1. \(36-6\)

By right-hand rule, \(\mathbf{B}\) points \textit{north}.

\[
\text{Magnitude } = \frac{E_0}{c} = 2.0 \times 10^{-9} \text{ T}
\]

2. \(36-8\)

(a) In \textit{minus} \(z\)-direction \[
\text{from } \sin \left(\omega t + \frac{\omega z}{c}\right), \text{ since waves in form } f(z + vt) \text{ travel in minus direction.}
\]

(b) From diag., \(\mathbf{E}\) makes angle \(\theta = \tan^{-1} \left(\frac{2}{1}\right)\)

\[
= 63^\circ \text{ with } x\text{-direction,} \quad 27^\circ \text{ with } y\text{-axis, and} \quad 90^\circ \text{ with } z\text{-axis.}
\]

(c) Since \(\mathbf{B} = \frac{E_0}{c}\) in magnitude,
and \(\mathbf{B}\)-field is related to \(\mathbf{E}\) by right-hand rule, and since wave is propagating in minus-\(z\)-direction, we have

\[
\mathbf{B} = i \frac{E_0}{c} \sin \left(\omega t + \frac{\omega z}{c}\right) + 2i \frac{E_0}{c} \sin \left(\omega t + \frac{\omega x}{c}\right)
\]
\( \lambda = \frac{c}{v} = \frac{3 \times 10^8}{4 \times 10^6} = 75 \text{ Hz} \)

Sensitivity is:
Maximum for \( 5.5 \times 10^{-7} \text{ m} \) (3500 \( \text{Å} \))
One-half for \( 5 \times 10^{-7} \text{ m} \) (5000 \( \text{Å} \)) and \( 6 \times 10^{-7} \text{ m} \) (6000 \( \text{Å} \))
One-quarter for \( 4.8 \times 10^{-7} \text{ m} \) (4000 \( \text{Å} \)) and \( 6.2 \times 10^{-7} \text{ m} \) (6200 \( \text{Å} \))

By (36), \( S = \frac{1}{2} \mu_0 c E_0^2 \Rightarrow F_0 = \sqrt{2} \mu_0 c S \)
\[ = \sqrt{2 \times 1.26 \times 10^{-6} \times 3 \times 10^3 \times 1.4 \times 10^3} \text{ V/m} = 1030 \text{ V/m} \]

\[ B = \frac{E_0}{c} = 3.4 \times 10^{-6} \text{ T} \]

Power received by the Arecibo telescope = Flux Area
\[ = 4.1 \times 10^{-25} \times \pi(150)^2 = 2.9 \times 10^{-20} \text{ W} \]
Total power emitted by the quasar = Flux \( \times 4\pi R^2 \) (\( R \) is the earth-quasar distance)
\[ = 4.1 \times 10^{-25} \times 4\pi(2.8 \times 10^9 \times 9.46 \times 10^{15})^2 = 3.6 \times 10^{27} \text{ W} \]
Energy flux = \( \frac{\text{power}}{\text{area}} = \frac{6000 \ W}{\pi (10^{-3} \ m)^2} = 1.9 \times 10^9 \ W/m^2 = \text{time} \)

average Poynting vector. By (36), \( E_0 = \sqrt{2\mu_0 c S} = 1.2 \times 10^6 \ V/m \)

and \( B = \frac{E_0}{c} = \frac{1.2 \times 10^6 \ V/m}{8 \times 10^8 \ ms^{-1}} = 4.0 \times 10^{-3} \ T \)

\[ \text{Flux} = S = \frac{\text{power}}{\text{area}} = \frac{P}{4\pi r^2}. \text{By (36),} \ S = \frac{1}{2\mu_0 c} E_0^2 \text{, therefore,} \frac{1}{2\mu_0 c} E_0^2 = \frac{P}{4\pi r^2}. \]

therefore, \( r = \left( \frac{P}{4\pi \ 2\mu_0 c} \right)^{1/2} \frac{1}{E} = \sqrt{\frac{10,000 \ W}{4\pi}} \times 756 \frac{1}{2 \times 10^{-4}} = 3.9 \times 10^6 \ m \)
(a) The potential difference between the shell and wire must be \( \Delta V = \varepsilon \). By symmetry, field lines point radially outward, and for flux to remain constant throughout the inside, \( E(r) = \frac{1}{r} \); therefore, \( E(r) = \frac{k}{r} \), where \( k \) is some constant, and

\[
\int_a^b E \cdot dr = \Delta V = \varepsilon; \text{ therefore,}
\]

\[
\int_a^b \frac{k}{r} dr = k \ln \frac{b}{a} = k \ln b - k \ln a
\]

therefore, \( k = \frac{\varepsilon}{\ln(b/a)} \).

Hence \( E(r) = \frac{\varepsilon}{\ln(b/a)} \frac{1}{r} \) for \( a < r < b \).

(b) Current \( = \frac{\text{Power}}{\text{voltage}} = \frac{P}{\varepsilon} \). By Ampere's law, treating wire as a "long" wire,

\[
B = \frac{I}{2\pi R} = \frac{\varepsilon}{2\pi} \frac{P}{\varepsilon} \frac{1}{r}
\]

(c) Therefore, Poynting vector

\[
S = \frac{1}{\mu_0} E \times B. \text{ From diag.}
\]

\( S \) points from \( \varepsilon \) or \( R \) along axis of cylinder.

Since \( B, E \) perpendicular, \( S = \frac{1}{\mu_0} EB = \frac{1}{\mu_0} \frac{\varepsilon}{\ln(b/a)} \frac{1}{r} \frac{P}{\varepsilon} \frac{1}{r} \)

\[
= \frac{P}{\mu_0 \varepsilon \ln(b/a) 2\pi r^2}
\]

(d) Power \( = \int S \cdot dA \). Take area elements \( dA = \text{circular disc} \), radius \( r \), thickness \( dr \),

therefore, \( dA = 2\pi r dr \),

therefore,

\[
\text{Power} = \int_a^b \frac{P}{\mu_0 \varepsilon \ln(b/a)} \frac{1}{2\pi} \frac{1}{2\pi r} 2\pi r dr
\]

\[
= \frac{P}{\mu_0 \varepsilon \ln(b/a)} \int_a^b \frac{1}{r} dr = \frac{P}{\mu_0 \varepsilon \ln(b/a)} \ln(b/a) = P
\]
The force of gravity is given by $F_g = \frac{GM_e m}{R^2}$. The force of pressure of sunlight is $2 \frac{S}{c} A^*$. 

$s = \text{Poynting vector}, A = \text{area of sail};$ therefore, we want: $2 \frac{S}{c} A > \frac{GM_e m}{R^2}$

\[ A > \frac{c \left( \frac{GM_e m}{2R^2} \right)}{1.4 \times 10^3 \times 6.67 \times 10^{-11} \times 1.99 \times 10^8 \times 70,000} \]

\[ A > \frac{4.4 \times 10^7 \text{ m}^2}{44 \text{ km}^2} \]

The factor 2 involved because mirror reflects, not absorbs, the light. (If circular, sail radius = 3.74 km.)

36-34

Component of force in vertical direction is zero because momentum in that direction does not change. Therefore, only component we care about is horizontal. By (45),

\[ F_x = 2 \frac{S}{c} A \cos 30^\circ, \text{ factor } 2 \]

Inserted because the ray is reflected.

Change in momentum

\[ \Delta p = F_x \Delta t = 2 \frac{S}{c} A \Delta t \cos 30^\circ \]

\[ S \Delta A t = \frac{\text{Energy}}{(\text{area} \times \text{time})} \times \text{Area} \times \text{time} = \text{Energy}. \]

Therefore,

\[ \Delta p = F_x \Delta t = \left( \frac{2}{c} \text{Energy} \right) \cos 30^\circ = \left( \frac{2}{3 \times 10^8 \times 10^5 \cos 30^\circ} \right) \text{kg m s}^{-1} \]

\[ = 5.8 \times 10^{-4} \text{ kg m s}^{-1} \]
(b) The Poynting vector at a particular point is given by $S = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$. Since $\mathbf{E}$ is perpendicular to $\mathbf{B}$, $|S| = \frac{1}{\mu_0} \mathbf{E} \cdot \mathbf{B} = \frac{A^2}{\mu_0} \sin^2 \left( \frac{\omega}{c} x - \omega t \right)$. Flow of power through an area $= \int S \cdot dA$; take area elements $dA = 2\pi r \, dr$, where $r$ is radius from center, $dr = \text{width}$. Then

$$\int S \cdot dA = \int_a^b \frac{A^2}{\mu_0} \sin^2 \left( \frac{\omega}{c} x - \omega t \right) 2\pi r \, dr = \frac{2\pi A^2}{\mu_0} \int_a^b \frac{1}{r} \, dr$$

$$= \frac{2\pi A^2}{\mu_0 c} \ln \left( \frac{b}{a} \right) \sin^2 \left( \frac{\omega}{c} x - \omega t \right)$$

At $x = L$ this gives Instantaneous Power

$$= \frac{2\pi A^2}{\mu_0 c} \ln \left( \frac{b}{a} \right) \sin^2 \left( \frac{\omega L}{c} - \omega t \right)$$

The average of $\sin^2 \left( \frac{\omega L}{c} - \omega t \right)$ is $\frac{1}{2}$. Therefore,

Average power $= P = \frac{\pi A^2}{\mu_0 c} \ln \left( \frac{b}{a} \right)$

(c) $F_x = \frac{S}{c} \times \text{Area} = \frac{\pi A^2}{\mu_0 c} \ln \left( \frac{b}{a} \right)$ [from part (b) above].

Therefore $F_x = \frac{\pi A^2}{\mu_0 c} \ln \left( \frac{b}{a} \right)$
(a) Since $\lambda = \frac{c}{u}$, we have from (51) $\frac{u}{u_0} = \frac{\lambda_0}{\lambda} = \sqrt{\frac{1 - u/c}{1 + u/c}}$, gives $\frac{u}{c} = \frac{1 - (\lambda_0 \lambda)^{1/2}}{1 + (\lambda_0 \lambda)^{1/2}}$.

![Graph showing distances and velocities with labeled galaxies: Virgo, Ursa Major, Corona Borealis, and Bootes.]

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>$\frac{u}{c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virgo</td>
<td>$3.90 \times 10^{-3}$</td>
</tr>
<tr>
<td>Ursa Major</td>
<td>0.0488</td>
</tr>
<tr>
<td>Corona Borealis</td>
<td>0.0694</td>
</tr>
<tr>
<td>Bootes</td>
<td>0.122</td>
</tr>
<tr>
<td>Hydra</td>
<td>0.183</td>
</tr>
</tbody>
</table>

(b) Since velocity $\times$ time = distance, this time $\Delta t = \frac{d}{v} = \text{slope of graph}$

\[
\frac{4000 \times 10^6 \text{ light yr}}{0.183 \, c} \left( \frac{\text{distance}}{\text{velocity}} \text{ for Hydra cluster} \right)
\]

\[= \frac{4000 \times 10^6 \text{ light yr} \times c}{0.183 \, c} = 2 \times 10^{10} \text{ yr} \]
\[
\frac{u}{v_0} = \frac{\lambda_0}{\lambda} = \sqrt{\frac{1 \pm u/c}{1 \pm u/c}} = \frac{u}{c} = \frac{1 - \left(\frac{\lambda_0}{\lambda}\right)^2}{1 + \left(\frac{\lambda_0}{\lambda}\right)^2} \quad \text{for} \quad \frac{\lambda_0}{\lambda} < 1 \quad \text{or} \quad \frac{\lambda_0}{\lambda} > 1.
\]

One surface has \( u_1/c = \frac{1 - \left(\frac{4101.74}{4101.77}\right)^2}{1 + \left(\frac{4101.74}{4101.77}\right)^2} = 7.3 \times 10^{-6} \),

\( u_1 = 7.3 \times 10^{-6} \) cm (receding) = \( 2.2 \times 10^3 \) m/s.

Other surface has \( u_2/c = \frac{\left(\frac{4101.74}{4101.71}\right)^2 - 1}{\left(\frac{4101.74}{4101.71}\right)^2 + 1} = 7.3 \times 10^{-6} \),

\( u_2 = 7.3 \times 10^{-6} \) cm (approaching) = \( 2.2 \times 10^3 \) m/s.

Angular velocity \( \omega = \frac{v}{r} = \frac{7.3 \times 10^{-6} \times 3.0 \times 10^8 \text{ ms}^{-1}}{7.0 \times 10^8 \text{ m}} = 3.1 \times 10^{-6} \text{ s}^{-1} \)

\( I(30^\circ) = \cos^2 30^\circ = 0.75 \)

\( I(45^\circ) = \cos^2 45^\circ = \frac{1}{2} \)

\( I(60^\circ) = \cos^2 60^\circ = \frac{1}{4} \)