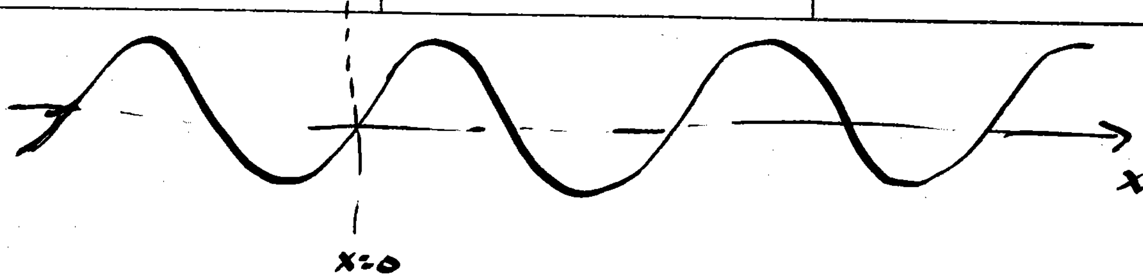


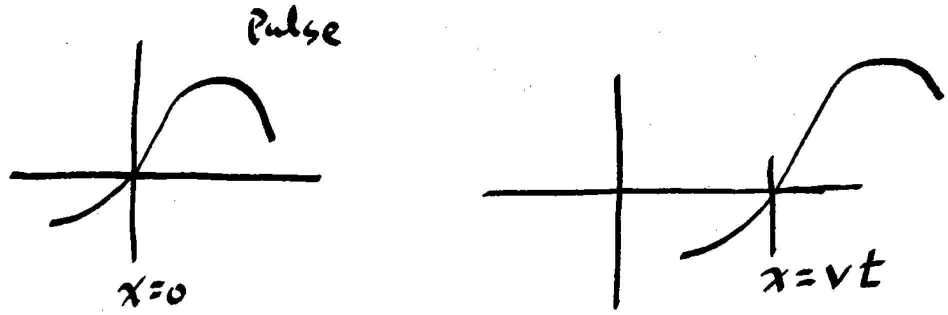
$y(x) = A \sin(kx)$

where A gives the Amplitude

$k = \frac{2\pi}{\lambda}$ because if $x = \lambda$ or 2λ or $n\lambda$ The fn must look the same

$k \equiv$ Wave Number

let's let the wave travel for a time ~~t~~ t



$y = f(x)$

time $= t$

$y = f(x - vt)$

So – Mathematically we can take a "frozen" (in time) equation of a wave and make it travel to the right by letting $x \rightarrow x - vt$

Similarly can make it go to the left w/ $x \rightarrow x + vt$

let's make our frozen periodic wave above travel to the right

$$y(x) = A \sin[k(x - vt)]$$
$$= A \sin(kx - kvt)$$

$$kvt = \frac{2\pi}{\lambda} v\, t = (2\pi \nu)\, t$$

$$\frac{v}{\lambda} = \nu \qquad 2\pi\nu = \omega = \frac{2\pi}{T}$$

$V = \lambda \nu$ — just as in SHM

So
Periodic wave moving to the right

$$y(x,t) = A \sin(kx - \omega t)$$

two variables now!

Could have arbitrary starting phase

use either Sin or Cosine

general eqn for Harmonic Wave —

$$y(x,t) = A \sin(kx - \omega t + \phi)$$

$$[\text{or} \quad A \cos(kx - \omega t + \phi)]$$

Transverse waves moving on a string are a useful example for demonstrating wave characteristics

What properties of a wave depend on the Medium??

Amplitude?

λ?

ν?

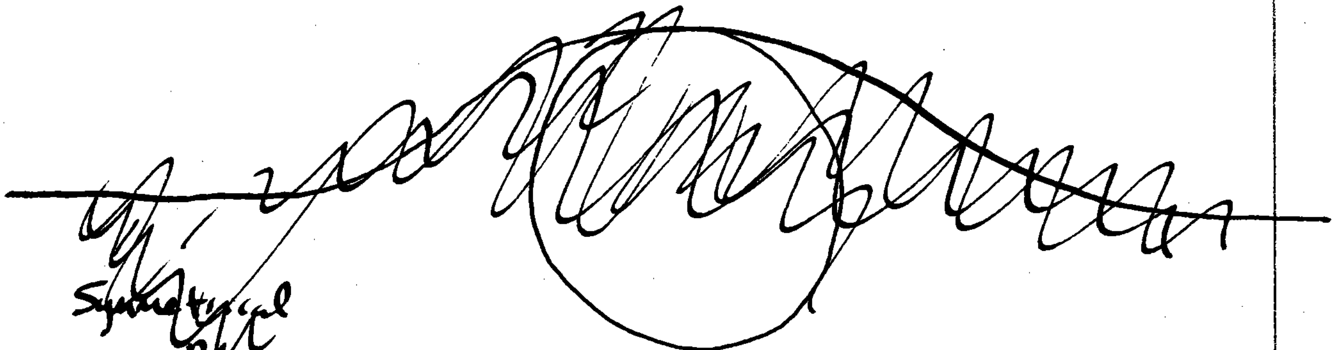
Velocity?

Know about medium should be able to calc. properties of wave

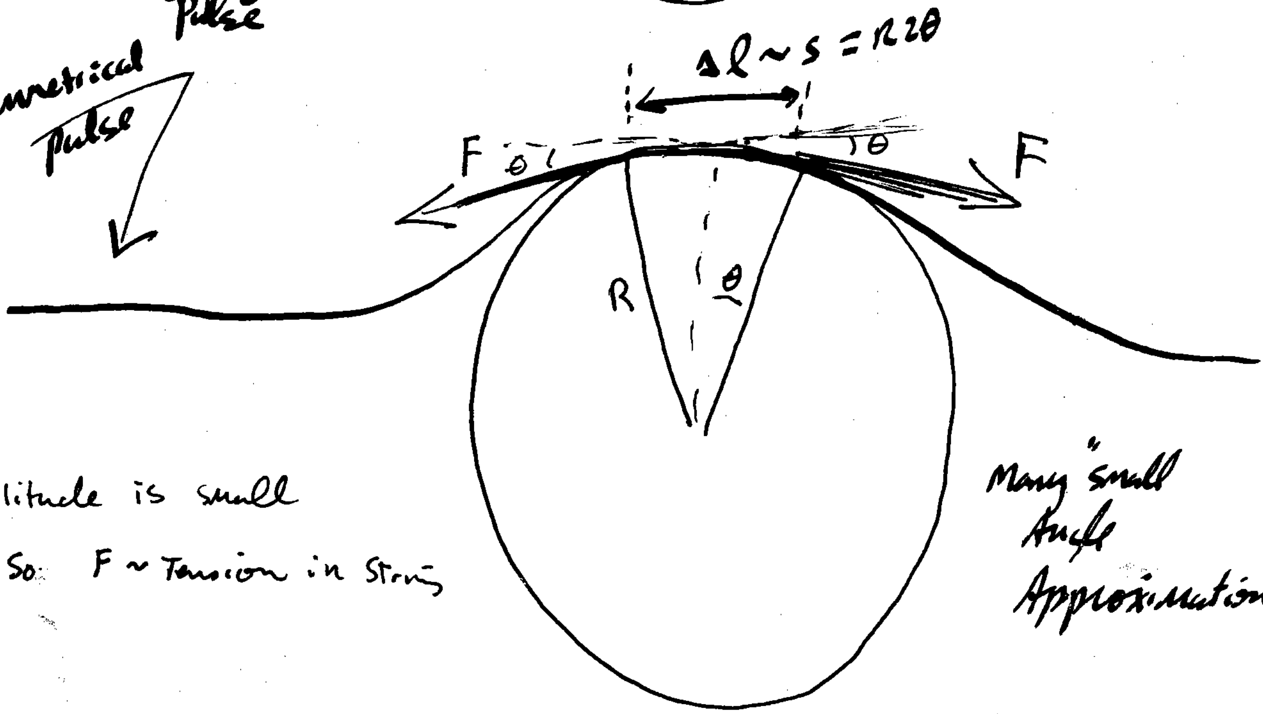
Calculate velocity of wave on a string w/ Tension $\equiv F$

and Mass/length $\equiv \mu$

To avoid confusion w/ Period



Symmetrical Pulse



Amplitude is small

So: $F \sim$ Tension in String

Many "small Angle Approximations"

Horizontal components of 2 F vectors cancel

Vertical restoring force $\sim 2F\sin\theta \sim 2F\theta \sim F\dfrac{\Delta l}{R}$

Mass of string segment $\Delta m = \mu \Delta l$

Mass Segment is Moving on a Circle for the moment

$\therefore \quad m\dfrac{v^2}{r} =$ vertical force

$$\sqrt{\frac{\text{elastic force factor}}{\text{inertia factor}}}$$

$$F\frac{\Delta l}{R} = \frac{\mu \Delta l}{R} v^2$$

or $\quad V = \sqrt{\frac{F}{\mu}}$ or $\sqrt{\frac{T}{\mu}}$ ~ Tension

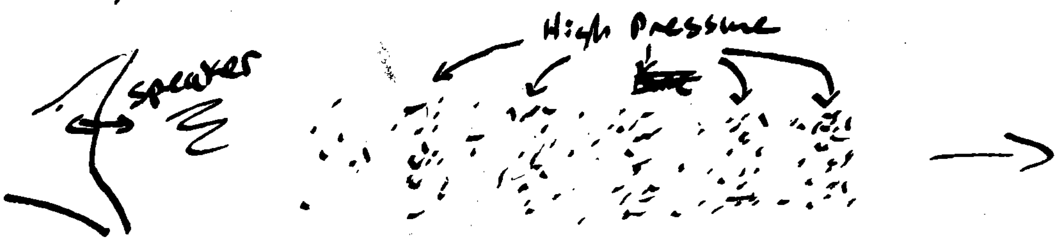
heavier rope → v smaller

larger Tension → V faster

for <u>longitudinal</u> <u>waves</u> have a similar thing

Such as Sound waves

longitudinal pressure variations in medium



$$V_{\text{longit. wave in liquid or gas}} = \sqrt{\frac{B}{\rho}}$$ ~ Bulk Modulus

just a Constant for given Medium

We did NOT cover → p. 342 of text

B ~ Tells how compressible a gas, fluid is

for ~~a~~ a given pressure

wave on a string $\quad v = \sqrt{\dfrac{\text{Tension}}{\mu = \text{mass/length}}}$

sound waves

$V = \sqrt{\dfrac{B}{\rho}}$ — Bulk modulus

ρ: volume density

Bulk modulus: defines the compressibility of medium

Transverse wave

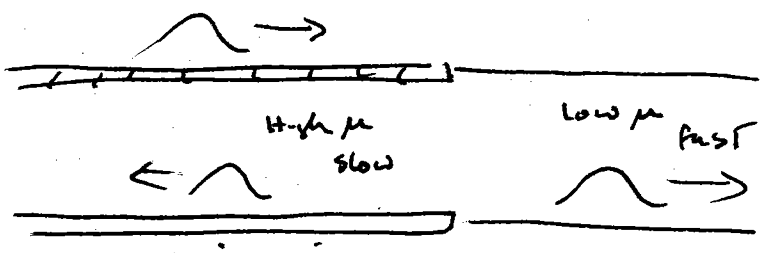
Fixed end

phase change of 180° at reflection

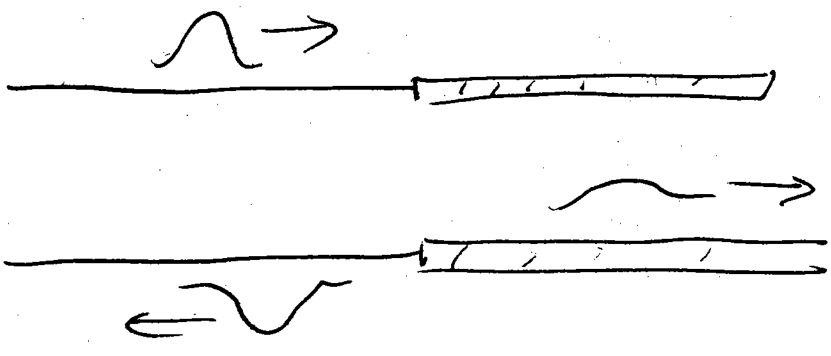
loose end

No phase change at reflection

Interface



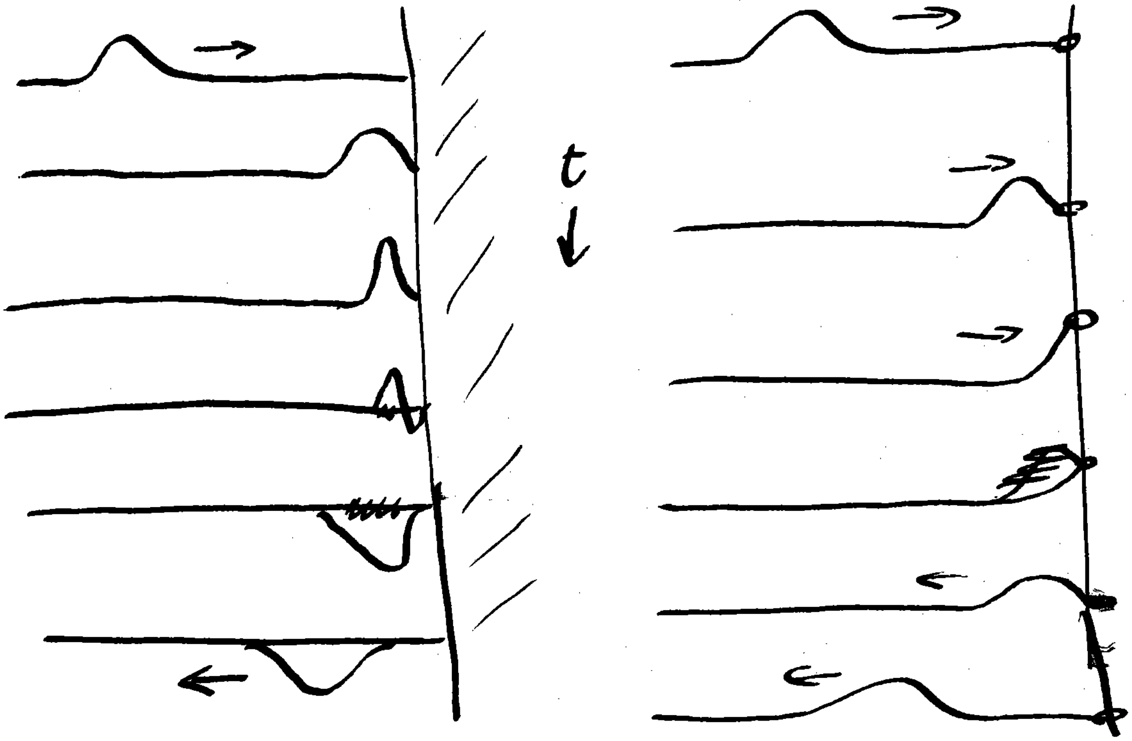
Slow to fast reflected wave has NO inversion



fast to slow reflected wave has inversion

Waves hitting obstacles

All reflected ... what is phase of reflected wave?



Fixed end

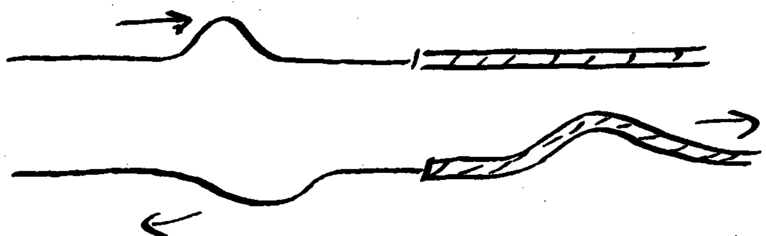
→ phase change of 180° at reflection

→ phase inversion

loose end

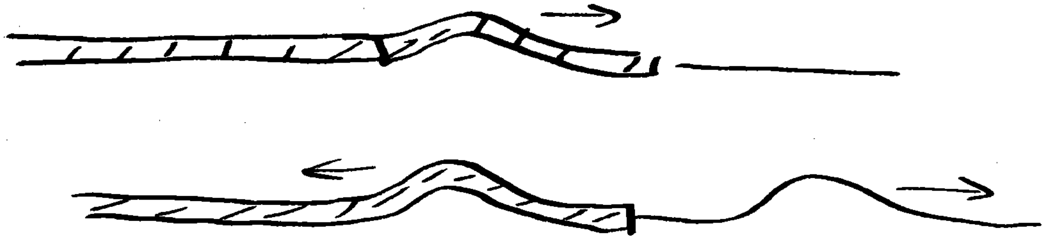
→ No phase change at reflection

Waves encountering an interface between Media w/ different characteristics have reflected and transmitted components



Wave incident on boundary

Travelling from "fast" to "slow" Medium



"slow to fast" media

TRANSmitted wave ... both cases → No phase inversion

Reflected wave

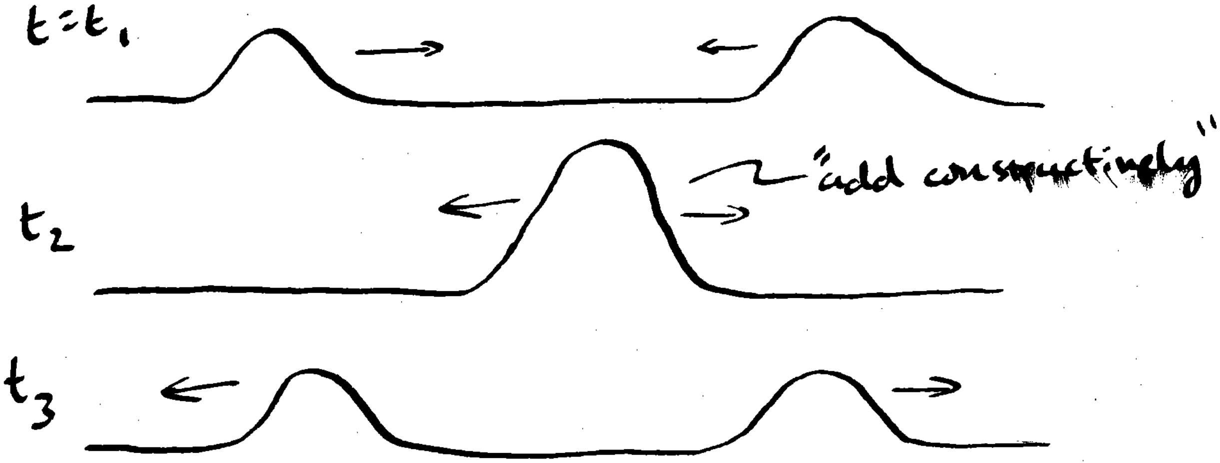
{ fast to slow / low μ to high μ } → $180°$ phase inversion

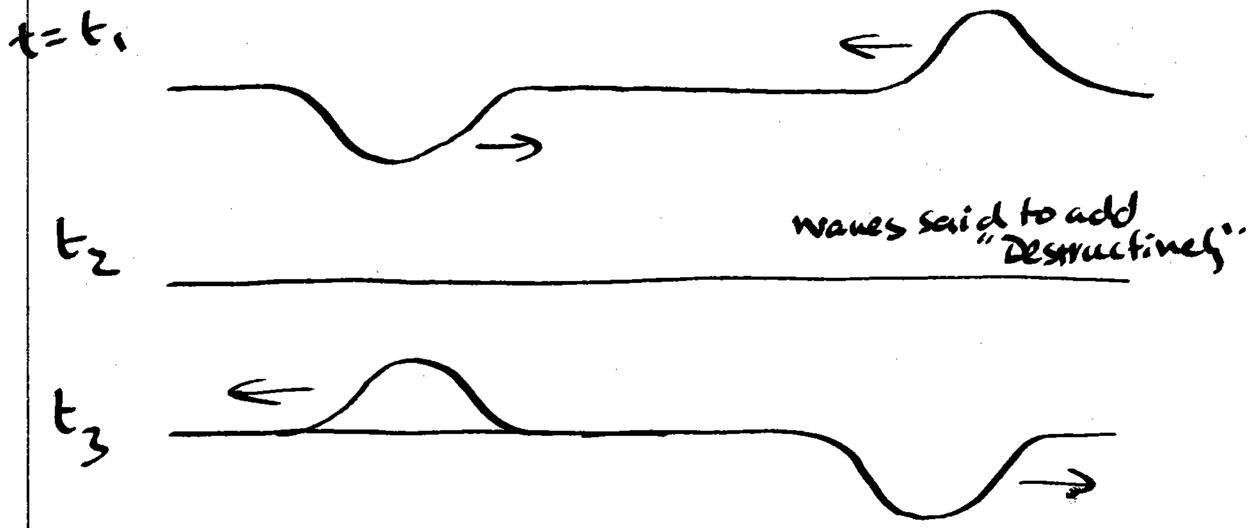
{ slow to fast / high μ to low μ } → No phase inversion

What happens when two waves meet one another

Principle of Superposition

When two or more waves pass thru a given point simultaneously the resultant displacement is the sum of the individual displacements





Two periodic waves superimposed on one another in some way are said to Interfere.

$\Rightarrow$ This is an incredibly rich + useful phenomenon!

Music
Antireflective coatings
best techniques for measuring small distances
Allows us to split light into colors
etc.

Consider 2 Harmonic waves w/ Amplitude A and frequency ω
Traveling opposite directions in a string
$\rightarrow$ one being the reflection of the other!

We've learned that two waves are out of phase by $180°$ due to the "boundary condition" of the fixed end at the wall

$$Y_1(x,t) = A \sin(kx - \omega t)$$

$$Y_2(x,t) = A \sin(kx + \omega t + \phi)$$

Bound condition $\rightarrow$ $Y_2(x,t) = A \sin(kx + \omega t + \pi)$

Trig identity $\quad A \sin(X + \pi) = -A \sin X$

$$Y_2(x,t) = -A \sin(kx + \omega t)$$

Principle of Superposition

$$\begin{aligned} Y(x,t) &= Y_1(x,t) + Y_2(x,t) \\ &= A \sin(kx - \omega t) - A \sin(kx + \omega t) \\ &= A \sin(kx - \omega t) + A \sin(-kx - \omega t) \end{aligned}$$

Trig identity

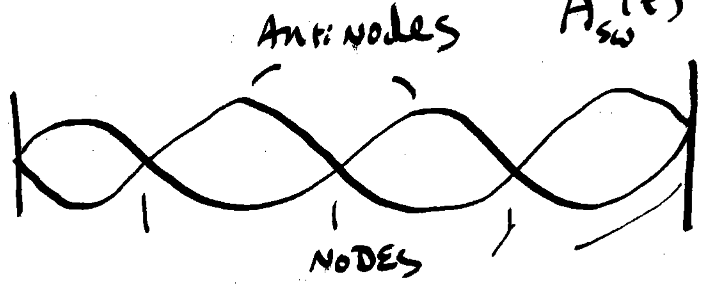
$$\sin A + \sin B = 2 \sin\left[\tfrac{1}{2}(A+B)\right] \cos\left[\tfrac{1}{2}(A-B)\right]$$

$$Y(x,t) = 2A \sin\left(-\frac{2\omega t}{2}\right) \cos\left(\tfrac{1}{2} 2kx\right)$$

$$Y(x,t) = \underbrace{-2A}_{\equiv A_{sw}} \sin(\omega t) \cos(kx)$$

$\equiv A_{sw} \equiv$ Amplitude of "STANDing Wave"

$$Y(x,t) = \underbrace{A_{sw} \sin(\omega t)}_{A_{sw}(t)} \cos(kx)$$



STANDing Waves