Soln's to Prob set #7 posted (2 days?)

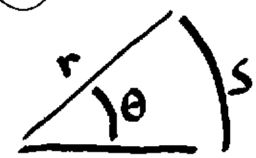
Prob set 8 is posted

Worry about Procrastination

Rotational Motion NOT easy to learn

Motivation Slide → Think Circular

Think



$s = r\theta$

$v = r\omega$

$a = r\alpha$

→ const a eqns

⇓

const α eqns

$F = ma$

$F = mr\alpha$

$rF = rmr\alpha$

$rF = mr^2\alpha$

τ I

Angular force   Angular Mass

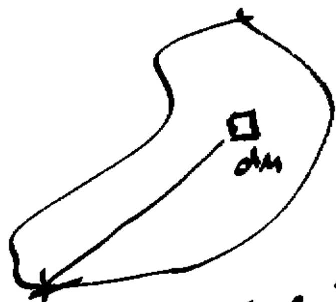
Torque   Moment of Inertia

$$\boxed{\tau_{NET} = I\alpha}$$

$$I = \int_{volume} r^2\,dm$$

↑

depends on Mass distribution



see Table p.278

SOLUTION a) The particle at point A lies *on* the axis. Its distance r from the axis is zero, so it contributes nothing to the moment of inertia. Equation (9–16) gives

$$I = \Sigma m_i r_i^{\,2} = (0.10\ \text{kg})(0.50\ \text{m})^2 + (0.20\ \text{kg})(0.40\ \text{m})^2$$
$$= 0.057\ \text{kg}\cdot\text{m}^2.$$

b) The particles at B and C both lie *on* the axis, so for them $r = 0$, and neither contributes to the moment of inertia. Only A contributes, and we have

$$I = \Sigma m_i r_i^{\,2} = (0.30\ \text{kg})(0.40\ \text{m})^2 = 0.048\ \text{kg}\cdot\text{m}^2.$$

Since this moment of inertia is less than in part (a), it's easier to make the body rotate about this axis than about the axis through point A.

c) From Eq. (9–17),

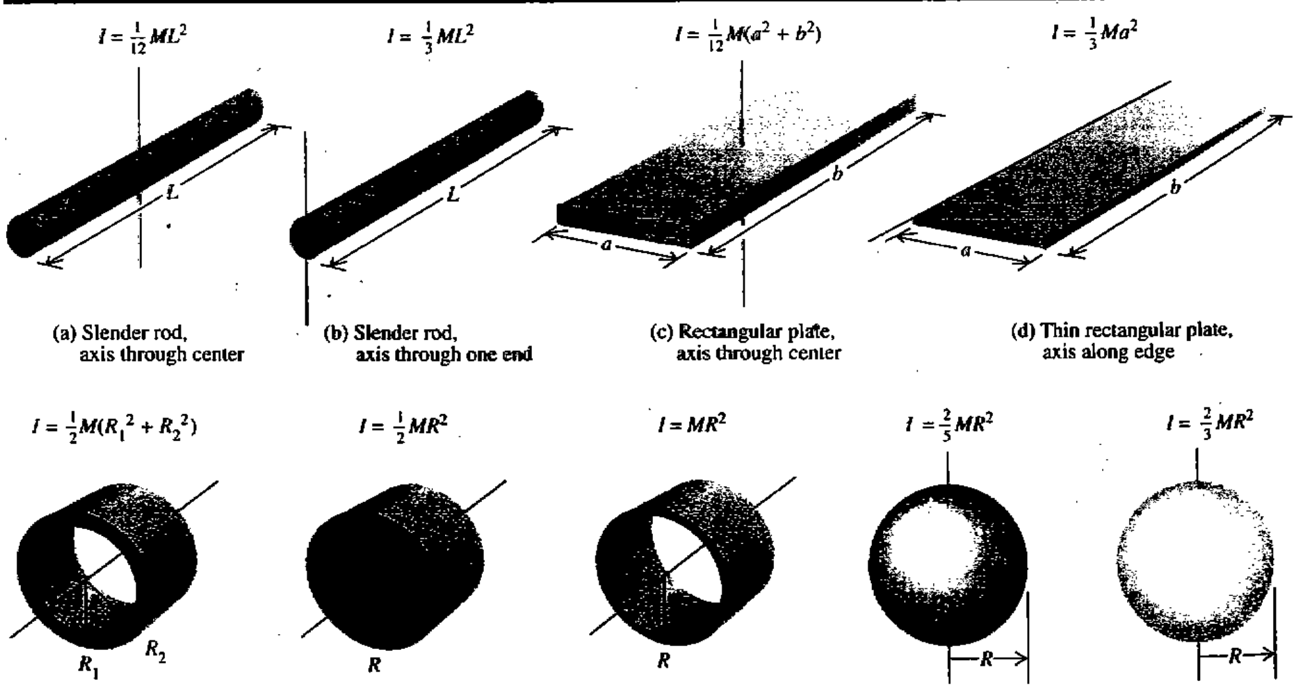
$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.057\ \text{kg}\cdot\text{m}^2)(4.0\ \text{rad/s})^2 = 0.46\ \text{J}.$$

7.7 Rotational Inertia

CAUTION▶ The results of parts (a) and (b) of Example 9–7 show that the moment of inertia of a body depends on the location and orientation of the axis. It's not enough to just say, "The moment of inertia of this body is $0.048\ \text{kg}\cdot\text{m}^2$." We have to be specific and say, "The moment of inertia of this body *about axis BC* is $0.048\ \text{kg}\cdot\text{m}^2$." ◀

In Example 9–7 we represented the body as several point masses, and we evaluated the sum in Eq. (9–16) directly. When the body is a *continuous* distribution of matter, such as a solid cylinder or plate, the sum becomes an integral, and we need to use calculus to calculate the moment of inertia. We will give several examples of such calculations in Section 9–7; meanwhile, Table 9–2 gives moments of inertia for several familiar shapes in terms of the masses and dimensions. Each body shown in Table 9–2 is *uniform;* that is, the density has the same value at all points within the solid parts of the body.

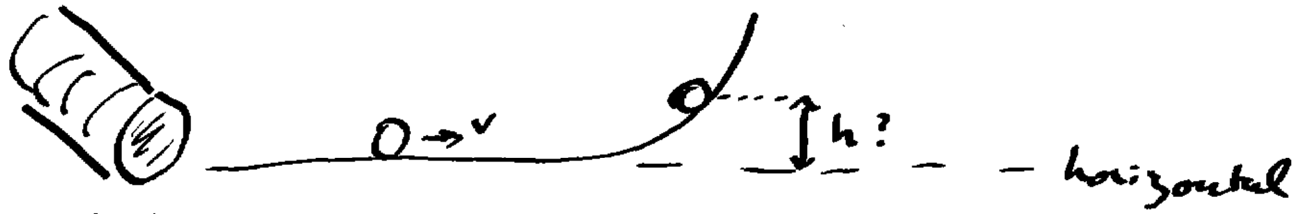
TABLE 9–2
MOMENTS OF INERTIA OF VARIOUS BODIES



(a) Slender rod, axis through center
(b) Slender rod, axis through one end
(c) Rectangular plate, axis through center
(d) Thin rectangular plate, axis along edge
(e) Hollow cylinder
(f) Solid cylinder
(g) Thin-walled hollow cylinder
(h) Solid sphere
(i) Thin-walled hollow sphere

$$\boxed{\text{rotational } KE = \tfrac{1}{2} I \omega^2}$$

Example Problem



Solid homogeneous cylinder

mass $M = (50\ \text{kg})$

radius $R = (15\ \text{cm})$

rolls w/ linear velocity V toward ramp $= (6\ \text{m/s})$

How far up ramp does cylinder climb?

Use Energy conservation

$$E_{TOT} = KE_{init} + PE_{init}$$

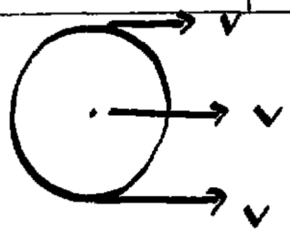
$$\cancel{KE_{final} - KE_{initial} = \Delta PE}$$

$$E_{TOT} = KE_{fin} + PE_{final}$$

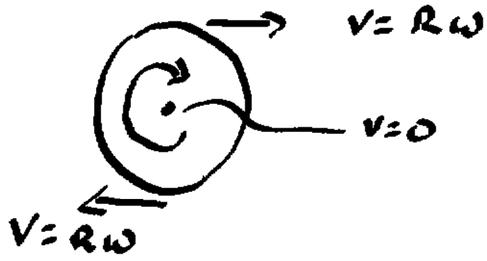
$$0 = \underbrace{KE_{final} - KE_{init}}_{\Delta KE} + \underbrace{PE_{final} - PE_{init}}_{\Delta PE}$$

$$-\Delta KE = \Delta PE$$

$$\underset{?}{K.E_{init}} - \underset{0}{KE_{final}} = \cancel{PE_{End}}\ \Delta PE = mgh$$

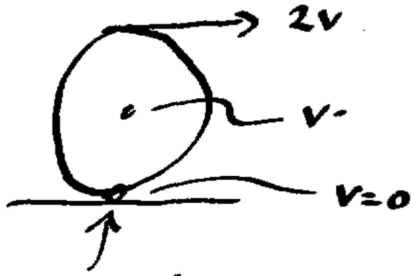


Translation only



Rotation only

combine the two



actual Axis of rotation at a given moment!

Think of motion as

1) ~~...~~ rotational motion about an axis
2) Translational motion of the ~~...~~ Axis + Associated mass

Typically ... axis passes thru center of mass
So you can think of part two as linear translation of C. M.

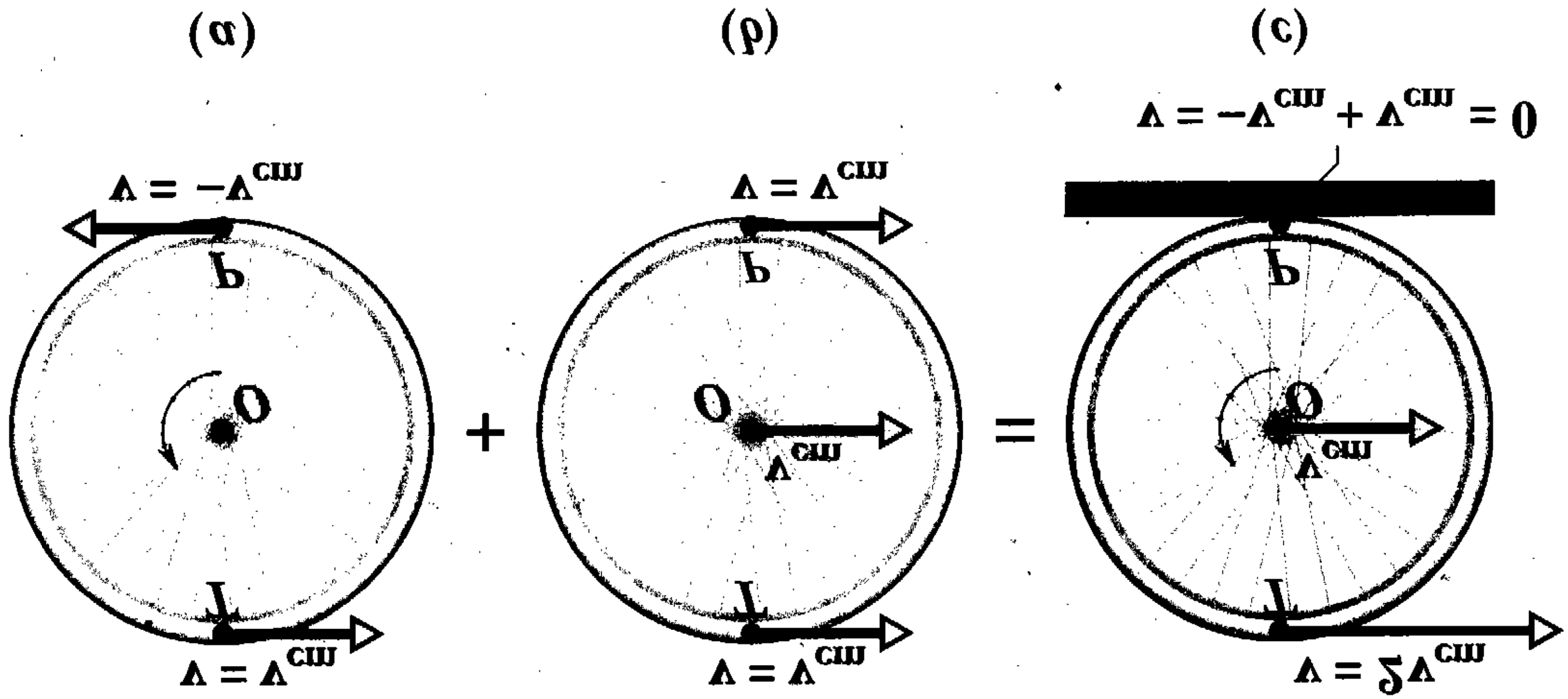
So

$$K.E. = \frac{1}{2} I \omega^2 + \frac{1}{2} M V^2 = Mgh$$

$V = R\omega$ so, know ω

What is I?

Must ~~...~~ evaluate or look at table $I = \frac{1}{2} MR^2$ in this case


(a)
(b)
(c)
$v = -v_{cm} + v_{cm} = 0$
$v = -v_{cm}$
$v = v_{cm}$
P
O
v_{cm}
T
$v = v_{cm}$
$v = v_{cm}$
$v = 2v_{cm}$

$$\frac{1}{2}\frac{1}{2}MR^2 \frac{v^2}{R^2} + \frac{1}{2}Mv^2 = Mgh$$

$$\frac{1}{4}Mv^2 + \frac{1}{2}Mv^2 = Mgh$$

$$\frac{3}{4}Mv^2 = Mgh$$

$$\boxed{\frac{3}{4}\frac{v^2}{g} = h}$$

$$\frac{m^2/s^2}{m/s^2} = m \quad \text{units} \checkmark$$

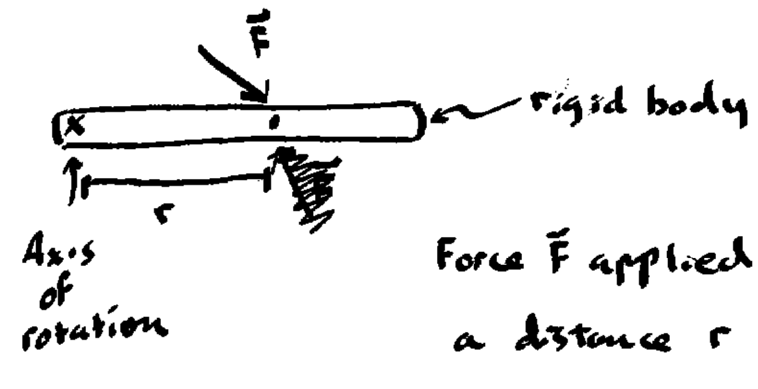
If sliding (non rotating)

$$\frac{1}{2}Mv^2 = Mgh$$

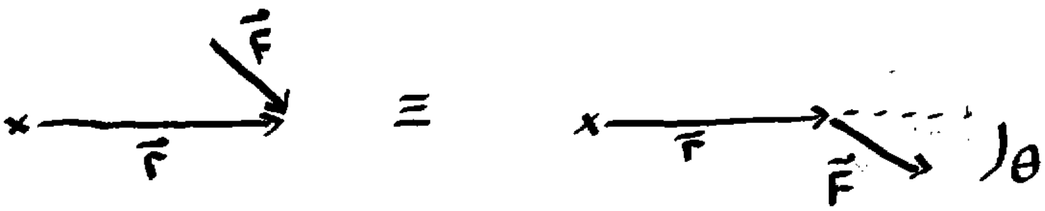
$$h = \frac{1}{2}\frac{v^2}{g}$$

Goes higher if moving at V and rotating because system starts w/ more KE.

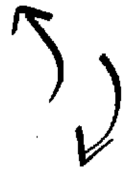
Rotational Motion and Vectors



Force $\vec{F}$ applied to rigid body at a distance r from axis of rotation



Does $\vec{\tau}$ have a direction?



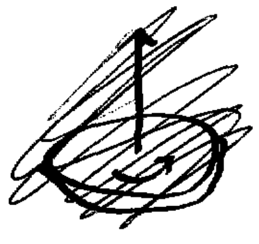
Does α have a direction?

$$\boxed{\vec{\tau} = I\vec{\alpha}}$$

How do we define these vectors

Specify $\begin{cases} \text{Axis of rotation} \\ \text{Magnitude of rotation (force / accel)} \\ \text{direction of rotation (force / accel.)} \end{cases}$

simultaneously



~~Right hand rule~~

~~fingers along direction of rotation~~

~~Thumb points along direction~~

Think about it ... we'll come back to it

let's continue to think in terms of scalars

rotating ball $\vec{V} = 0$

Is there kinetic energy associated with ~~rotation~~ this system?

small m_i at r_i

$KE_i = \frac{1}{2} m_i v_i^2 \qquad v = r\omega$

$K.E_i = \frac{1}{2} m_i r_i^2 \omega^2 = \frac{1}{2} I_i \omega^2$

$\Sigma KE_i = KE = \frac{1}{2}\omega^2 \Sigma m_i r_i^2 = \frac{1}{2} I \omega^2$