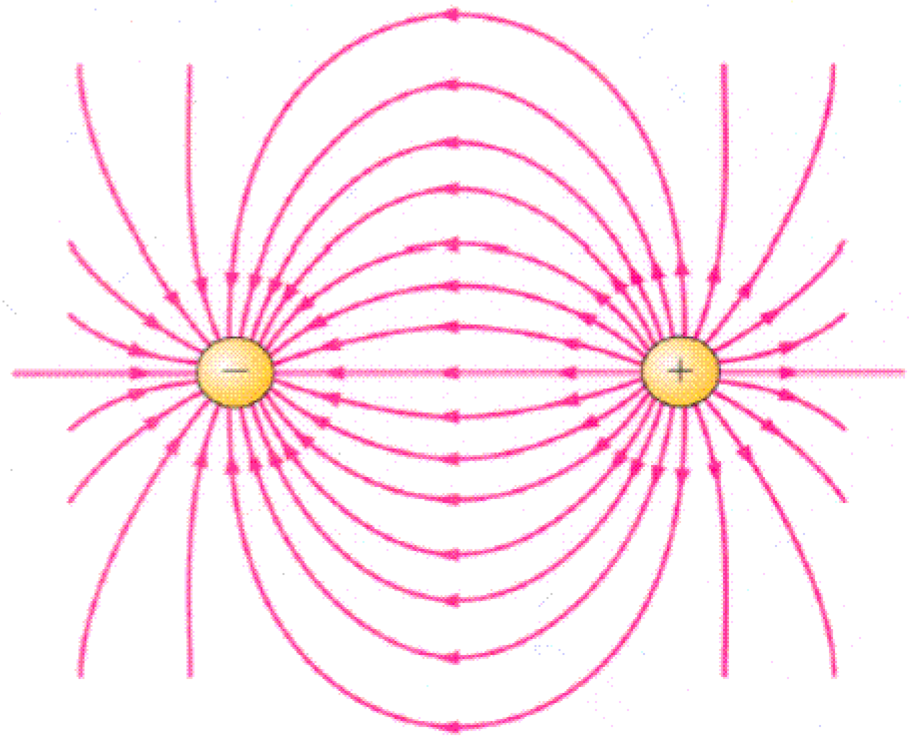
Physics 114 – February 2, 2006

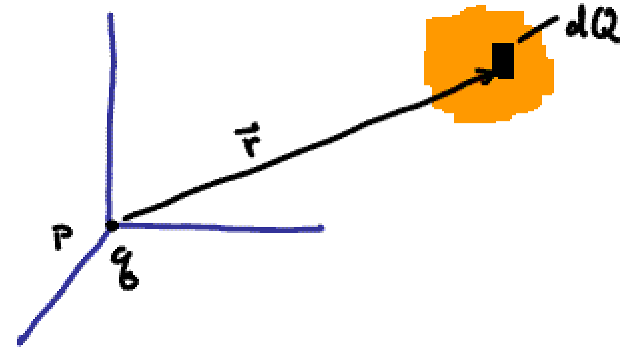
Last time:

Lines of Force



$\vec{E}_{\text{at } p}$ due to system of discrete charges $= \sum_{i=1}^{N} \frac{kQ_i}{r_i^2}\hat{r}_i$

$\vec{E}_{\text{at } p}$ due to continuous charge distribution



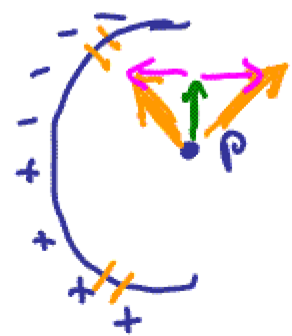
$$= \int_{\text{vol}} \frac{k\,dQ}{r^2}\hat{r} = \int_{\text{volume}} \frac{k\,\rho(\vec{r})}{r^2}\hat{r}$$

All of these things might change as you integrate over the volume

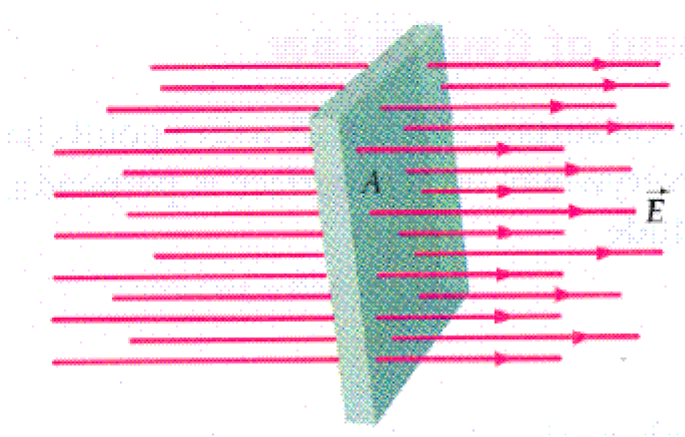
- might be hard.
- might not be so bad.

Show TEAL Demos for

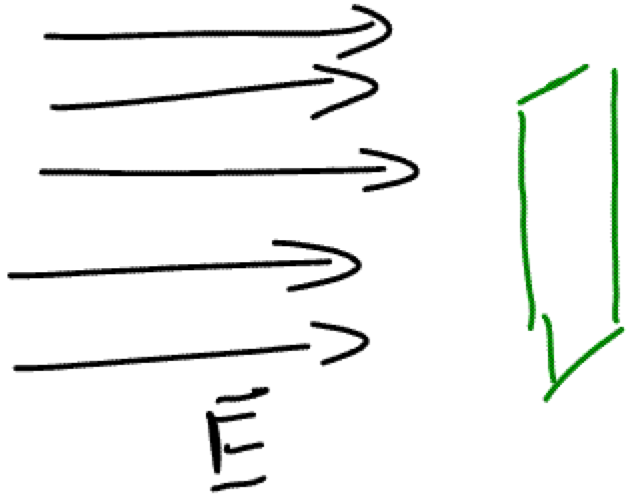
- constant λ line segment integration.
- constant λ circle integration.

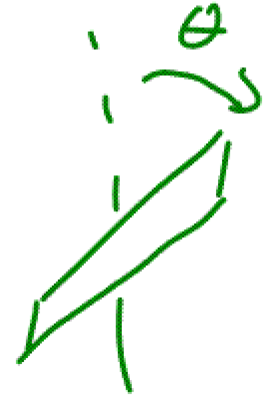


Electric flux

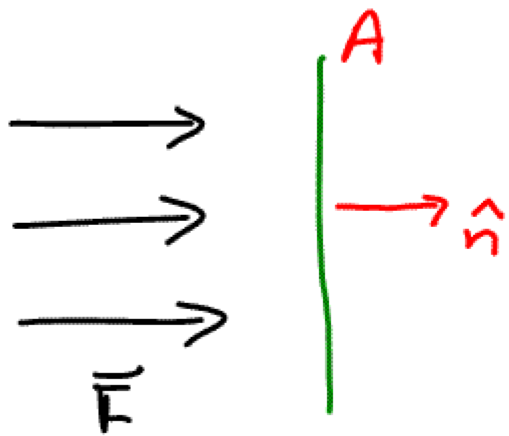


$EA = \text{flux}$



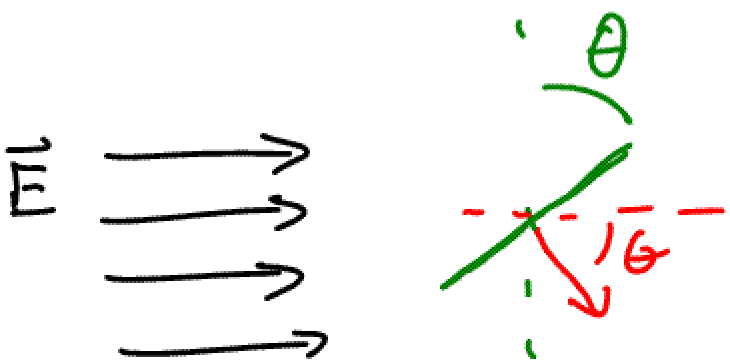


$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$$



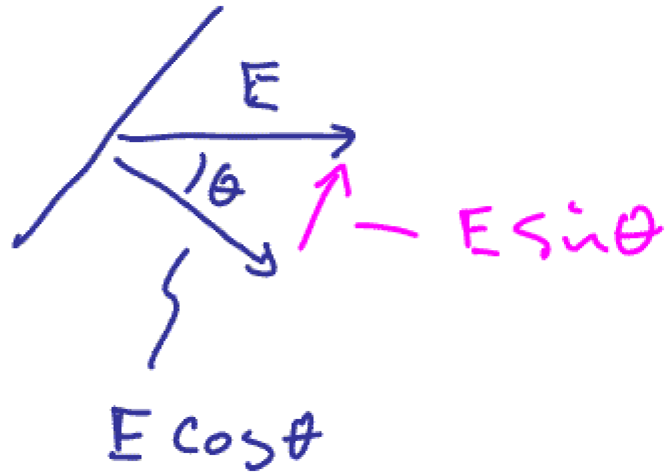
$$\text{Flux} = \vec{E}\cdot\hat{n}\,A$$

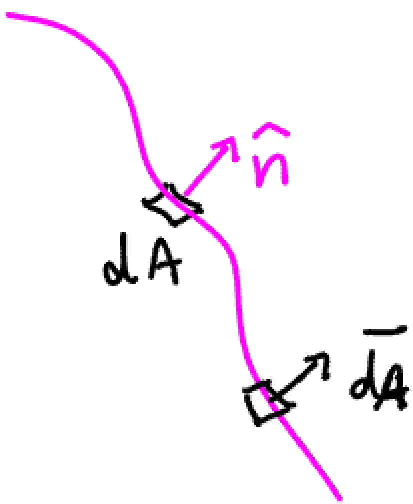
$$|\vec{E}|\,A$$

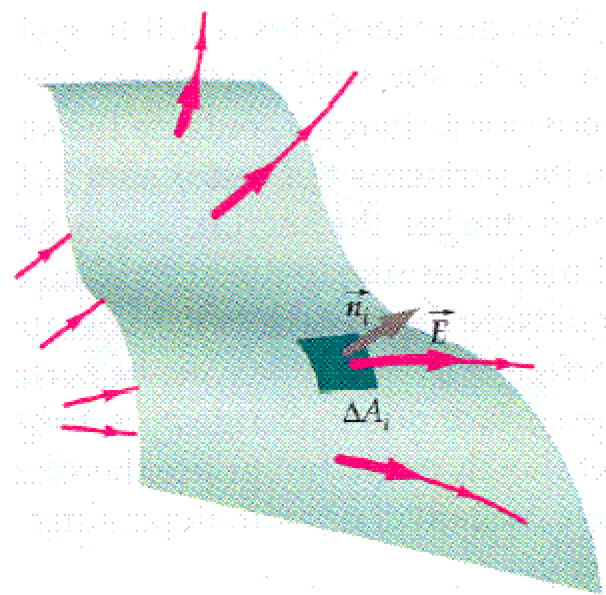


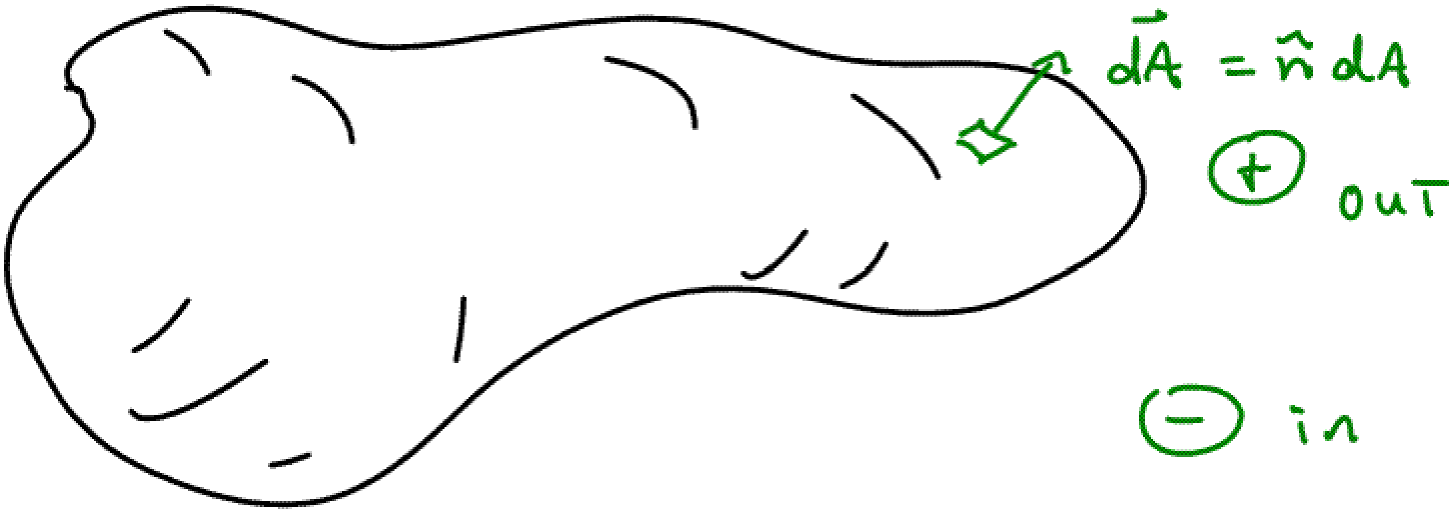
$$\text{Flux} = \vec{E}\cdot\hat{n}\,A$$

$$= |\vec{E}|\,A\cos\theta$$


E
θ
E sin θ
E cos θ

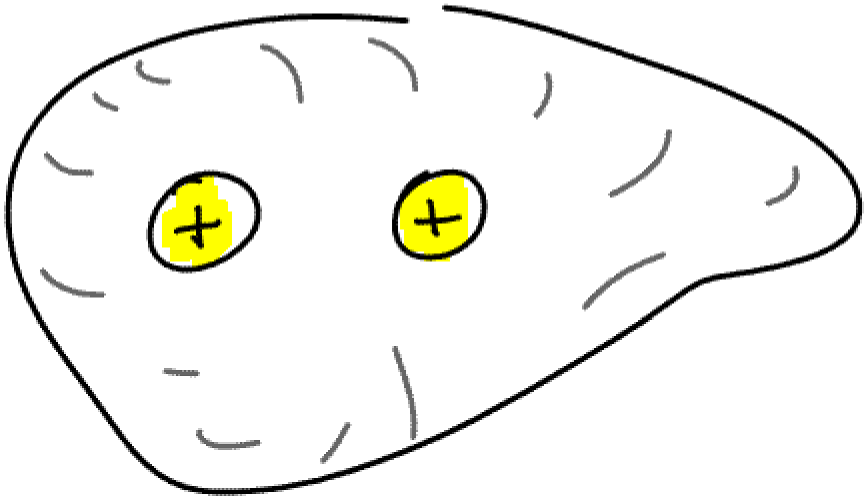

n̂
dA
d̄A


$\vec{n}_i$
$\vec{E}$
ΔA_i

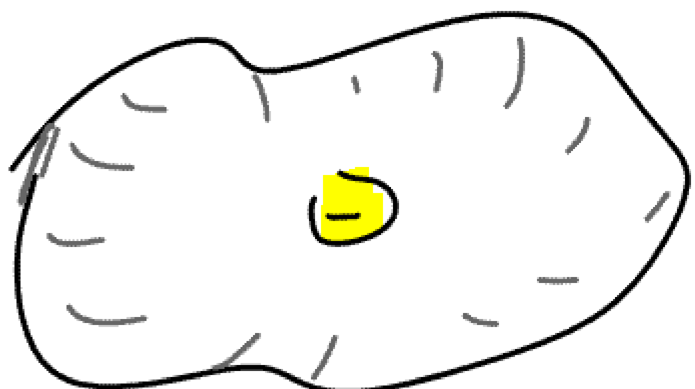


What is net flux thru surface

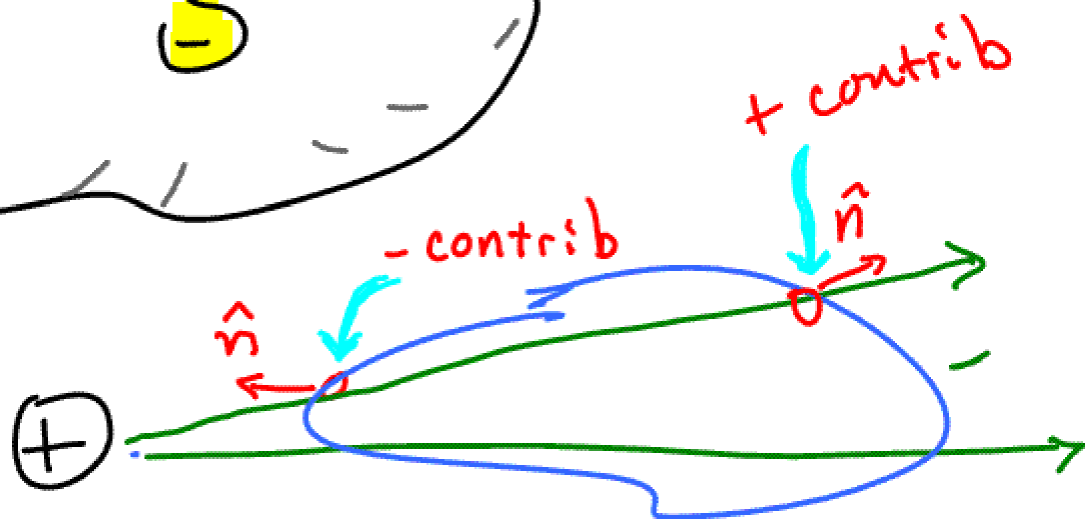
$$\text{Flux} \equiv \phi = \oint_{\text{surf}} \vec{E} \cdot \hat{n}\, dA$$



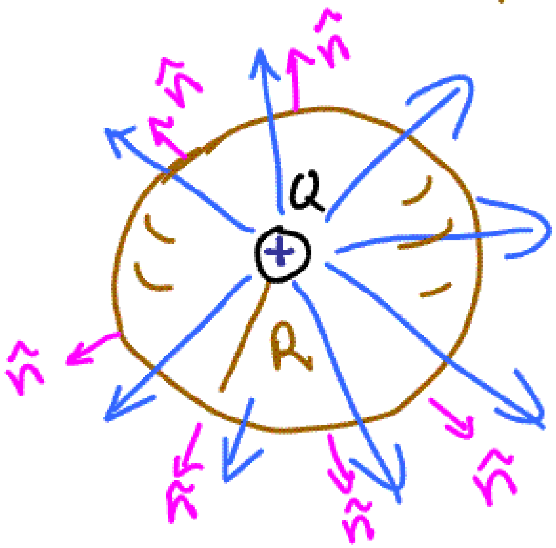
+ net flux



- net flux



Spherical surface
surrounding pt. chg.



$\vec{E}\cdot\hat{n}$ simple
$= |\vec{E}|$

$$\phi = \oint E\cdot\hat{n}\,dA = \int_S |\vec{E}|\,dA = |\vec{E}|\underbrace{\int dA}_{4\pi R^2}$$

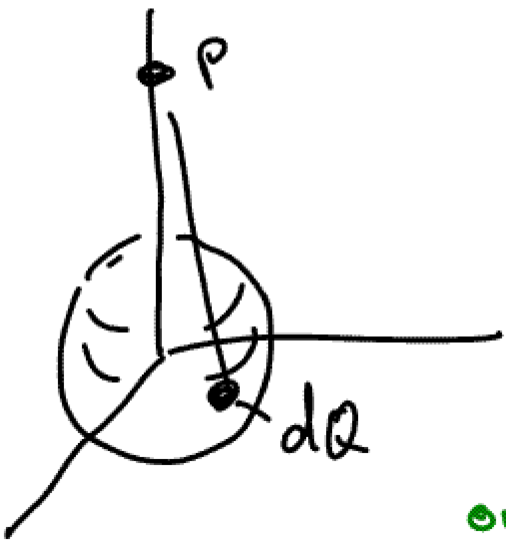
surf. of sphere

$$\phi = |\vec{E}|\,4\pi R^2$$

$$\phi = \frac{kQ}{R^2}4\pi R^2 = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}4\pi R^2 = \frac{Q}{\epsilon_0}$$

$$\boxed{\phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{Enclosed}}{\epsilon_0}}$$

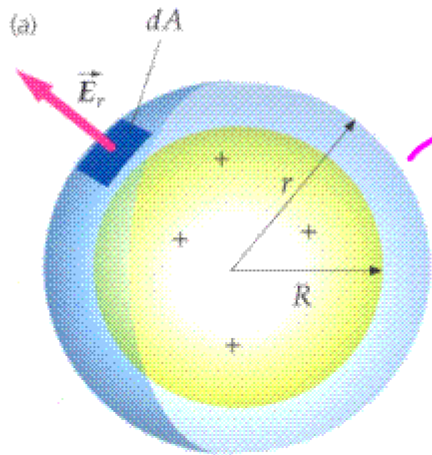
Gauss' Law



recall our tough example at end of last class

What is $\vec{E}$ at point P outside of uniform spherical charge distribution.

Let's go one better what is $\vec{E}$ in all of space?



Spherically symmetric Gaussian Surface centered on dist

Total charge = Q

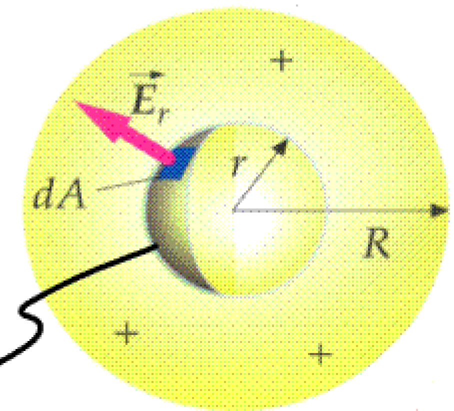
<u>r > R</u>

$$\underbrace{\oint \vec{E} \cdot d\vec{A}} = \frac{Q_{encl}}{\epsilon_0}$$

$$|\vec{E}| 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$|\vec{E}| = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{kQ}{r^2}$$

$r < R$



Gaussian Surface

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$\downarrow$

$$\oint |\vec{E}|\, dA = |\vec{E}| \int dA = |\vec{E}|\, 4\pi r^2 = \frac{Q_{encl}}{\epsilon_0}$$

charge inside Gaussian surf $= \int \rho \, dv$

ρ is constant $= \rho \int dv = \rho \frac{4}{3}\pi r^3$

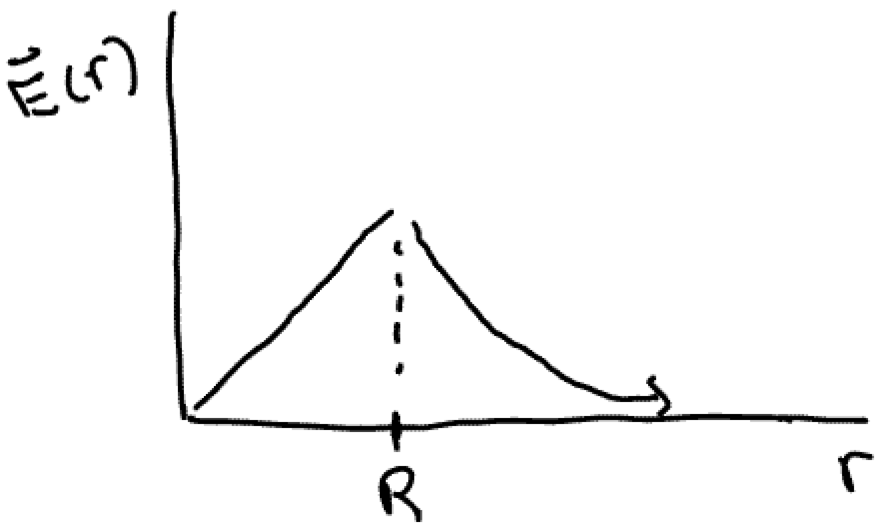
ρ = volume chg density

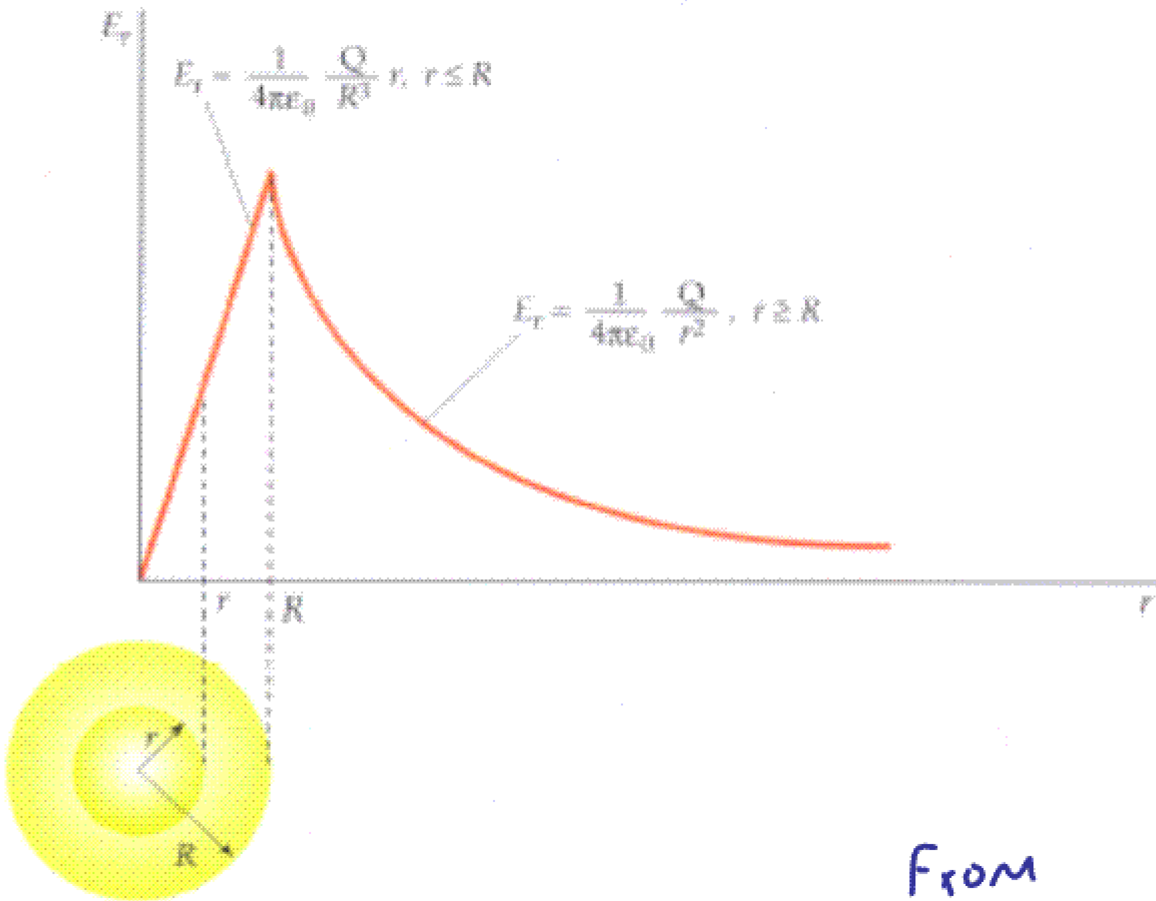
$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$|\vec{E}| 4\pi r^2 = \frac{Q}{\frac{4}{3}\pi R^3} \frac{4}{3}\pi r^3 \frac{1}{\epsilon_0}$$

$$|\vec{E}| = \frac{Q}{\epsilon_0 4\pi} \frac{r}{R^3}$$

direction
radially
outward
-> symmetry




E_r
$E_r = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3}r,\ r \le R$
$E_r = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2},\ r \ge R$
r
R
r
r
R

From Tipler

Gauss' law is useful under certain situations of symmetry

Spherical Symmetry

cylindrical symmetry

planar Symmetry