Example

Seawater
Specific gravity = 1.03

What fraction of an iceberg is submerged?

bouyant force = weight of displaced fluid = $\overset{\text{TOTAL}}{\text{Weight}}$ of iceberg

specific gravity = $\dfrac{\rho_{\text{material}}}{\rho_{H_2O \text{ at } 4^\circ C}}$

let V = Volume iceberg
x = submerged fraction

$$(\rho_{\text{seawater}})\, x\, V = (\rho_{\text{ice}})\, V$$

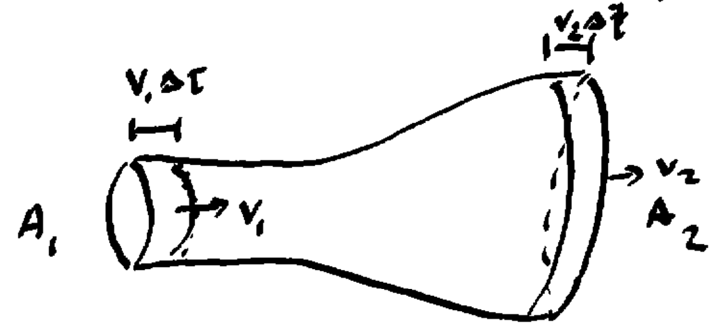
÷ both sides by ρ_{H_2O} at $4^\circ C$

$$(Sg_{\text{seawater}})\, x\, V = (Sg_{\text{ice}})\, V$$

$$x = \frac{\text{Spec. grav ice}}{\text{Spec. grav seawater}} = 89\%$$

Hydrodynamics or Fluid Dynamics

Consider an ideal Fluid
(No internal friction → viscosity
and is NOT compressible)



A pipe

fluid is incompressible so ρ is constant

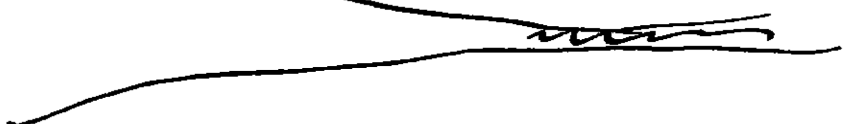
Amount of Fluid to flow in at ①
= Amount of fluid to flow out at ②

$$A_1 V_1 \Delta t = A_2 V_2 \Delta t$$

$$A_1 V_1 = A_2 V_2$$ equation of Continuity

River

when broad + deep ... water flows slowly



when narrow or shallow water flows fast!

height variable can be confusing

Example

Blood flow

radius Aorta = 1.0 cm

$|\vec{\text{Velocity}}|$ of blood thru Aorta is 30 cm/s

Capillary (Typical) $r = 4 \times 10^{-4}$ cm

$V_{blood} \sim 5 \times 10^{-4}$ m/s

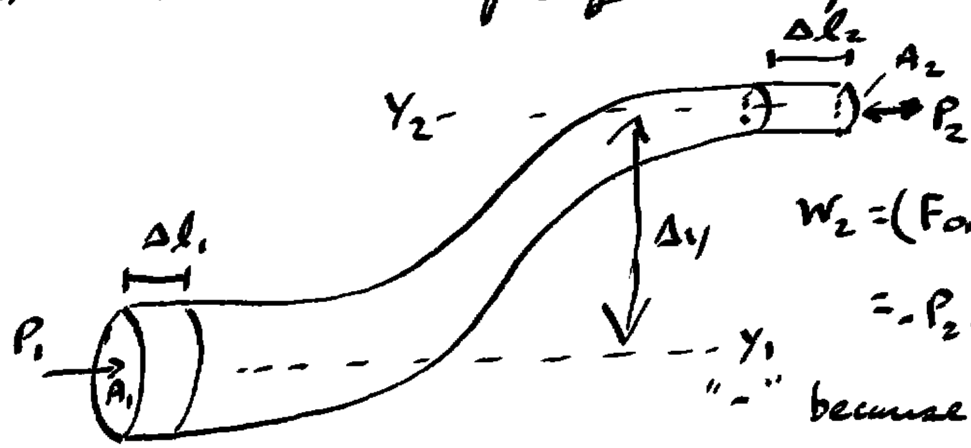
Estimate how many capillaries there are in body

$$A_1 V_1 = A_2 V_2$$

$$\left(\pi (.01)^2 \, m^2\right)(.3 \text{ m/s}) = N \pi (.04 \times 10^{-4} \, m)^2 (5 \times 10^{-4} \text{ m/s})$$

$$N = 4 \times 10^9 = 4 \text{ billion!}$$

let's be a little more careful about grav. P.E. + work, etc



$$W_2 = (F \text{ on } A_2)\, \Delta l_2$$

$$= -P_2 A_2 \Delta l_2$$

"$-$" because $\vec{F}$ and $\vec{\Delta x}$ in opposite directions

$$W_1 = (F \text{ on } A_1)\, \Delta l_1$$

$$W_1 = P_1 A_1 \Delta l_1$$

$W_1, W_2 \equiv$ work done on fluid by pressure

Also have work done on fluid by gravity

$$W_3 = -mg\Delta y = -mg(y_2 - y_1)$$

E_{TOT} is conserved ... so

$\Delta KE =$ net work done on fluid

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = W_1 + W_2 + W_3$$

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 + mgy_1 - mgy_2$$

$$m = \rho \underbrace{A_1 \Delta l_1}_{\text{volume of fluid element}} = \rho \underbrace{A_2 \Delta l_2}$$

$$\frac{1}{2}\rho A_2 \Delta l_2 v_2^2 - \frac{1}{2}\rho A_1 \Delta l_1 v_1^2 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 + \rho A_1 \Delta l_1 g y_1 + \rho A_2 \Delta l_2 g y_2$$

rearrange

$\div$ by $A_1 \Delta l_1 = A_2 \Delta l_2$

$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2}$$

– or –

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g y = \text{CONSTANT}}$$

Known as Bernoulli's Eqn

governs fluid dynamics for ideal (incompressible) flu

Why do you care?

Air foil on wing of airplane

Air on top moves faster than air on bottom

$$y_{top} \approx y_{bottom}$$

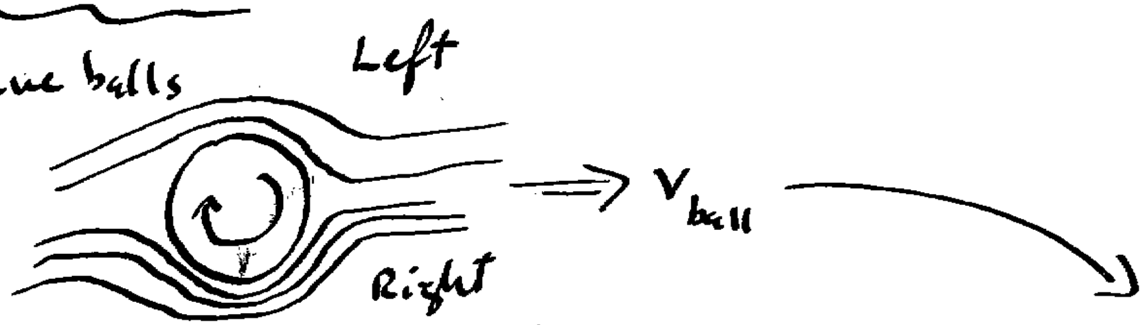
$$P_b + \frac{1}{2}\rho v_b^2 = P_{Top} + \frac{1}{2}\rho v_{Top}^2$$

$$V_{Top} > V_{bottom} \quad \therefore \quad P_{top} < P_{bottom}$$

This difference in pressure provides "Lift" (an upward force)!!

This is one component of why airplanes can fly!

Curve balls



Rotating ball: on ~~bottom~~ Right friction between ball surface and air causes increase in velocity of air passing by

on ~~top~~ left friction slows velocity of air passing by

$$\therefore \quad P_{\cancel{top}\ left} > P_{\cancel{bottom}\ right} \ \ldots \Rightarrow \text{ball curves left!}$$