Physics 142 - Sept. 11, 2007

- Workshops
- email distribution
- Problem Set

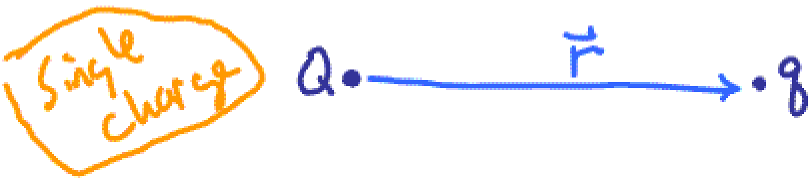
Mon 2-4 Mel 218

Tues 5-7 CSB 209

Wed 5-7 B+L 270

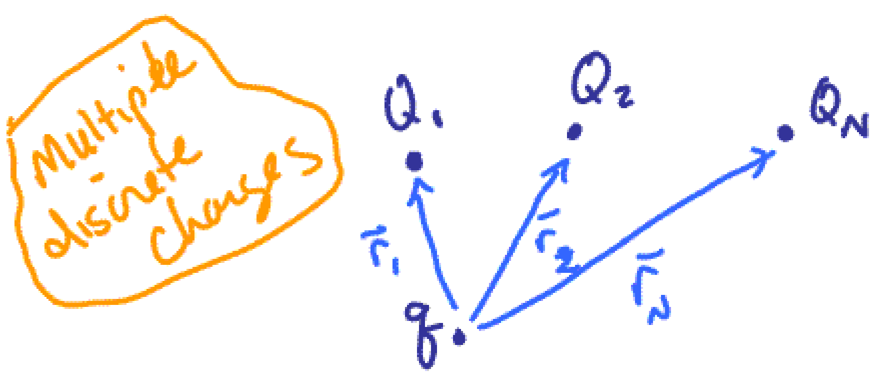
Thurs 6:15-8:15 B+L 208

Last Time



Force of Q on q

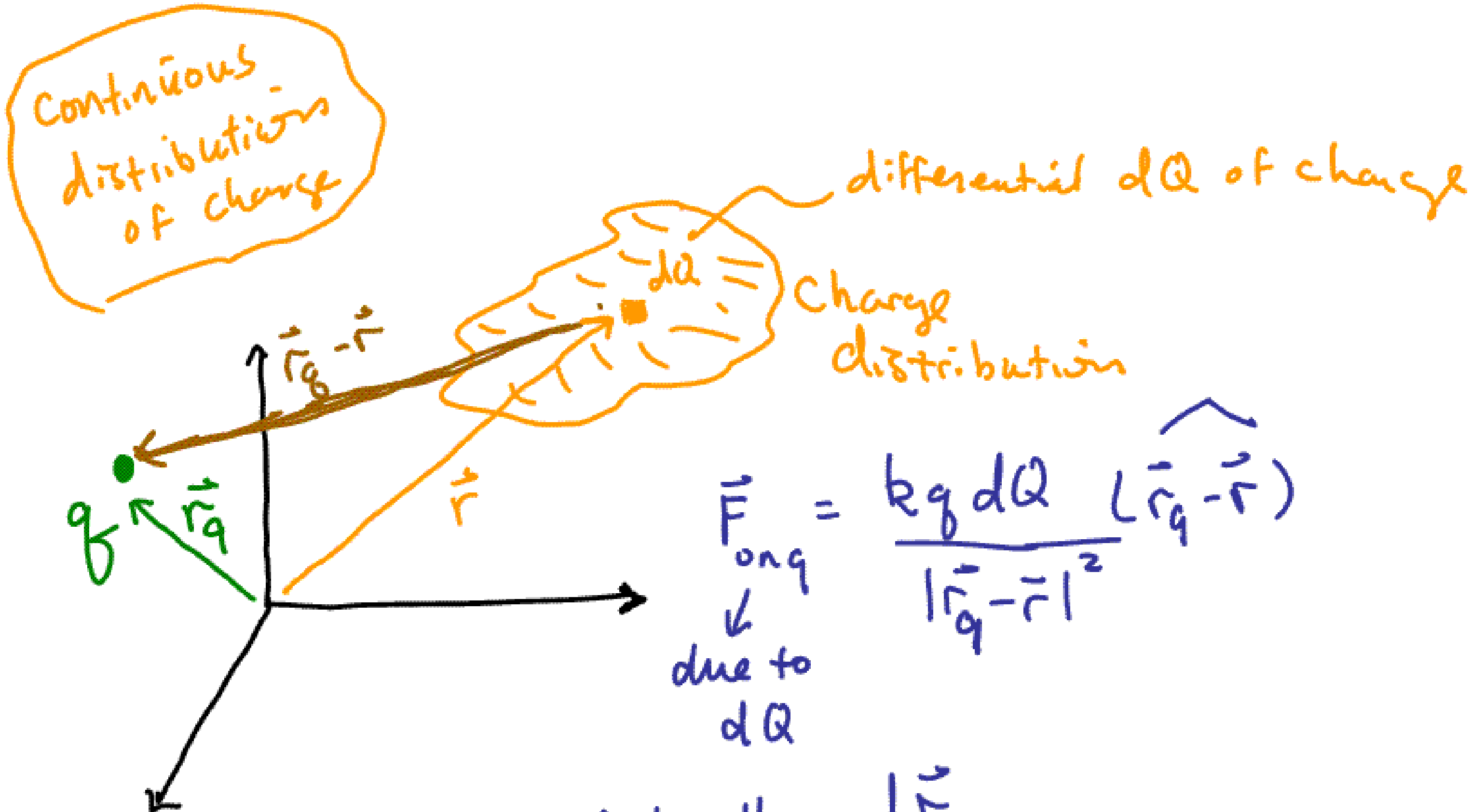
$$\vec{F} = \frac{kQq}{r^2}\,\hat{r}$$



vector superposition

Force of $Q_1, Q_2, \cdots Q_N$ on q

$$\vec{F} = \sum_i \frac{k\,Q_i\,q}{r_i^2}\,\hat{r}_i$$



$$\vec{F}_{on\,q} = \frac{kq\,dQ}{|\vec{r}_q - \vec{r}|^2}\,\widehat{(\vec{r}_q - \vec{r})}$$

due to dQ

Actually $d\vec{F}_{on\,q}$

$$dQ = \underbrace{\rho(\vec{r})}_{\text{volume charge density}}\,dv$$

$$\vec{F}_q = \int_{\text{vol of charge}} \frac{kq\,\rho(\vec{r})\,dv}{|\vec{r}_q - \vec{r}|^2}\,\widehat{(\vec{r}_q - \vec{r})}$$

Electric field

q

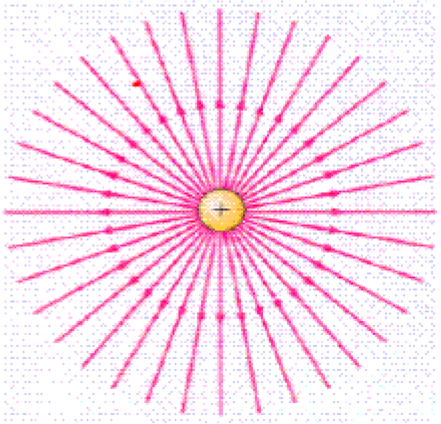
$\cdot P$

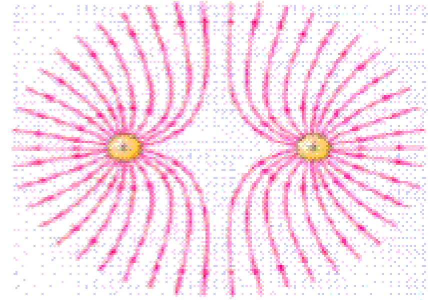
Q •

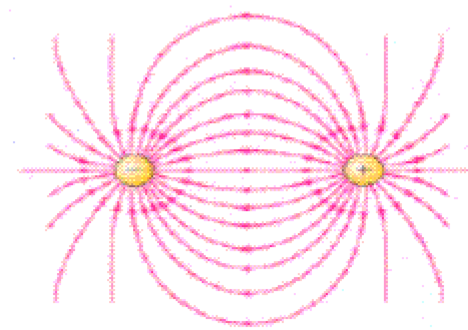
$$\vec{E}_P = \frac{\vec{F}_q}{q}$$

Imagine placing positive test charge q at point P $\rightarrow$ $\vec{F}_{on\,q} = q\,\vec{E}_P$

use this to visualize $\vec{E}$



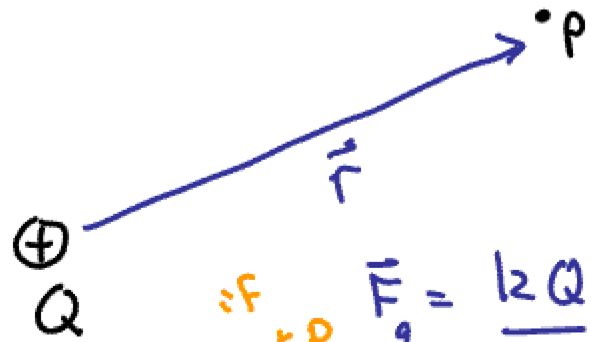




use "lines of force" to visualize the electric field

- go from ⊕ to ⊖ or have an endpoint at ∞
- lines never cross
- density of lines $\propto |\vec{E}|$
- $\vec{F}, \vec{E}$ always tangent to line of force

$\vec{E}$ for point charge

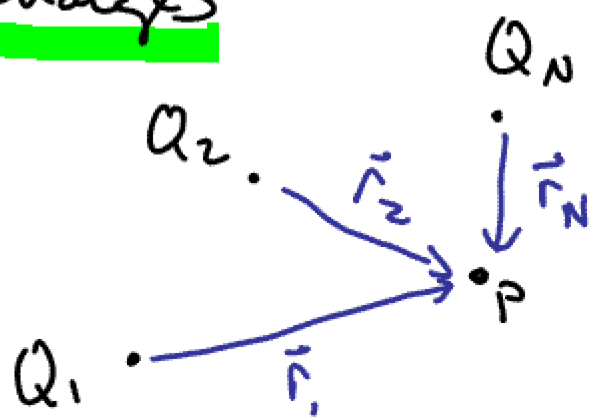


if q at P: $\vec{F}_q = \dfrac{kQq}{r^2}\hat{r}$

E field of Q

$$\vec{E}_P = \frac{\vec{F}_q}{q} = \frac{kQ}{r^2}\hat{r}$$

discrete charges

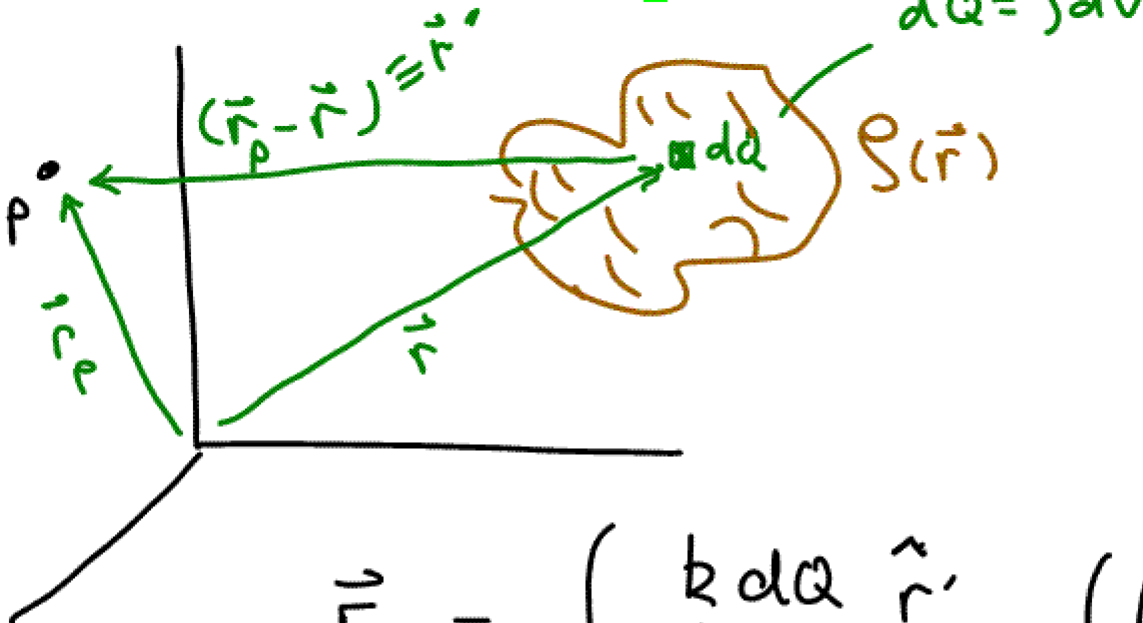


put q at each pt

$$\vec{F}_q = \sum_i \frac{kQ_i q}{r_i^2}\hat{r}_i$$

$$\vec{E}_P = \frac{\vec{F}_q}{q} = \sum_i \frac{kQ_i}{r_i^2}\hat{r}_i$$

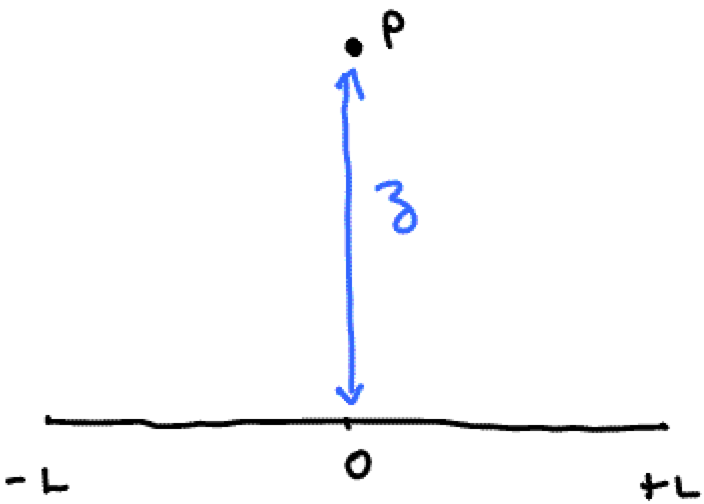
Continuous dist of charge

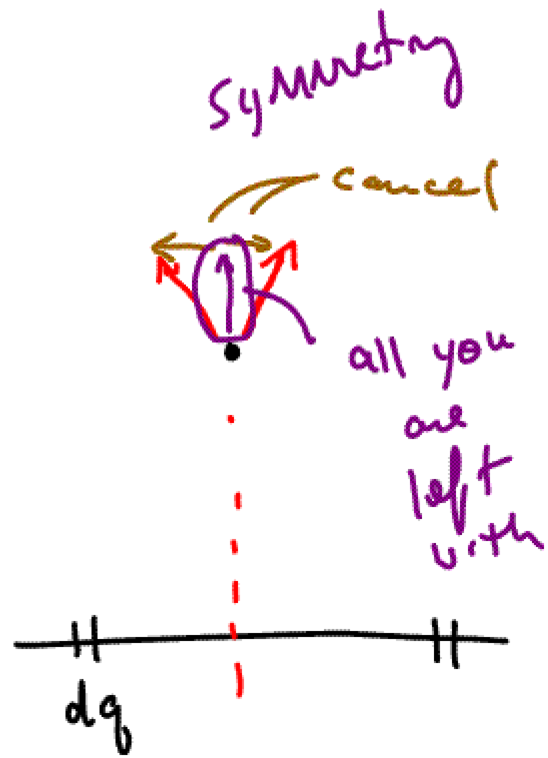


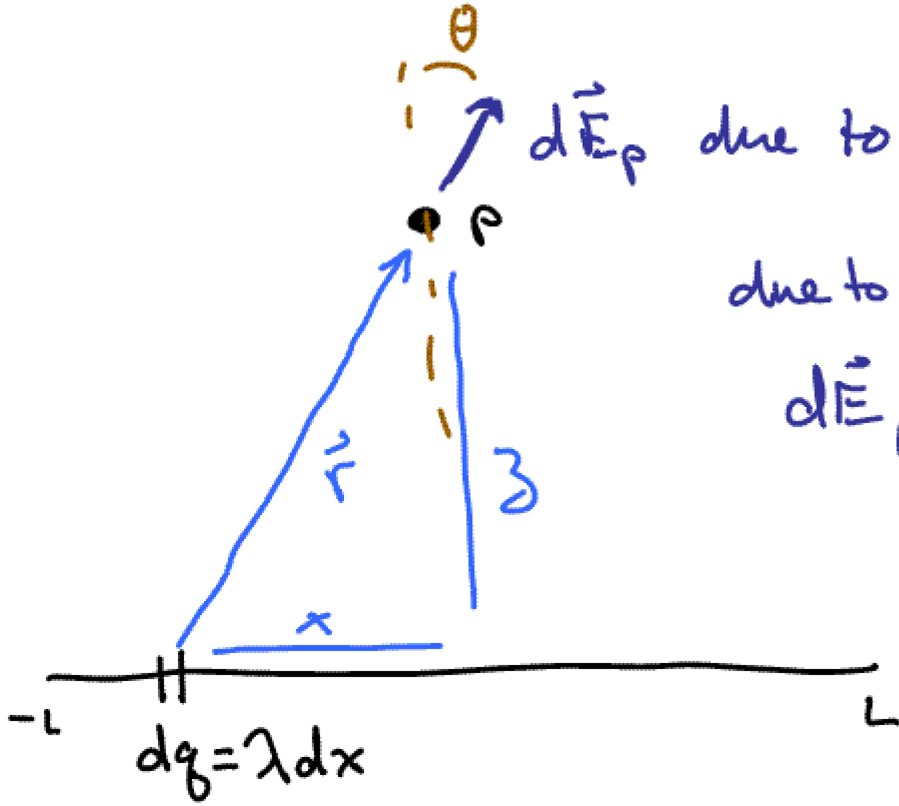
$$\vec{F}_p = \int_{\text{charge dist}} \frac{k\, dQ}{r'^2}\, \hat{r}' = \int_{\text{chg dist}} \frac{k\, \rho(\vec{r})\, dv}{r'^2}\, \hat{r}'$$

Example

Find $\vec{E}$ at distance z above the midpoint of a line segment of length $2L$ which carries a uniform line charge of λ



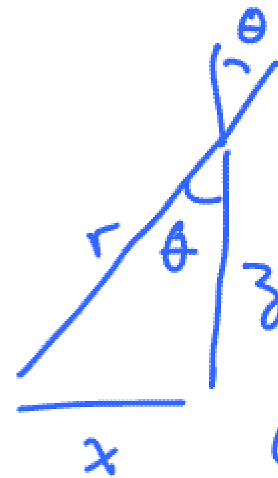




$d\vec{E}_p$ due to $dq = \lambda dx$

due to symmetry

$$d\vec{E}_p = d\vec{E}_z = dE\cos\theta\,\hat{z}$$



$$\cos\theta = z/r$$

$$E = \int_{-L}^{+L} \frac{k\,dq}{r^2}\,\hat{r}$$

$$\vec{E} = 2\int_0^L \frac{k\,\overset{\lambda dx}{dQ}\cos\theta}{r^2}\,\hat{z}$$

$$r^2 = x^2 + z^2$$

$$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{x^2+z^2}}$$

$$\vec{E} = 2\int_0^L \frac{k\lambda\, dx\, z}{(x^2+z^2)(x^2+z^2)^{1/2}}\hat{z} = 2\int_0^L \frac{k\lambda z\, dx\, \hat{z}}{(x^2+z^2)^{3/2}}$$

$$\vec{E} = 2k\lambda z \int_0^L \frac{dx}{(x^2+z^2)^{3/2}}\hat{z}$$

$z \equiv$ const in this problem

$$\vec{E} = 2kz\lambda \left[\frac{x}{z^2(x^2+z^2)^{1/2}}\right]_0^L \hat{z}$$

$$\boxed{\vec{E}_p = \frac{2kL\lambda}{z\,(L^2+z^2)^{1/2}}\hat{z}}$$

how do you know Answer makes sense?

dimensional Analysis

$$F = \frac{kqq}{r^2} \qquad \frac{N\,m^2}{c^2}$$

$$E \sim \frac{F}{Q} \sim \frac{N}{coul}$$

units ✓

$$E \sim \frac{\frac{N\,m^2}{c^2}\; m\; c/m}{m(m)} \sim \frac{N}{coul}$$

Limiting cases

$$z \to \infty \qquad E \to \frac{k\lambda}{z^2}$$

form as pt chg

$L \to \infty$

$$\frac{d(\text{top})}{dL} = 2k\lambda \qquad \frac{d(\text{bott})}{dL} = \sim z$$

$$E \to \frac{2\lambda k}{z}\,\hat{z} \quad \text{as } L \to \infty$$

Field of ∞ line chg