Physics 114 - January 31, 2006

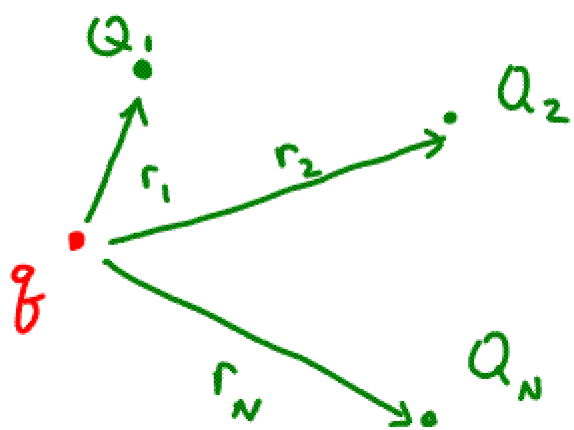
Last Time:

Charge is Conserved

Conductors

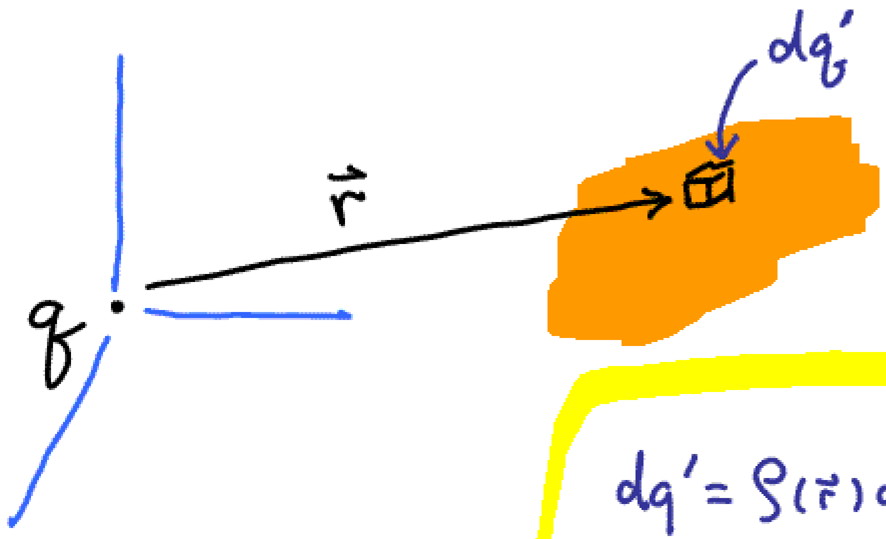
insulators

Coulomb's Law for system of discrete charges



$$\vec{F} = \sum_{i=1}^{N} \frac{k q Q_i}{r_i^2} \hat{r}_i$$

Coulomb's Law for a continuous charge dist.



$$\vec F = \int \frac{k\, q\, dq'}{r^2}\,\hat r$$

$dq' = \rho(\vec r)\,dv$

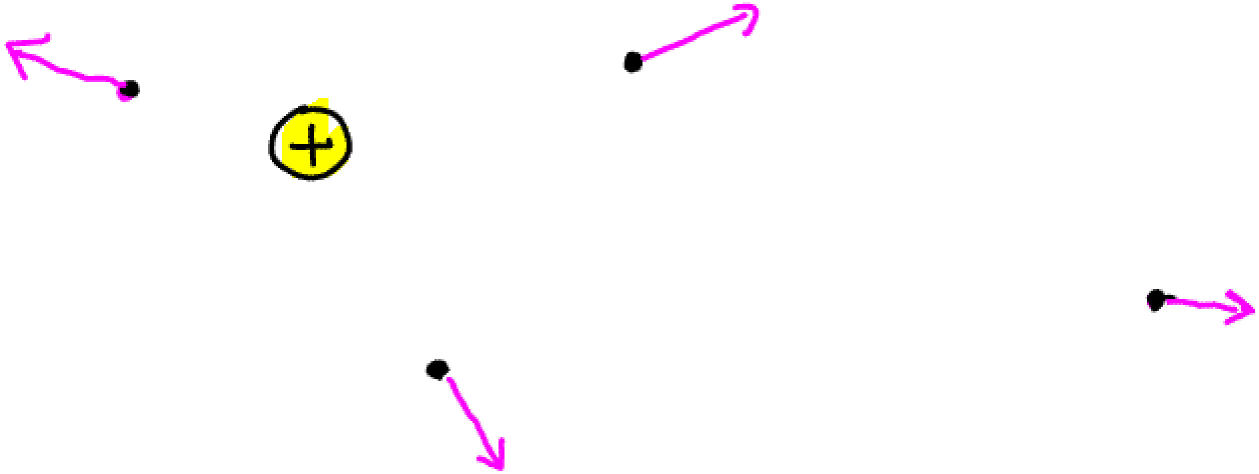
$dq' = \sigma(\vec r)\,da$

$dq' = \lambda(\vec r)\,dx$

doable ... Sometimes nasty

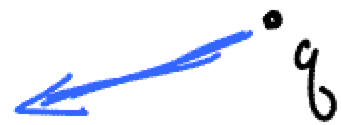
usually work w/ Electric field instead

$$\boxed{\vec E(\vec r) = \frac{\vec F(\vec r)}{q}}$$



$$\vec{F} = \frac{k q_1 q_2}{r^2} \hat{r}$$

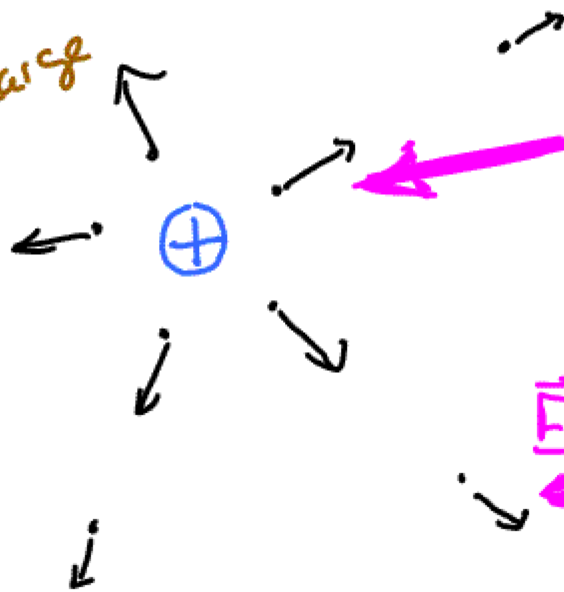
$$\vec{E} = \frac{\vec{F}}{q_1} = \frac{k q_2 \hat{r}}{r^2}$$



$$\vec{E} = \vec{F}/q$$

$\vec{E}$ vectors point radially outward (as does force)

$\vec{E}$ vectors About a ⊕ Point charge

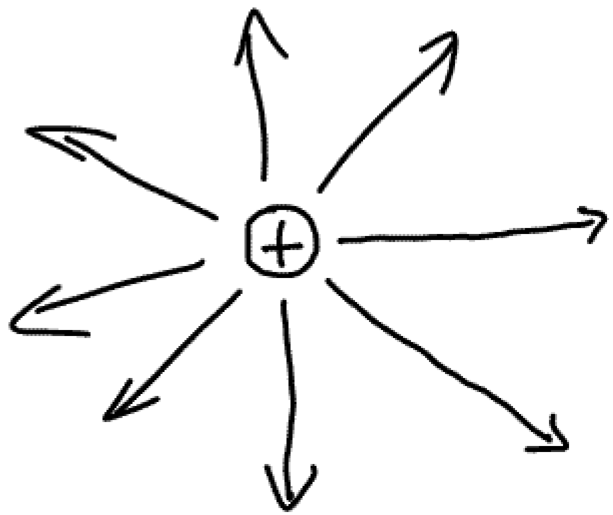


Closer ... $\vec{E}$ vector long ... Strong field

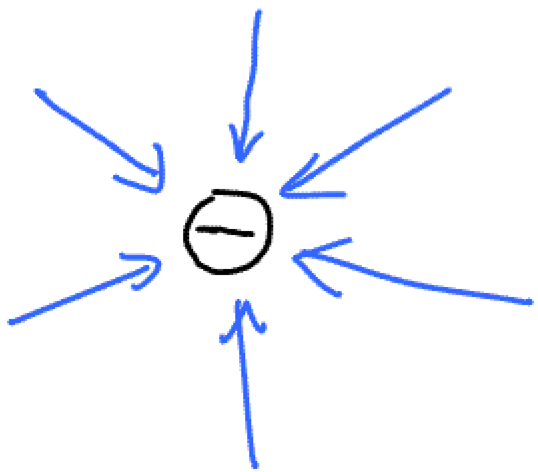
Farther Away ...

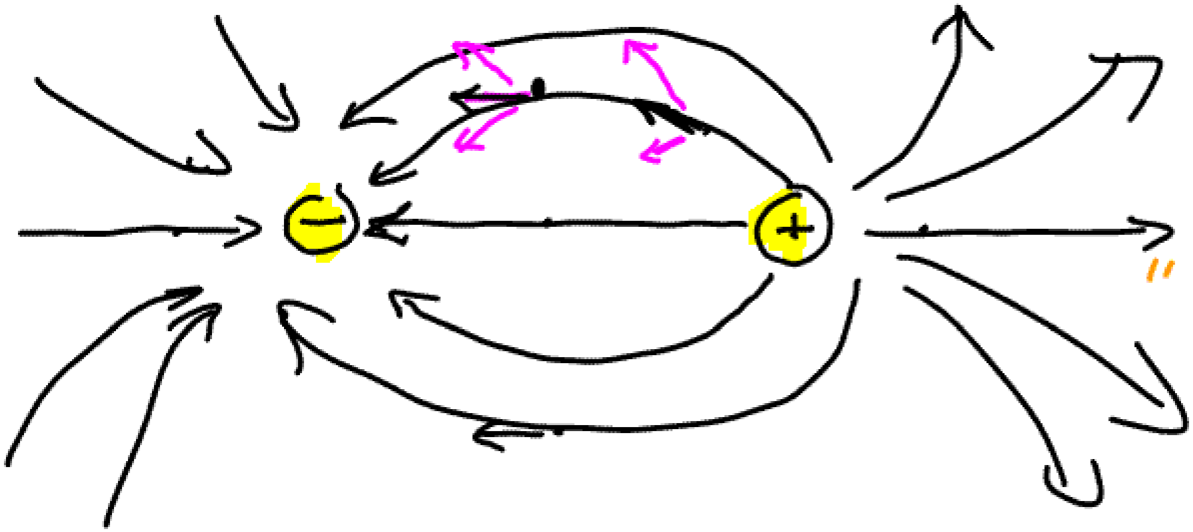
$\vec{E}$ vector Shorter here because field is weaker ... less force/charge

Lines of force



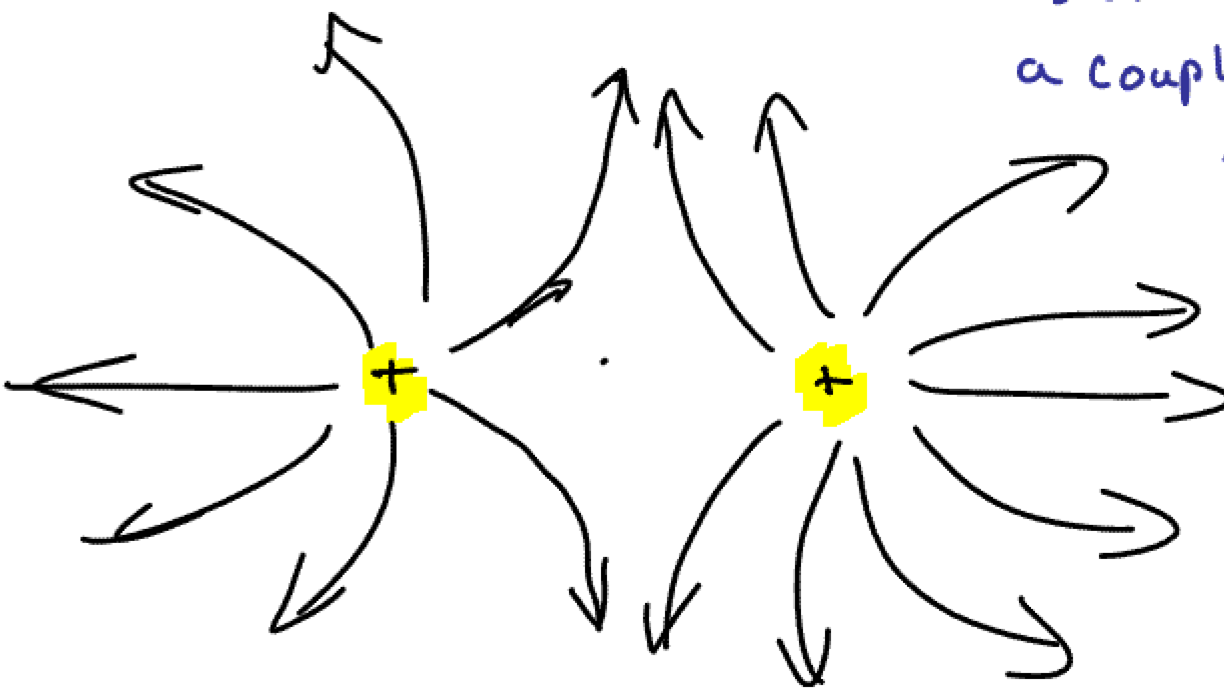
- Go in direction of + Test chg.
- density of lines → strength of force/Field
- Never cross

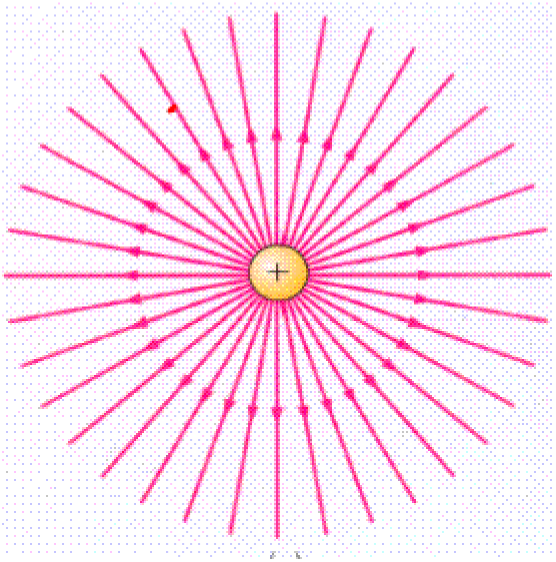


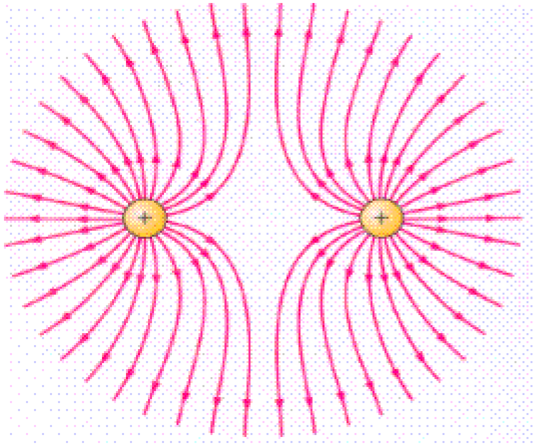


" Electric Dipole Field "

Lines of force around a couple of 2-charge Distributions

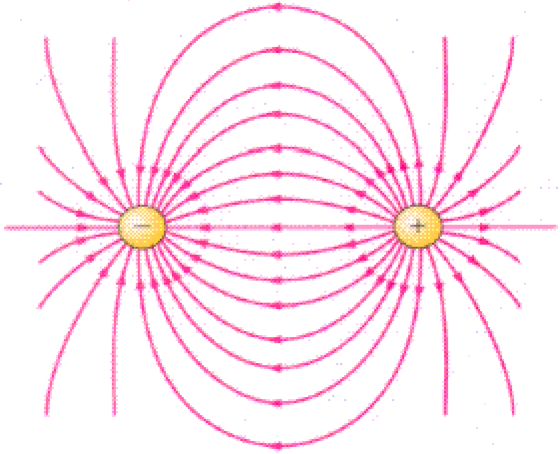






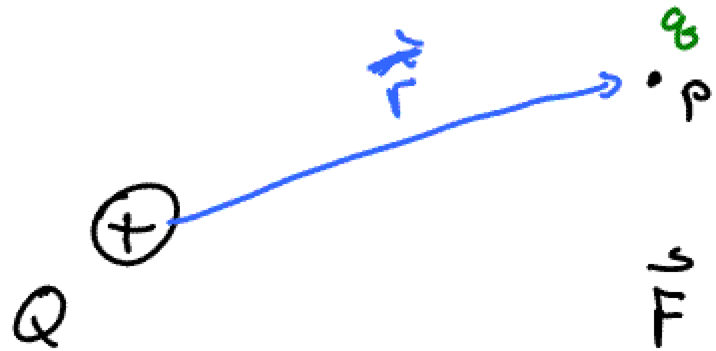
Lines of force

Nicer figures
from Textbook
by Tipler



Lines of force are a useful
way to visualize Electric Fields

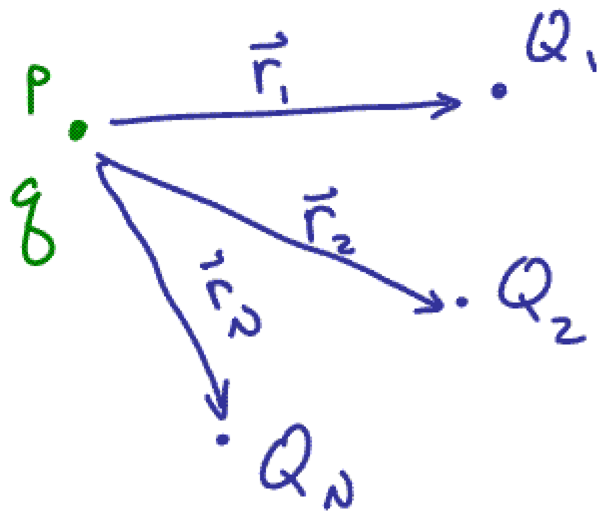
$\vec{E}$ for a point charge



$$\vec{F}_{\substack{\text{at } p \\ \text{due to } Q}} = \frac{kQq}{r^2}\hat{r}$$

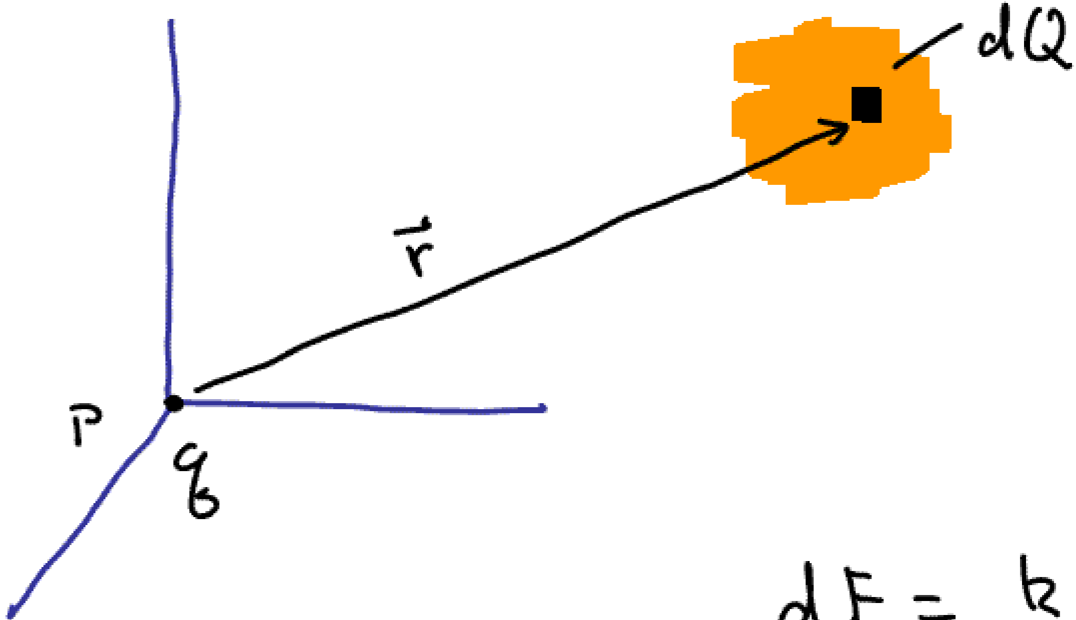
$$\vec{E} = \vec{F}/q$$

$$\vec{E}_{\substack{\text{at } p \\ \text{due to } Q}} = \frac{kQ}{r^2}\hat{r}$$



$$\vec{F}_{at\,p} = \sum_{i=1}^{N} \frac{kQ_i q}{r_i^2} \hat{r}_i$$

$$\vec{E}_{at\,p} = \sum_{i=1}^{N} \frac{kQ_i}{r_i^2} \hat{r}_i$$

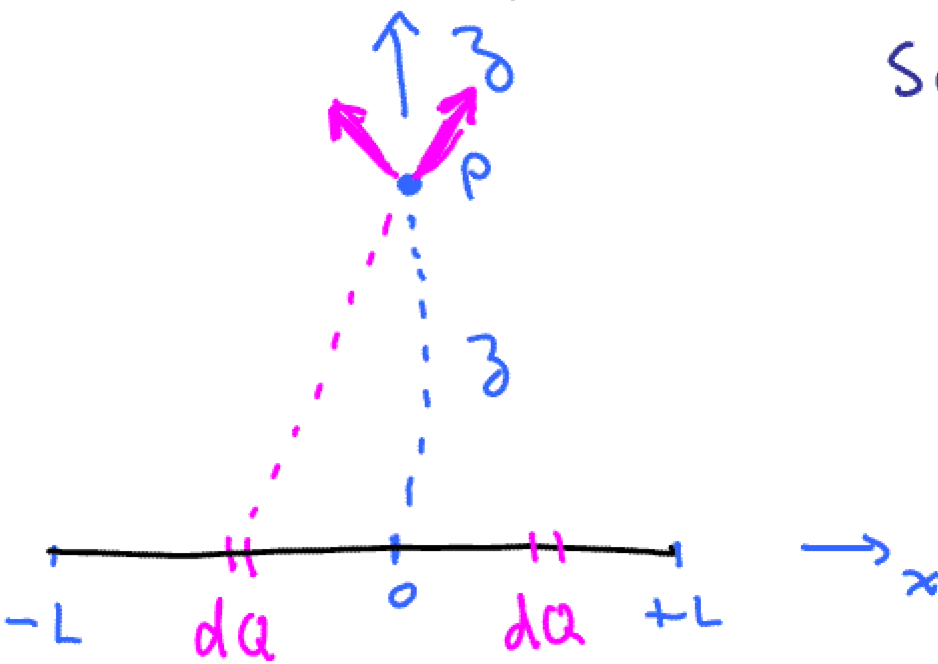


$$dF = \frac{k q \, dQ}{r^2} \hat{r}$$

$$d\vec{E} = \frac{k \, dQ}{r^2} \hat{r}$$

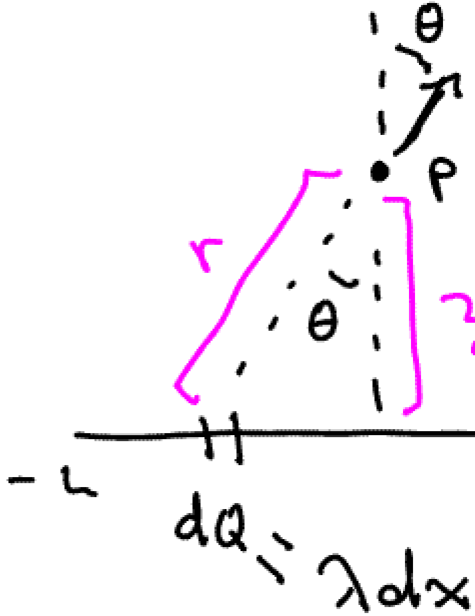
$$\vec{E}_{\text{at } p} = \int_{\substack{\text{chg} \\ \text{dist}}} \frac{k \, dQ}{r^2} \hat{r}$$

Find $\vec{E}$ at a distance z above the midpoint of a line segment of Length $2L$ that carries a uniform Line charge of $+\lambda$



Symmetry

$\longrightarrow \vec{E}$ straight up



$$d\vec{E} = \frac{k\,dQ}{r^2}\,\hat{r}$$

$\cos\theta = z/r$

$$\vec{E} = \int_{-L}^{L} \frac{k\,dQ}{r^2}\,\hat{r}$$

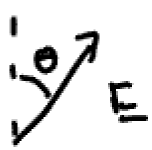
$$\bar{E} = \int_{-L}^{L} \frac{k\lambda\,dx}{r^2}\,\hat{r}$$

$E\cos\theta$

Symmetry

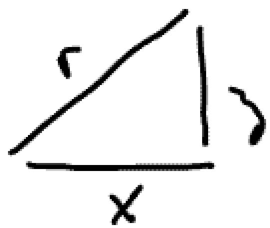
only worry abt straight up component

in $\hat{z}$ direction



$$\vec{E} = \int_{-L}^{L} \frac{k\lambda}{r^2}\, dx\, \cancel{\hat{r}} \; \cos\theta \qquad \hat{up} \leftarrow \text{or } \hat{z}$$

$$\vec{E} = \int_{-L}^{L} \frac{k\lambda}{r^2}\, dx\, \frac{z}{r}\, \hat{z}$$



$$r = (x^2 + z^2)^{1/2}$$

$$\vec{E} = \int_{-L}^{L} \frac{k\lambda\, dx\, z}{(x^2+z^2)^{3/2}}\, \hat{z}$$

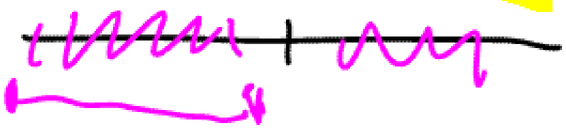
By symmetry of problem change limits from $-L \to L$ to $0-L$ and mult by 2

$$\vec{E} = k\lambda z \int_{-L}^{L} \frac{dx}{(x^2+z^2)^{3/2}}\, \hat{z} = 2k\lambda z \int_{0}^{L} \frac{dx}{(x^2+z^2)^{3/2}}\, \hat{z}$$

Check units

check limiting cases

!!



$$\boxed{\vec{E} = \frac{2k\lambda L}{z\left(L^2+z^2\right)^{1/2}}\,\hat{z}}$$

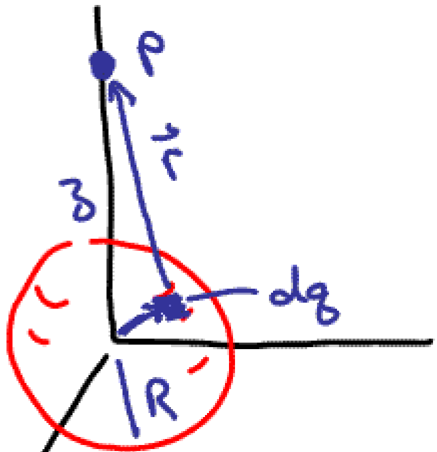
$$\vec{E} = \frac{F}{q} \Rightarrow \frac{\text{Newton}}{\text{Coul}} \qquad \frac{kq^2}{r^2} \sim \text{Newton}$$

charge

$$z \to \text{large} \qquad \frac{2k\lambda}{z\left(L^2+z^2\right)^{1/2}} \to \boxed{\frac{2k\lambda L}{z^2}}$$

Field of a point charge with total charge λL viewed From a distance z

✓ Limiting case Makes Sense



Sphere of uniform charge

$$\rho = \frac{Q_{TOT}}{\frac{4}{3}\pi R^3}$$

$$\vec{E} = \int_{vol} \frac{k\,dq}{r^2}\,\hat{r}$$

$= \rho\, dv$

Holy Crap

$$= \int_{vol} \frac{k\,\rho\, dv}{r^2}\,\hat{r} = \int_0^R \int_0^{\pi} \int_0^{2\pi} \frac{k\rho}{r^2}\, r^2 \sin\theta \, d\theta\, d\phi\, dr$$

This can be quite difficult. There are easier ways. I drag you through the mud here so you can appreciate the usefulness of Gauss' Law (Next Time)