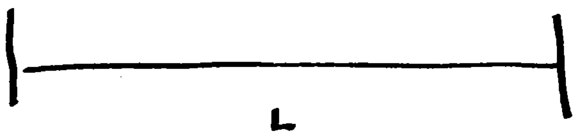
Amplitude at antinodes oscillates w/ time

at greatest it is $= 2A$

at least it is zero!

Nodes are every $\frac{1}{2}\lambda$

for a string tied on both ends ... each end must be a node



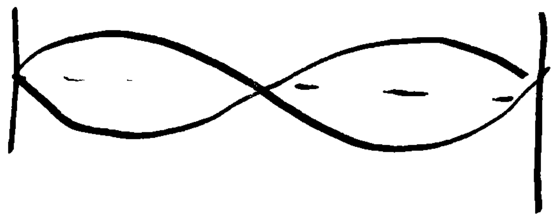
1st harmonic

These are so called Normal Modes!

fundamental frequency ... mode — ~~harmonic~~

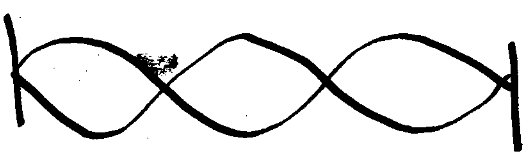
$\lambda = 2L$

overtones ↓



2nd harmonic

$\lambda = L$



3rd harmonic

$\lambda = \frac{2}{3}L$

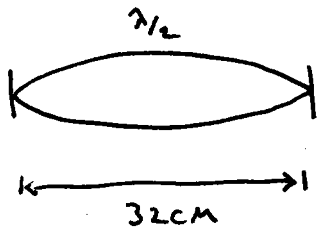
↗ $\lambda_n = \frac{2L}{n}$ $n = 1, 2, 3 \ldots$

wavelengths of the harmonics

ν_n = frequency of n^{th} harmonic $= \frac{v}{\lambda_n} = \frac{nv}{2L}$

Example

The G string on a violin has a fundamental frequency of 196 Hz. The length of the vibrating portion is 32 cm and has a mass of 0.68 g. What is the tension in the string?



$\lambda = 2(32\text{ cm}) = 64\text{ cm}$

$\nu = \frac{v}{2L}$

$(64)(196) = v$ in cm/s

$\mu = \frac{0.68\text{ g}}{32\text{ cm}} = $ g/cm $\quad v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\bigcirc}}$

$T = v^2 \mu$

<u>Vibrating Strings</u> → violins, guitars, cellos etc...
pianos

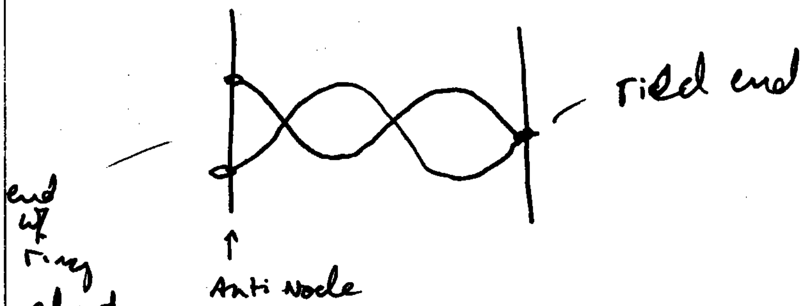
We have "normal modes" for vibrating sound waves in tubes. Now, instead of string displacement we have air displacement or pressure variation

Got to watch <u>Boundary conditions</u>!

String tied at each end ⟹ has a node at each end

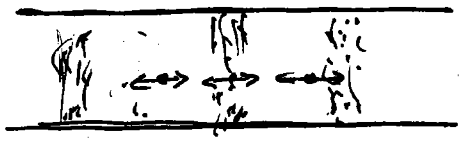
→ usual, but not only case

Could have



because string is free to move up and down

Consider standing sound waves in a tube



Tube w/ open end – Molecules ~~Air~~ can be easily moved

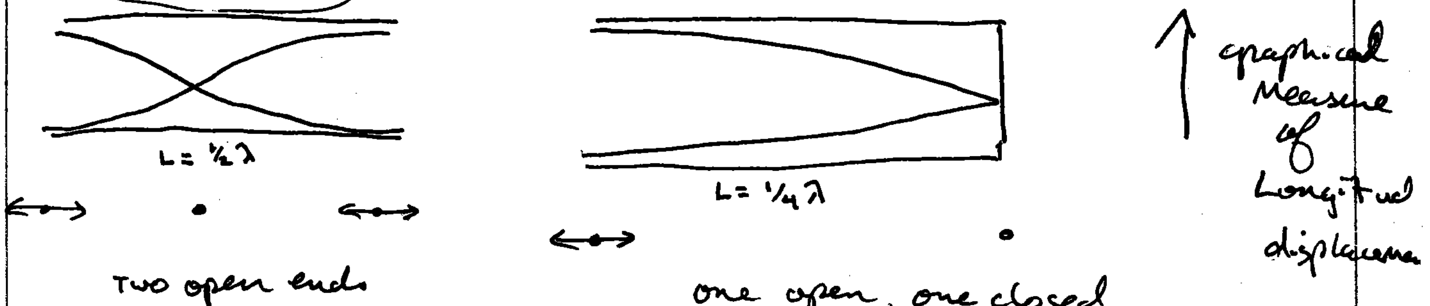
⇒ Displacement Anti Node

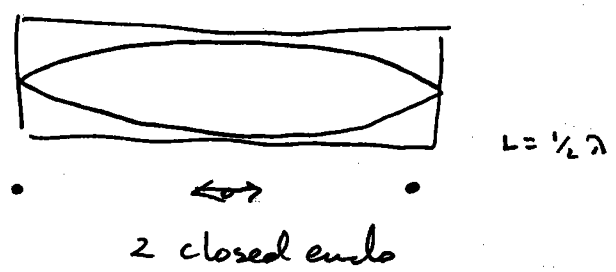
Tube w/ closed end

⇒ Displacement constrained

Displacement NODE

Fundamental Modes



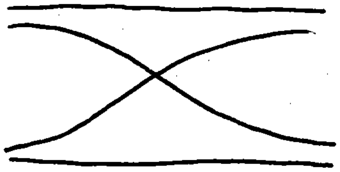


EXAMPLE What frequencies are available to an organ w/ 2 pipes of length L?
one open ended + one w/ 1 closed end

open end

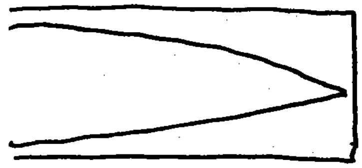
1 closed end

fundamental



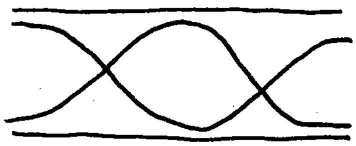
$L = \frac{1}{2}\lambda$

Fund.



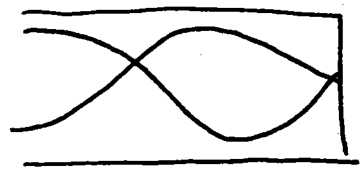
$L = \frac{1}{4}\lambda$

1ST harmonic



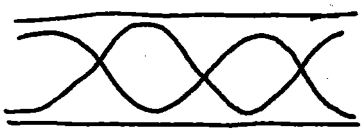
$L = \lambda$

1ST harmonic

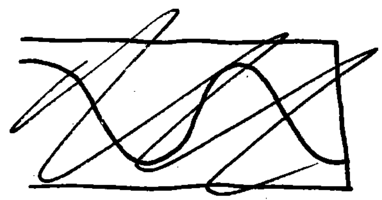


$L = \frac{3}{4}\lambda$

2ND harmonic



$L = \frac{3}{2}\lambda$



$L = \frac{n}{2}\lambda_n$

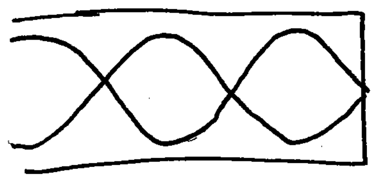
$\lambda_n = \frac{2L}{n} \quad n = 1, 2, 3 \ldots$

$v = \nu_n \lambda_n$

$\frac{v}{\lambda_n} = \nu_n$

$\nu_n = \frac{vn}{2L} \quad n = 1, 2, 3 \ldots$

2ND harmonic



$L = \frac{5}{4}\lambda$

$L = \frac{n\lambda_n}{4}$

$\lambda_n = \frac{4L}{n}$

$\frac{v}{\lambda_n} = \nu_n$

$\nu_n = \frac{vn}{4L} \quad n = 1, 3, 5$

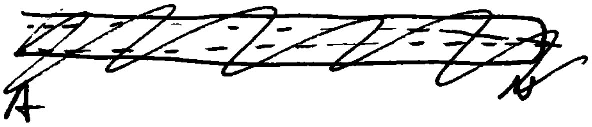
Only odd harmonics Available!

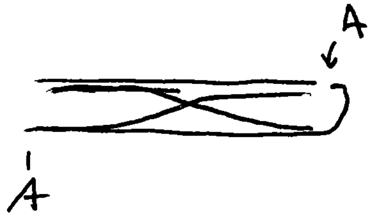
Example

Flute middle C (262 Hz)

as fund frequency

when all holes covered!





How long from mouthpiece to end.

Assume Temp = 20°c

V_{sound} Air at 20°c = 343 m/s

~~$\nu = \frac{V}{4L}$ $L = \frac{V}{4\nu} = \frac{343}{2}$~~

$\nu = \frac{V}{2L}$ $L = \frac{V}{2\nu} = \frac{343 \text{ m/s}}{2 (262 \text{ 1/s})}$ $= 0.655 \text{ m}$

Suppose the Temp is only 20°c

What is the fundamental frequency of the flute?

$\nu = \frac{V}{2L} = \frac{337}{2(.655)}$

V_{sound} at 10°c = 337 m/s

$\nu = 257$ Hz

Consider two waves passing a fixed point $(x=0)$

Let them differ slightly in frequency.

Let Amplitudes be equal.

$$x_1(t) = A \sin(k_1 x - \omega_1 t) = A \sin(\omega_1 t) \quad [\text{at } x=0]$$

↑ Displacement due to wave 1 + wave 2

$$x_2(t) = A \sin \omega_2 t$$

$$x(t) = x_1(t) + x_2(t) = A\left[\sin(\omega_1 t) + \sin(\omega_2 t)\right]$$

↑ Total Displacement <u>Principle of superposition</u>

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B)$$

$$x(t) = A\,2 \sin\left[\left(\frac{\omega_1+\omega_2}{2}\right)t\right] \cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t\right]$$

$\omega_1 + \omega_2 \approx 2\bar{\omega}$ Small frequency Diff $\bar{\omega}$ = Ave freq.

$$x(t) = A\,2 \sin(\omega t) \cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t\right]$$

wave vibrates at Average frequency

Amplitude Modulated w/ Time at a frequency that depends on the difference between ω_1 and ω_2

Beat frequency!

Very Sensitive way to detect small frequency differences