Last Time:

Charge is Conserved

Conductors

Insulators

Coulomb's Law for system of discrete charges:

\[
\vec{F} = \sum_{i=1}^{N} \frac{k q_i q_j}{r_i^2}
\]
Coulomb's law for a continuous charge dist.

\[ \vec{F} = \int \frac{k q dq'}{r^2} \hat{r} \]

\[ dq' = S(r) dv \]
\[ dq' = \nabla(r) da \]
\[ dq' = \lambda(r) dx \]

"doable ... sometimes nasty"

usually work w/ Electric field instead
\[ \vec{F} = \frac{k_q q_2}{r^2} \hat{r} \]

\[ E = \frac{\vec{F}}{q_1} = \frac{k q_2}{r^2} \]
\( \vec{E} = \frac{\vec{F}}{q} \)

\( \vec{E} \) vectors point radially outward (as does force)

Closer \( \vec{E} \) vector long \( \rightarrow \) Strong field

Farther Away \( \vec{E} \) vector Shorter here because field is weaken \( \rightarrow \) less force/charge
- Go in direction of $+$ Test charge.
- Density of lines $\rightarrow$ Strength of force/field
- Never cross
"Electric Dipole Field"

Lines of force around a couple of 2-charge distributions
Lines of force

Vicor figures from Textbook by Tipler

Lines of force are a useful way to visualize Electric Fields
Electric field, $\mathbf{E}$, for a point charge, $Q$.

\[\mathbf{F}_{at\ p} = \frac{kQq}{r^2} \hat{r}\]

due to $Q$.

\[\mathbf{E} = \frac{\mathbf{F}}{q} \]

\[\mathbf{E}_{at\ p} = \frac{kQ}{r^2} \hat{r}\]

due to $Q$. 

\[\mathbf{r} \]
\[\mathbf{r} \]
\[ \vec{F}_{at\rho} = \sum_{i=1}^{N} \frac{kQ_i \vec{g} \cdot \vec{r}_i}{r_i^2} \]

\[ \vec{E}_{at\rho} = \sum_{i=1}^{N} \frac{kQ_i \vec{r}_i}{r_i^2} \]
\[ dF = \frac{k q dQ}{r^2} \hat{r} \]

\[ dE = \frac{k dQ}{r^2} \hat{r} \]

\[ \overrightarrow{E}_{at \ p} = \int \frac{k dQ}{r^2} \hat{r} \]
Find $E$ at a distance $z$ above the midpoint of a line segment of length $2L$ that carries a uniform line charge of $+\lambda$.

Symmetry: $\rightarrow E$ straight up.
\[ d\vec{E} = \frac{k \, d\Omega \, \hat{r}}{r^2} \]

\[ \vec{E} = \int_{-\infty}^{\infty} \frac{k \, d\Omega \, \hat{r}}{r^2} \]

Symmetry only worry abt straight up component in \( \hat{z} \) direction
\[ E = \int_{-L}^{L} \frac{k^2 \lambda}{r^2} \, dx \cos \theta \]

\[ \vec{E} = \int_{-L}^{L} \frac{k^2 \lambda}{r^2} \, dx \frac{\hat{z}}{r} \]

\[ r = (x^2 + \hat{z}^2)^{1/2} \]

By symmetry of problem change limits from -L to 0 to L and mult.

\[ E = k^2 \lambda \hat{z} \int_{-L}^{L} \frac{dx}{(x^2 + \hat{z}^2)^{3/2}} = 2k^2 \lambda \hat{z} \int_{0}^{L} \frac{dx}{(x^2 + \hat{z}^2)^{3/2}} \]
Field of a point charge with total charge $\lambda L$ viewed from a distance $z$.

$$E = \frac{F}{q} = \frac{\text{Newton}}{\text{Coul}} \quad \frac{1}{r^2} \approx \text{Newton}$$

$$3 \to \text{large} \quad \frac{2k\lambda^2}{3(L^2 + z^2)^{1/2}} \longrightarrow \frac{2k\lambda L}{3^2}$$

Check units
Check limiting cases
This can be quite difficult. There are easier ways. I drag you through the mud here so you can appreciate the usefulness of Gauss' Law (Next Time)