Physics 142 - September 20, 2005

Electric Potential

Project - show skateboarding video

Last time -
- review of curvilinear coordinates and scale factors that come into integrations
- Gauss' Law, cylindrical example
- Start of Electric potential

\[ W = \int_A^B \mathbf{E} \cdot d\mathbf{s} = -\int q_0 \mathbf{E} \cdot d\mathbf{s} = -\int_{B_0}^B E dr \]

\[ W = -q_0 \Phi \int_A^B \frac{1}{r^2} dr = kq_0 \frac{1}{r_0} - \frac{1}{r_f} \]
\[
\frac{\text{Work}}{q_0} = \text{Potential difference} \quad \Delta U = V_B - V_A = V_{AB}
\]

\[\Delta \text{in potential energy} \equiv \text{work}\]

\[\text{Potential Diff} \equiv \frac{\Delta \text{PE}}{q_0}\]

"is defined as"

can define "zero" of potential

\[W_{B_0} = 12Q \left[\frac{1}{r_B} - \frac{1}{r_A}\right]\]
define potential to be zero at \( r \to \infty \)

Absolute potential at \( r \)

Potential at \( r \):

\[
V(r) = \frac{kQ}{r}
\]

Important

\[
V_P = \sum_{i} V_i = \sum_{i} \frac{kQ_i}{r_i}
\]
Important

Path Independent

Conservative Force

What is the potential at pt A

\[ v = \frac{\int F \cdot ds}{q_0} \]

Arbitrary distribution

\[ \Delta Q \]

\[ r \]

\[ A \]

\[ v_A \text{ due to } dQ = dV_A = \frac{k}{r} dQ \]

\[ V_A = \int_{vol} \frac{k}{r} \frac{dQ}{vol} \]

Important
\[ V = \frac{W}{q_0} \]
\[ dv = \frac{dw}{q_0} = -E \cdot ds = -E_s ds \]
\[ E_s = -\frac{dv}{ds} \]

Often work in 3d

\[ V(x, y, z) \]

to get \( \vec{E} \) we need

\( E_x, E_y, E_z \)

\[ E_x = -\frac{dv}{dx}, \quad E_y = \frac{dv}{dy}, \quad E_z = \frac{dv}{dz} \]

\[ E_x = -\frac{\partial V}{\partial x} = \text{partial derivative of } V \text{ w.r.t. to } x \]

\[ \frac{\partial F}{\partial x} = \frac{dF}{dx} \text{ Hold all other variables constant} \]
\[ E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z} \]

\[ \vec{E} = -\vec{\nabla}V = -\text{grad}(V) \]

The vector operator \( \vec{\nabla} \) is called "del"

\[ \vec{\nabla} \equiv \text{grad} \equiv \text{gradient} \quad \text{3d vector operator} \]

\[ = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \]

If 1D

\[ E_x = -\frac{dV(r)}{dr} \]

Often it is easier to calculate the potential than \( \vec{E} \) directly. Then use \( \vec{E} = -\vec{\nabla}V \) to find \( \vec{E} \)
If $r_A = r_B$, then $V_A = V_B$.

Points all at same potential.

Equipotential surface.