Problem 1 (Ohanian 22-2)

\[ |F| = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} = 8.99 \times 10^9 \times \frac{1.60 \times 10^{-19} \times 1.60 \times 10^{-19}}{(2.82 \times 10^{-10})^2} \text{ N} \]
\[ = 2.89 \times 10^{-9} \text{ N} \]

Problem 2 (Ohanian 22-3)

\[ |F| = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} = 9 \times 10^9 \times \frac{1.60 \times 10^{-19} \times 1.60 \times 10^{-19}}{(2 \times 10^{-15})^2} \text{ N} = 58 \text{ N} \]
\[ a = \frac{F}{m} = \frac{58 \text{ N}}{1.67 \times 10^{-27} \text{ kg}} = 3.4 \times 10^{28} \text{ m/s}^2 \]

Problem 3 (Ohanian 22-9)

Proton has charge $1.60 \times 10^{-19}$ C. 1 mole has $6.02 \times 10^{23}$ particles. Therefore, Faraday’s constant is $1.60 \times 10^{-19} \times 6.02 \times 10^{23} \text{ C} = 96300 \text{ C}$

Problem 4 (Ohanian 22-10)

1 electron has $1.6 \times 10^{-19}$ C. Therefore

\[ 1.5 \text{ C/s} = \left( \frac{1.5 \text{ C}}{1.6 \times 10^{-19} \text{ C/electron}} \right) \text{s} = 9.4 \times 10^{18} \text{ electron/s} \]
Problem 5 (Ohanian 22-13)

Number of moles in copper penny = \( \frac{2.7 \text{ g}}{63.5 \text{ g mol}} = 0.0425 \text{ mol} \).

Number of electrons is

\[
0.0425 \text{ mol} \times 6.02 \times 10^{23} \text{ atoms mol}^{-1} \times 29 \text{ electrons atom}^{-1} = 7.42 \times 10^{23} \text{ electrons}
\]

Total charge is \( 7.42 \times 10^{23} \text{ electrons} \times (-1.60 \times 10^{-19} \text{ C/electron}) = -1.19 \times 10^5 \text{ C} \).

There will be an equal positive charge of \( +1.19 \times 10^5 \text{ C} \) due to protons. Therefore,

\[
|\mathbf{F}| = \left| \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \right| = 9 \times 10^9 \times \frac{1.19 \times 10^5 \times 1.19 \times 10^5}{(2.0)^2} = 3.2 \times 10^{19} \text{ N}
\]

Addendum to problem 5

\[
F_w = 3.13 \times 10^9 \text{ N}
\]

to lift a carrier, \( F_{up} = mg \) at least

\[
F_{up} = (1000 \text{ kg mass}) \times \frac{9.8 \text{ m/s}^2}{1.1 \times 10^{-3} \text{ m}^2} = 8.9 \times 10^8 \text{ N}
\]

\[
\# = \frac{F_w}{F_{up}} = 3.6 \times 10^{10} \text{ carriers}
\]
Problem 6 (Ohanian 22-18)

Let $Q$ be charge on Earth. Then charge on Moon will be

$$
\frac{1.74}{6.38} Q = 0.273 \frac{Q}{Q}
$$

Electric force $F_e = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} = \frac{1}{4\pi \varepsilon_0} \frac{0.273 Q^2}{r^2}$

Gravitational force $F_g = \frac{GMm}{r^2}$. We want $F_g = F_e$; therefore

$$
\frac{1}{4\pi \varepsilon_0} \frac{0.273 Q^2}{r^2} = \frac{GMm}{r^2}, \text{ or } Q^2 = \frac{GMm}{4\pi \varepsilon_0 / 0.273}
$$

$$
Q = \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 7.35 \times 10^{22}}{9 \times 10^9 \times 0.273}} C = 1.09 \times 10^{14} C
$$

At $1.6 \times 10^{-19} C$ per electron, this means

$$
\frac{1.09 \times 10^{14} C}{1.6 \times 10^{-19} C/\text{electron}} = 6.8 \times 10^{32} \text{ electrons on Earth}
$$

and $(0.273 Q) = 0.273 (6.8 \times 10^{32}) = 1.9 \times 10^{32} \text{ electrons on Moon}$

Problem 7 (Ohanian 22-25)

The following are impossible:

- $p + p \rightarrow n + n + \pi^+$ \quad (e + e \rightarrow 0 + 0 + e)
- $p + p \rightarrow n + p + \pi^0$ \quad (e + e \rightarrow 0 + e + 0)
- $p + p \rightarrow n + p + \pi^0 + \pi^-$ \quad (e + e \rightarrow 0 + e + 0 + (-e))

In all cases here, charge is not conserved.

Problem 8 … no solution to give
Problem 9

What is charge and placement of other charge?

Symmetry tells us that charge must be on line with other end. If we put a negative charge to left at both, that could stabilize \(-Q\), but definitely not \(-3Q\). If we use a positive charge, would stabilize \(-3Q\) but not \(-Q\). Similarly with putting a charge to right at \(b + r\). So let's put our charge in between, a distance \(x\) from \(-Q\).

\[ Q \rightarrow x \rightarrow -3Q \]

Call right positive.

Net force on \(-Q\) must be zero.

\[ 2\vec{F}_{-Q} = \vec{F}_{\text{net}} \]

\[ \vec{F}_{\text{net}} = -k(4/8) \]

\[ \vec{F}_{\text{net}} \rightarrow \vec{F}_{-Q} \rightarrow \]

\[ F_{-Q} = \frac{kQq}{x^2} - \frac{3kQ^2}{r^2} = 0 \]

\[ \frac{kQq}{x^2} = \frac{3kQ^2}{r^2} \quad \rightarrow \quad \frac{Qq}{x^2} = \frac{3Q^2}{r^2} \]

This equation is born our unknown, but we need another!
\[ 2F_{-0} = \vec{F}_{\text{from } q} + \vec{F}_{\text{from } -q} = 0 \]

\[ \vec{F}_{\text{from } q} = \frac{k(-3q)q}{(l-x)^2} \quad (\text{it points right here}) \]

\[ \vec{F}_{\text{from } -q} = \frac{k(-3q)(-q)}{l^2} \quad \text{(this is zero)} \]

\[ 2F_{-0} = -3kq^2 \frac{x}{(l-x)^2} + \frac{3kq^2}{l^2} = 0 \]

\[ \frac{3kq^2x}{(l-x)^2} = \frac{3kq^2}{l^2} \implies ql^2 = (l-x)^2 \frac{2}{3} \]

Answer equation! So we have:

\[ l^2 q = 3q x^2 \quad \text{and} \quad q l^2 = (l-x)^2 q \]

So,

\[ 3q x^2 = (l-x)^2 q \]
\[ l^2 = 2lx + x^2 \]
\[ 2x^2 + 2lx - l^2 = 0 \]
\[ x = \frac{-2l \pm \sqrt{(2l)^2 + 4 \cdot 2 \cdot l^2}}{2 \cdot 2} \]
\[ = \frac{-2l \pm \sqrt{4l^2}}{4} \]
\[ = \frac{-l \pm 2l}{2} \]
\[ x = -\frac{l}{2} \pm \frac{1}{2} \sqrt{3} l \]

We should choose +, otherwise we'll get a negative \( x \), which would correspond to putting our charge on the left side of each.
\[
x = \left( -\frac{1}{2} + \frac{\sqrt{3}}{2} \right) \ell = \left( \frac{\sqrt{3} - 1}{2} \right) \ell \approx 0.37 \ell
\]

Take one of earlier signs:

\[
\ell^2 q = 30 \times 2
\]

\[
q = \frac{32x^2}{\ell^2} = \frac{32 (0.37x^2)}{\ell^2}
\]

\[
q = 0.4Q \text{ positive, which makes sense!}
\]

So placement is 0.37 \ell from \(-q\),

charge is 0.4Q.
Problem 10 –

\[ F_{\text{from } y \text{ m}} = k \left( \frac{6 \times 10^{-6} \text{ C}}{1.2 \text{ m}} \right) (4 \times 10^{-6} \text{ C}) = 0.15 \text{ N} \]

\[ F_{\text{from } -8 \text{ m}} = k \left( \frac{6 \times 10^{-6} \text{ C}}{1.2 \text{ m}} \right) (8 \times 10^{-6} \text{ C}) = 0.30 \text{ N} \]

\[ F_{\text{from } 3 \text{ m}} = -0.30 \hat{x} \]

\[ F_{\text{from } y \text{ m}} = 0.15 \hat{\theta} \]

\[ \sum F = -0.30 \hat{x} + 0.08 \hat{\theta} + 0.15 \hat{\theta} \]

\[ \sum F = (0.28 \hat{x}) \hat{x} + (0.15 \hat{\theta}) \hat{\theta} \]

\[ \theta = \tan^{-1} \left( \frac{0.13}{0.23} \right) = 30^\circ \text{ of } \theta \]
Problem set 1 \#11 soln

\[ F = \frac{kqQ}{r^2} \]
\[ F_g = mg \]

At equilibrium, \( F_e = F_g \) and \( r = y_0 \)

\[ \frac{kqQ}{r^2} = mg \]

Think about Simple Harmonic Motion

\( F = -kx \)

\[ m \frac{d^2x}{dt^2} = -kx \]

Differential eqn of this form leads to SHM with frequency \( \omega \).

\[ \frac{d^2x}{dt^2} + \frac{k}{m} x = 0 \]

SHM

\[ \omega = \frac{k}{m} \]
Back to our problem

Make small displacement $y$

\[ F_{\text{NET}} = \frac{kgQ}{(y_0 - y)^2} - mg = m \frac{d^2y}{dt^2} \]

Want to look at \( \frac{1}{(y_0 - y)^2} = \frac{1}{y_0(1 - \frac{y}{y_0})^2} \)

$y$ is tiny for small displacements

\( \frac{y}{y_0} \)

Do Binomial Series

Expansion of \( \frac{1}{(1 - \frac{y}{y_0})^2} \)

From table \(- (1+x)^{-2} = 1 - 2x + 3x^2 - \ldots \)

\( -1 < x < 1 \)

\( \frac{1}{(1 - \frac{y}{y_0})^2} = 1 - 2\frac{y}{y_0} + 3\left(\frac{y}{y_0}\right)^2 \ldots \)

Neglect higher order terms

\( \frac{y}{y_0} \ll 1 \)
Substitute Binomial expansion into
eqn (1)  above

\[ F_{\text{net}} = \frac{kQq}{y_0^2} \left(1 - 2 \frac{y}{y_0}\right) - mg = m \frac{d^2y}{dt^2} \]

\[ y_0^2 = \frac{kQq}{mg} \]
Sub in

\[ m \frac{d^2y}{dt^2} = \frac{kQq \cdot mg \left(1 - 2 \frac{y}{y_0}\right)}{12Qq} - mg \]

\[ \frac{d^2y}{dt^2} + \frac{2g}{y_0} y = 0 \]

This is eqn for SHM with \( \omega^2 = \frac{2g}{y_0} \)