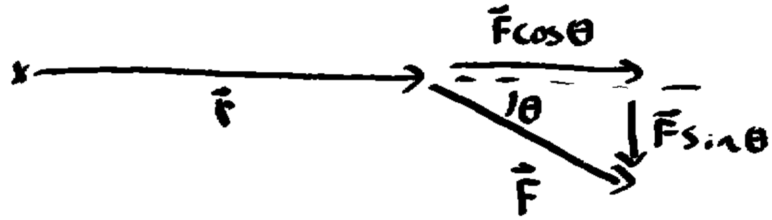
P121 Lecture

3/22/01



note: component of $\vec{F}$ along $\vec{r}$ cannot have an effect on the rotational motion!

... just pulls on axis more or less

component of $\vec{F}$ $\perp$ to $\vec{r}$ is the component that ~~rotates body~~ ... provides force (Torque)

Torque effective in producing rotational motion

↳ $rF \approx (\vec{r})(\vec{F}\sin\theta)$

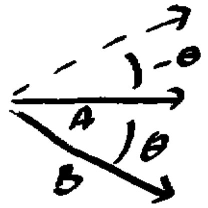
in dot product recall

$$\vec{A}\cdot\vec{B} = |\vec{A}|\,(\text{component of } \vec{B} \parallel \text{ to } \vec{A})$$

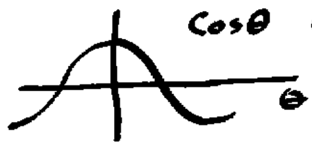
→ scalar ‖

Another way to say $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ $\quad |\text{comp of } \vec{A} \parallel \text{ to } \vec{B}|\,|\vec{B}|$

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta = |\vec{A}||\vec{B}|\cos(-\theta)$$



$|\vec{B}|\cos\theta = |\vec{B}|\cos(-\theta)$

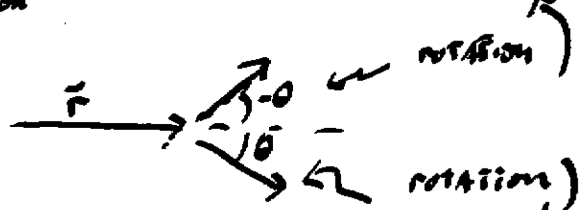


$\cos\theta$ is symmetric fn about 0

in our case this is NOT True

Torque $= |\vec{r}|$ (~~comp~~ component of F $\perp$ to $\vec{r}$)

But NOW

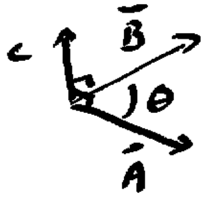


Sign of θ matters!

$\sin\theta$ is NOT a symmetric fn

Vector cross product $\Rightarrow$ (Two vectors $\rightarrow$ vector)

$\vec{A} \times \vec{B} = \vec{C} = |\vec{A}||\vec{B}| \sin\theta$ w direction given by Right hand rule



Right Hand Rule

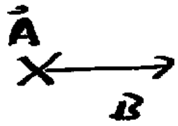
Put fingers of Right hand along vector $\vec{A}$ ... curl toward $\vec{B}$

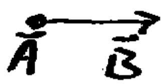
Thumb will point along $\vec{C}$

$\vec{B} \times \vec{A} = ?$

... in opposite direction!!

C Give me directions of $\vec{C}$ for practice





Tell Patti Eller story abt Right hand Screw rule

You can see this has all the parts we need for Making a Mathematical Model of Torque

$\vec{\tau} = \vec{r} \times \vec{F} = |\vec{r}||\vec{F}| \sin\theta$

↖ θ is opening angle between $\vec{r}$ and $\vec{F}$

$|\vec{F}|\sin\theta$ is $\perp$ component of $\vec{F}$ (relative to $\vec{r}$)

$\vec{A} \times \vec{B} = 0$ if $\vec{A}$ // to $\vec{B}$

$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|$ if $\vec{A}$ at $\angle$ (Angle) to $\vec{B}$

$\hat{i} \times \hat{i} = 0 = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

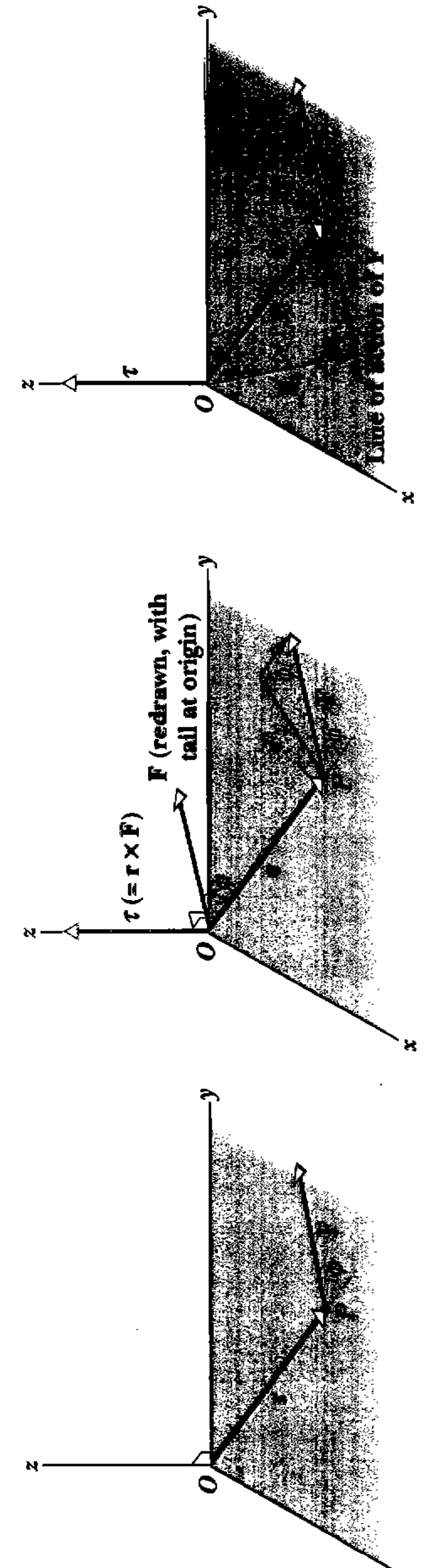
$\hat{k} \times \hat{i} = \hat{j}$

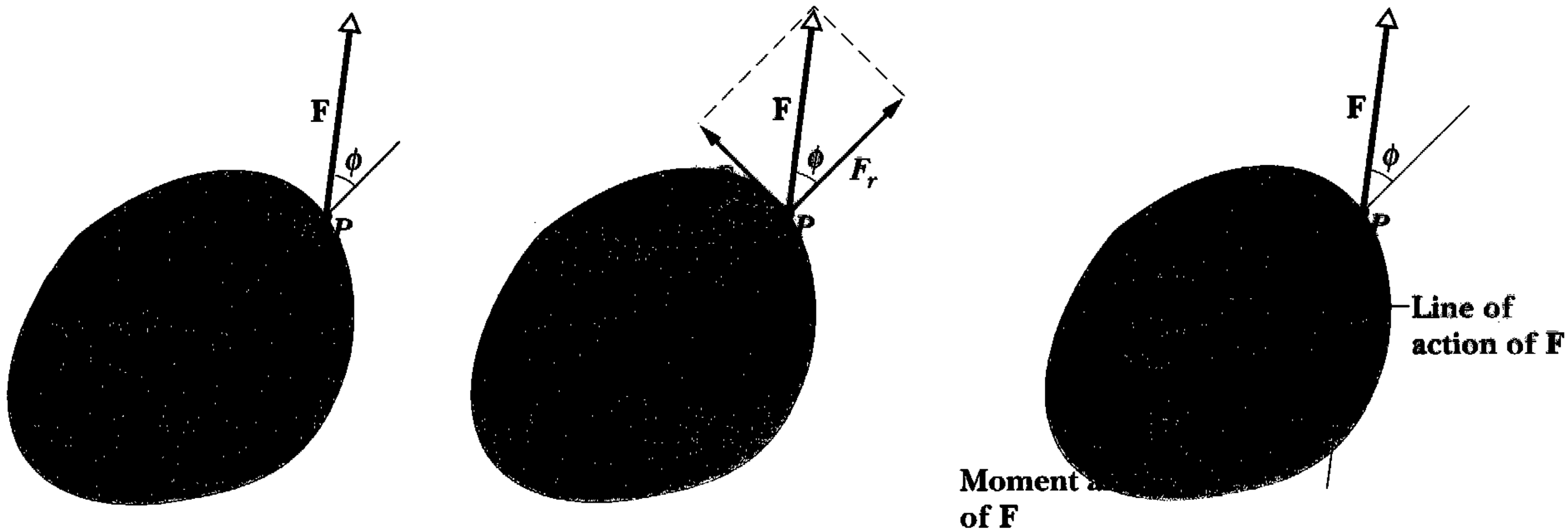
} what one means by right handed coordinate system

$\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}| \sin\theta$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

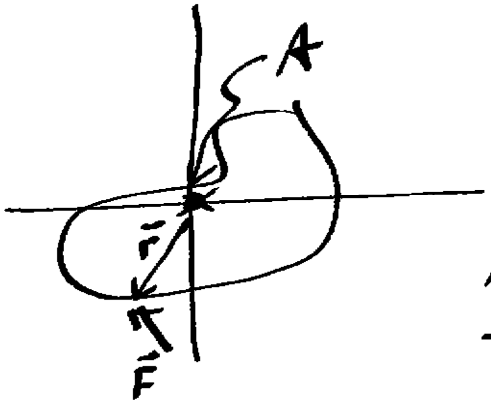
$$= \hat{i}(A_y B_z - A_z B_y) - \hat{j}(A_x B_z - A_z B_x) + \hat{k}(A_x B_y - A_y B_x)$$


z
y
x
O
(a)
τ (= r × F)
F (redrawn, with tail at origin)
(b)
τ
Line of action of F
(c)



(*a*) (*b*) (*c*)

rigid body able to rotate about an Axis A



A force $\vec{F}$ acts on the body at a position $\vec{r}$ relative to the Axis of rotation.

The Torque of $\vec{F}$ about A is given by

$$\vec{\tau} = \vec{r} \times \vec{F}$$

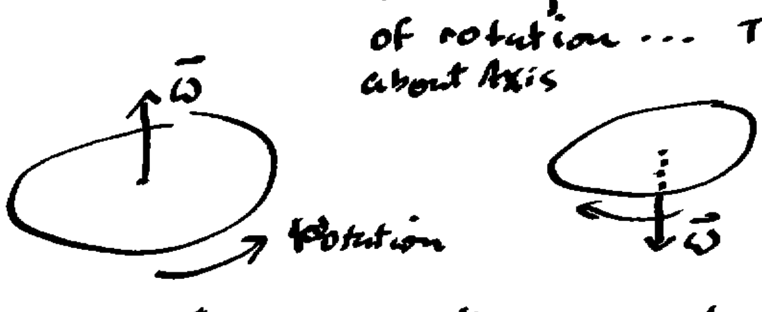
in example above $\vec{\tau}$ would be into paper

it would act to produce a clockwise Angular Acceleration

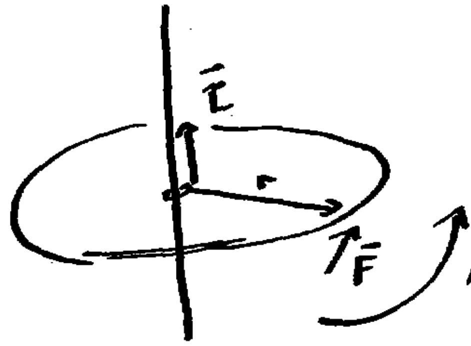
$\vec{\alpha}$, $\vec{\omega}$ are vectors, too! what are their directions?

determine direction w/ the second of our right hand rules

curl fingers of Right hand in direction of rotation about Axis ... Thumb will point in direction of ω

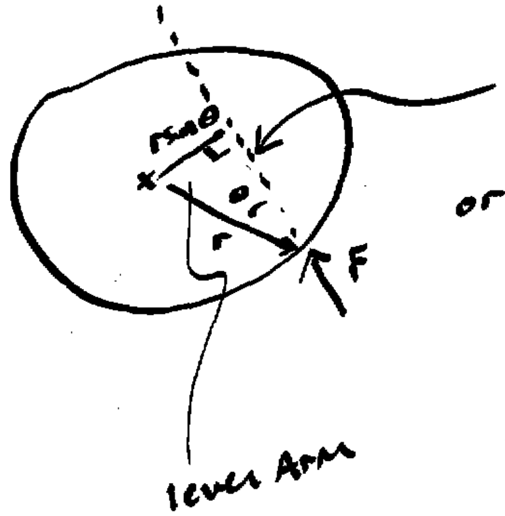


Same for $\vec{\alpha}$ but curl fingers in the direction of α increasing ω about axis. Thumb will point in direction of $\vec{\alpha}$

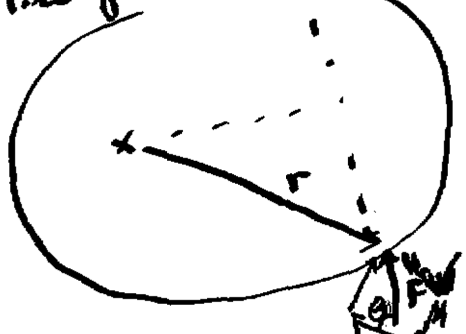


Torque is a vector up
usually drawn at axis of rotation

Motion
Counter clockwise

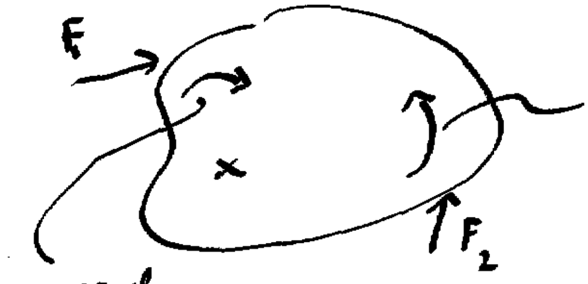


or



$\vec{r} \times \vec{F} =$
$|\vec{F}||\vec{r}|\sin\theta$

$\vec{r} \times \vec{F} = |\vec{r}||\vec{F}| \sin\theta$
Torque



sense of rotation is counterclockwise
due to force 2

⊕ Torque about Axis

sense of rotation caused by F_1 is Clockwise

⊖ torque about Axis

Unit of torque is N·M