parallel}$ to $\hat{n}$ oversurface
$\oplus$ if along $\vec{n}$
$\ominus$ in against $\hat{n}$





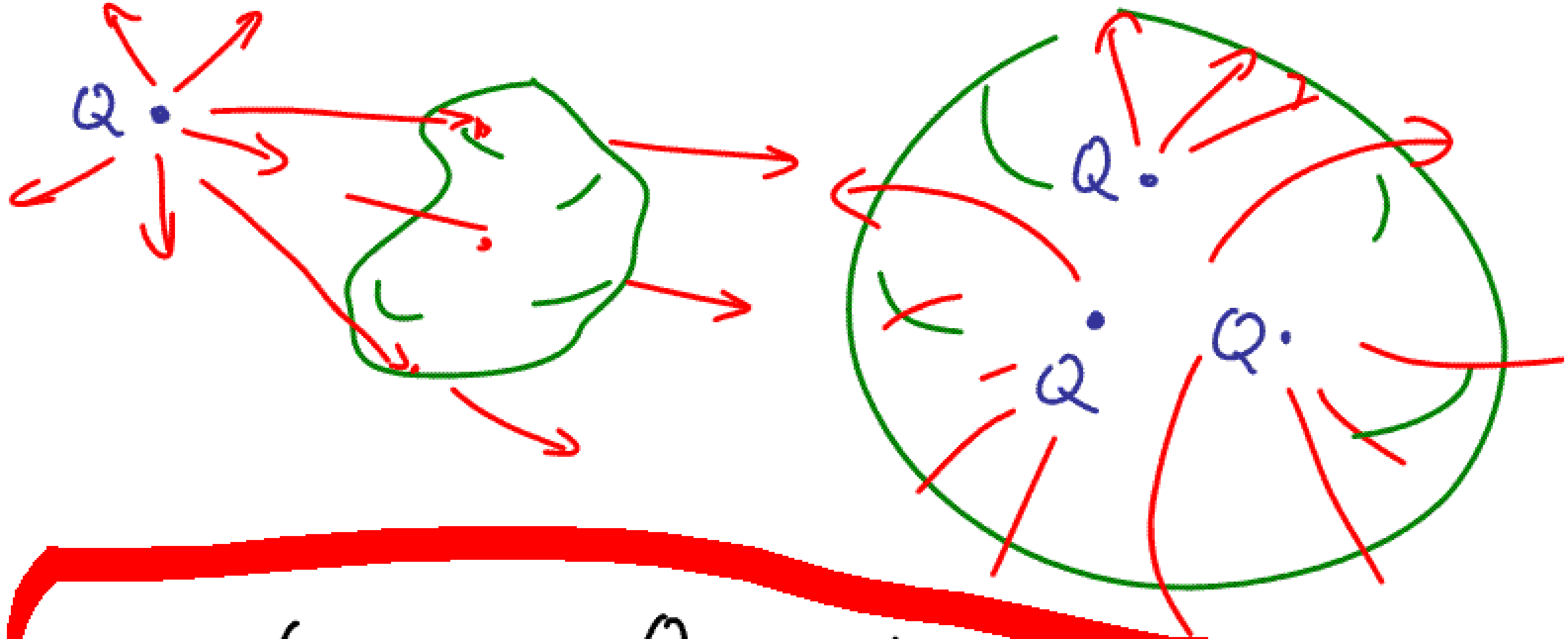
pt chg at origin
of spherical surface
centered at origin

$$\Phi_{surface} = \oint \vec{E}\cdot\vec{dA} = |\vec{E}|\oint dA = |\vec{E}|\, 4\pi R_0^2$$

$$= \frac{kQ}{R_0^2}\, 4\pi R_0^2 = \underbrace{k}_{\frac{1}{4\pi\epsilon_0}} Q\, 4\pi = \frac{Q}{\epsilon_0}$$

In this case

$$\oint \vec{E}\cdot\vec{dA} = \frac{Q}{\epsilon_0}$$



$$\phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

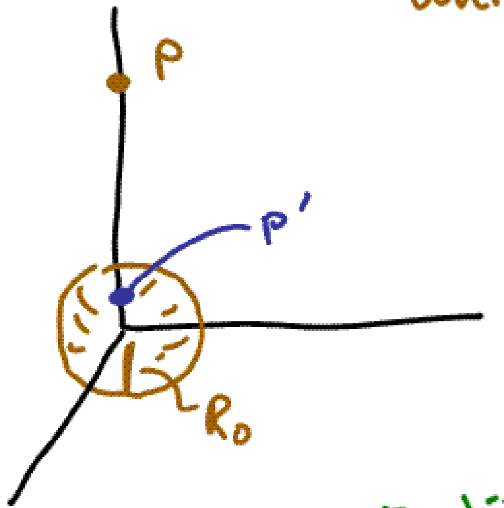
Enclosed by the surface

General

Gauss' Law

why do I care?

uniform spherical chg dist — tot Q_{TOT}



What is $\vec{E}$ at point P? at z

What is $\vec{E}$ at point P'? z'

Consider flux thru

Fictitious Gaussian surface

$r > R_o$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$\oint |\vec{E}|\, dA = |\vec{E}| \oint dA = |\vec{E}| 4\pi r^2$$

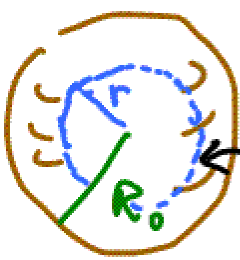
$$|\vec{E}|\, 4\pi r^2 = \frac{Q_{enc}}{\epsilon_0} = \frac{Q_{TOT}}{\epsilon_0}$$

$$|\vec{E}| = \frac{Q_{TOT}}{4\pi\epsilon_0 r^2} = \frac{k Q_{TOT}}{r^2}$$

$$\vec{E} = \frac{k Q_{TOT}}{r^2} \text{ outward}, \quad r \geq R_0$$

$$E \text{ at pt } p \text{ at } z = \frac{k Q_{TOT}}{z^2} \text{ out}$$

Find $\vec{E}$ for $r < R_0$



gaussian surface

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{Encl}}{\epsilon_0}$$

$|\vec{E}|$ at r (NOT R_0)

$$\downarrow$$

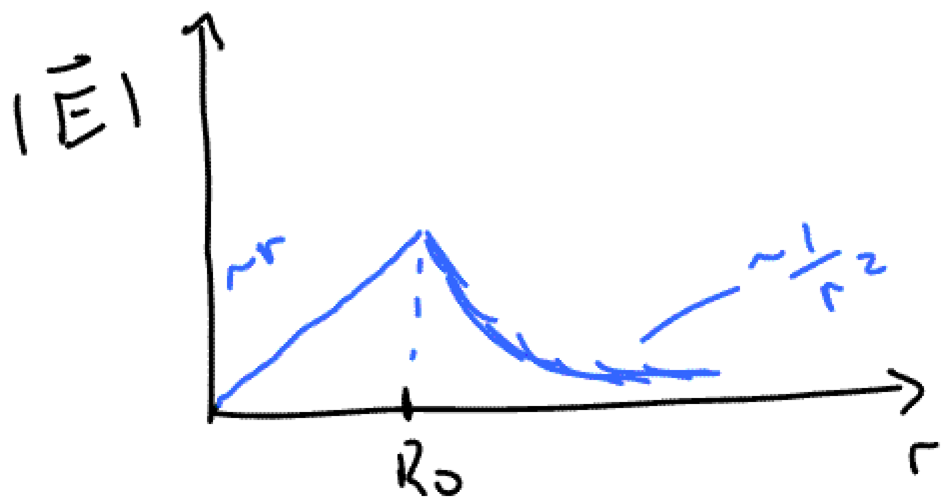
$$|\vec{E}| \oint dA = |\vec{E}| \, 4\pi r^2$$

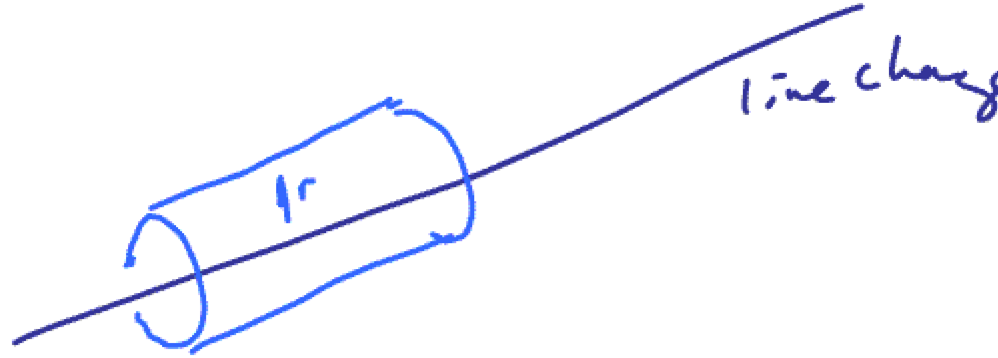
$$Q_{encl} = ? = \rho \, \tfrac{4}{3}\pi r^3 = \frac{Q_{TOT} \, \frac{4}{3}\pi r^3}{\frac{4}{3}\pi R_0^3} = Q_{TOT} \frac{r^3}{R_0^3}$$

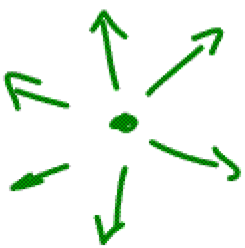
$$\rho = \frac{Q_{TOT}}{\frac{4}{3}\pi R_0^3}$$

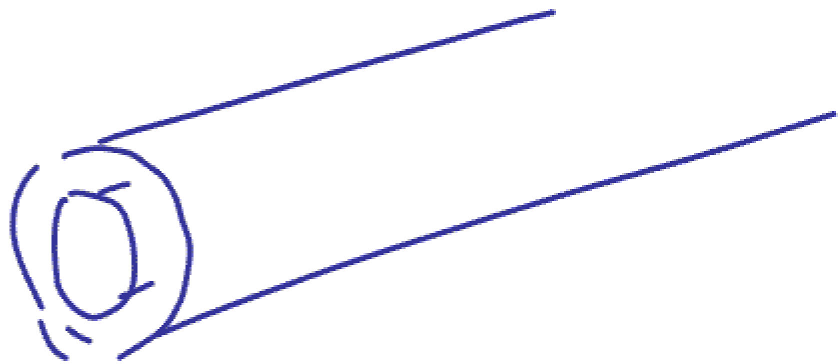
$$|\vec{E}|\, 4\pi r^2 = \frac{Q_{TOT}}{\epsilon_0} \frac{r^3}{R_0^3}$$

$$|\vec{E}| = \frac{Q_{TOT}}{4\pi \epsilon_0} \frac{r}{R_0^3}$$





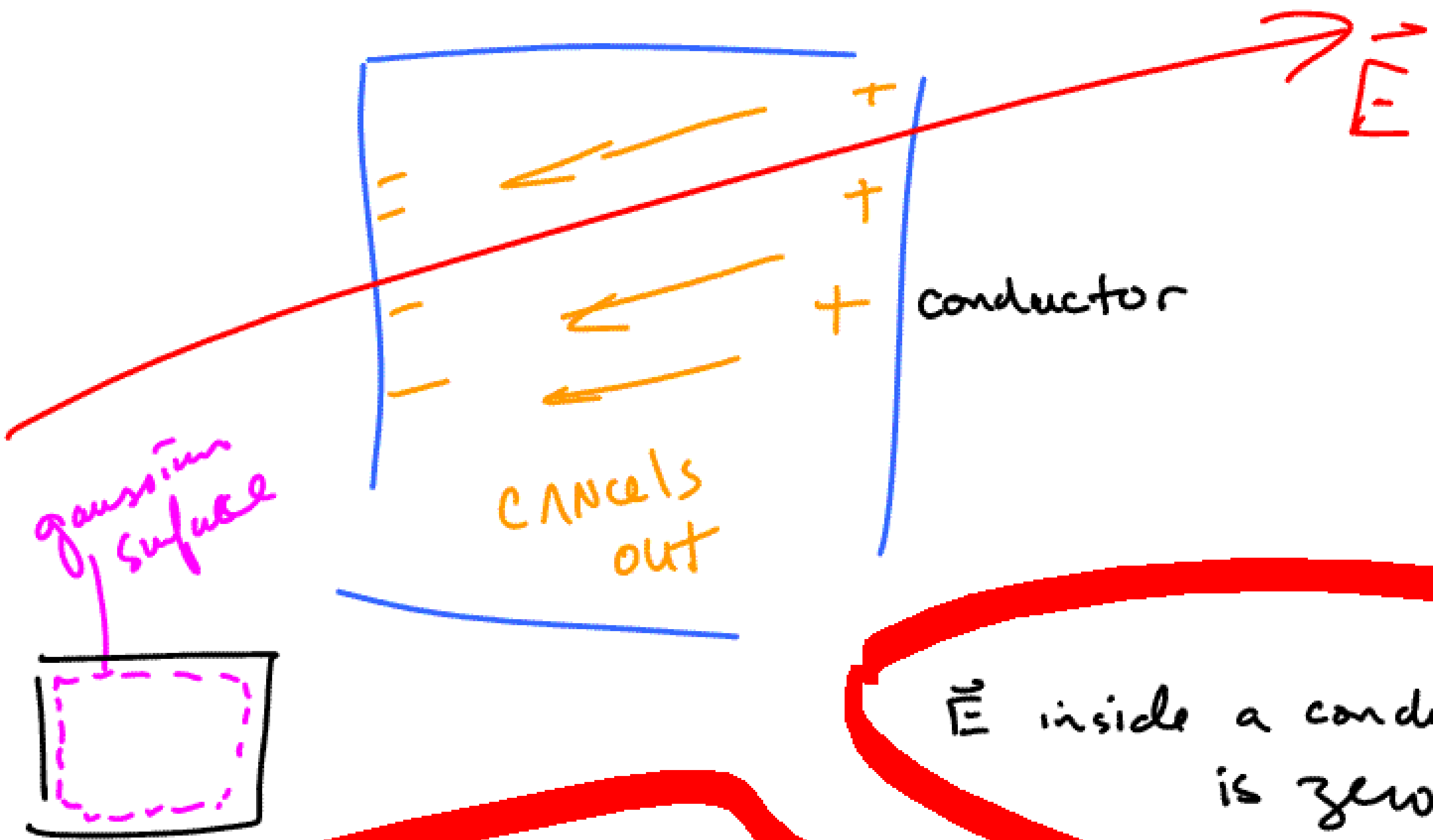




Gauss' law particularly useful when can choose geometry w/ simple dot product

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon}$$

+ E known or constant.



$\vec{E}$ inside a conductor is zero

Charge resides on surface of a conductor