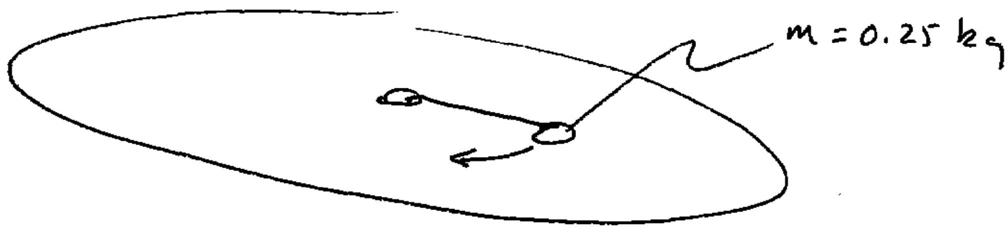
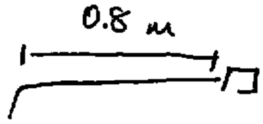
P121 Lecture
3/29/01

Angular Momentum Example Problem



initial condition



Tangential velocity = 4 m/s

Final situation
String is shortened until it breaks ($T = 30.0\,N$)
What is R when string breaks??

Two things come into play
1) Angular momentum cons.
$I\omega = \text{const}$
2) circular motion
$F_c = m\frac{V^2}{R}$

$s = r\theta$
$v = r\omega$

$$(I\omega)_{\text{Initial}} = mR^2\frac{V}{R} = mVR$$

$$= (I\omega)_{\text{Final}}$$

$\therefore$

$$mV_{\text{init}}R_{\text{init}} = mV_{\text{Final}}R_{\text{Final}} \Rightarrow \boxed{V_{\text{init}}R_{\text{init}} = V_{\text{Fin}}R_{\text{Final}}}$$

Also at break

$$\boxed{F_c = 30\,N = m\frac{V_{\text{Final}}^2}{R_{\text{Final}}}}$$

$$V_{Final} = \frac{V_i R_i}{R_f}$$

$$30 = m \frac{V_i^2 R_i^2}{R_f^2 R_f} = \frac{m V_i^2 R_i^2}{R_f^3}$$

$$\boxed{R_f^3 = \frac{m V_i^2 R_i^2}{30}}$$

$$R_f^3 = \frac{(0.25)(4)^2(0.8)^2}{30}$$

$$\boxed{R_f = 0.44\ m}$$

Static Equilibrium

A rigid body is in static equilibrium if $\sum \vec{F} = \vec{F}_{NET} = 0$

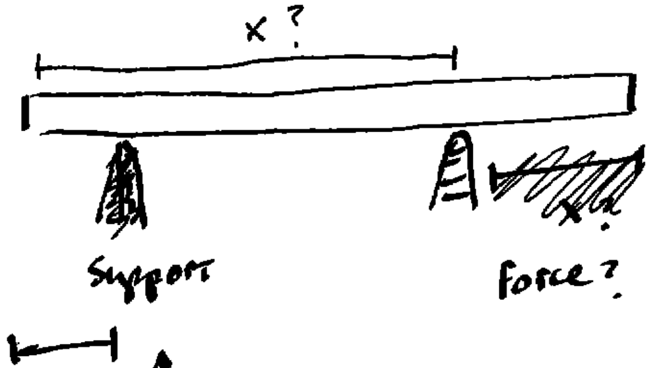
No linear Acceleration

And $\sum \vec{\tau} = \vec{\tau}_{NET} = 0$

No ROTATION

Both eqns are true at all points & all possible Axes of rotation

Example



uniform Beam
$M = 12$ kg
$L = 10$ M
$W = 117.6$ N

This one pushes up w/ $\left(\frac{1}{3}\right)(12\text{ kg})\,9.8\,\text{m/s}^2 = 39.2$ N

System is in static equilibrium.
Where is the 2ND support located?
How much weight does it support?

Recall:

We can treat the mass of an extended body as being concentrated at a single point called the center-of-mass.

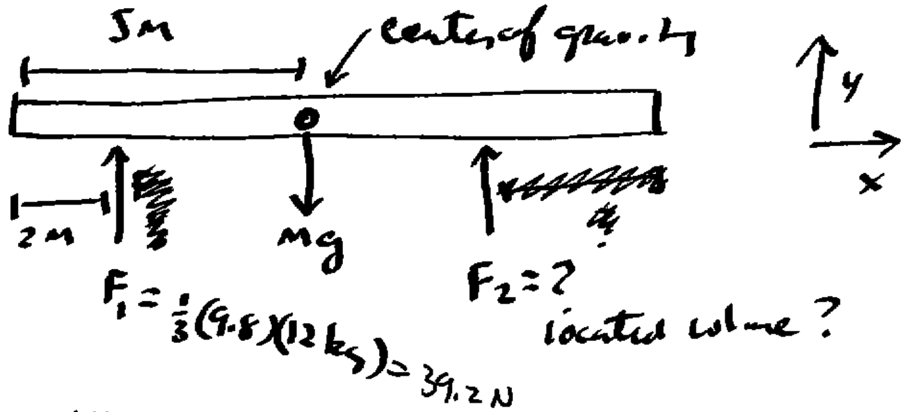
i.e., an external force acting on the body ~~is the same~~ will have the same effect as that force acting on the center of mass.

So ... When thinking about the effect of gravity in static equilibrium problems, it can be treated as if it acts on a single mass located at the center of mass of the extended body (usually obvious due to symmetry)

$\Longrightarrow$ called the center-of-gravity.

<u>SO</u> back to our problem -

Can consider the mass of the plank to be concentrated at a ~~length~~ length $= \frac{1}{2} L$ (center of the plank)



Linear equilibrium

$$\Sigma F_x = 0 = ma_x = 0 \quad \text{does not give us any information}$$

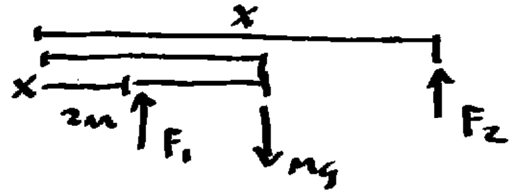
$$\Sigma F_y = 0 = F_1 + F_2 - Mg$$

Rotational Equilibrium

Pick an axis for rotation + be consistent

Where? ... Anywhere ... Make it somewhere easy for calculation

Let's sum Torques ~~about~~ around left end



$$\sum \tau = 0 = (2) F_1 - Mg(5) + x F_2$$

2 eqns

$$\begin{cases} F_1 + F_2 - mg = 0 \\ 2F_1 + xF_2 - Mg(5) = 0 \end{cases}$$

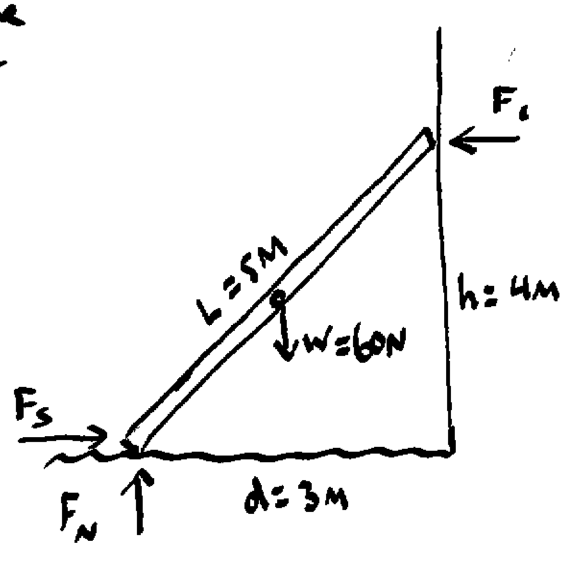
$$39.2\ N + F_2 - 117.6\ N = 0$$

$$F_2 = 78.4\ N$$

$$2(39.2\,N) + x(78.4\,N) - (117.6)5 = 0$$

$$x = 6.5\ m$$

Example



Ladder up against a frictionless ~~floor~~ wall

floor has friction

What is minimum coeffic. of static friction such that ladder will NOT slip?

Solve for μ_s such that ladder is in static equilibrium

$\Rightarrow$ this will be the minimum μ_s !

$$\Sigma F_x = m a_x = \underset{\text{static}}{0} = F_s - F_1 \quad \Rightarrow \quad F_s = F_1$$

$$\Sigma F_y = m a_y = \overset{\text{equilibrium}}{0} = F_N - W \quad \Rightarrow \quad F_N = W$$

$$\Sigma \tau_{\text{about base of ladder}} = F_1 h - W\frac{d}{2} = \underset{\text{static equilibrium}}{I\omega = 0}$$

Could choose any axis $\quad F_1 h = W\,d/2$

Choose about base of ladder because it means 2 forces $\rightarrow$ give zero torque $\rightarrow$ gives the simplest calculation

3 eqns

$F_s = F_1 \quad \longrightarrow \quad \mu_s F_N = F_1$

$F_N = W \quad\quad F_N = Mg = 60\,N$

$F_1 h = W\,d/2 \quad \longrightarrow \quad \mu_s F_N h = W\,d/2$

$$\mu_s = \frac{60\,\frac{3}{2}}{4\;60} = \frac{3}{8}$$

μ_s must be greater than $3/8$ for ladder to be stable

Note : μ_s is independent of M

Add a painter ... what happens

painter climbs ladder $\rightarrow$ creates a larger + larger torque

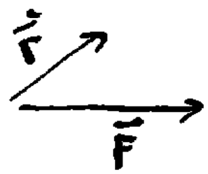
F_1 must become bigger to counterbalance ↑

eventually F_1 becomes larger than $\mu_s F_N$

and no longer have $\Sigma F_y = 0$

Static equilibrium broken - ladder falls

Practice w/ directions -



what direction is $\vec{r} \cdot \vec{F}$?

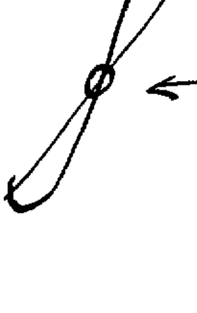
No direction $\rightarrow$ its a scalar

$\vec{r} \times \vec{F} = \vec{\tau} = 0$

Torque in what direction
$\vec{r} \times \vec{F}$ into paper

cause $\vec{\alpha}$ in what direction ?

into paper



$\leftarrow$ Propeller

direction of $\vec{\omega}$?

direction of L ?

Suppose Axis sticks out of paper +
I push on Axis to the left ...
What direction does the propeller system
go? $\leadsto$ Down