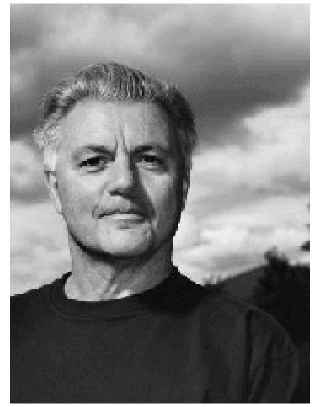
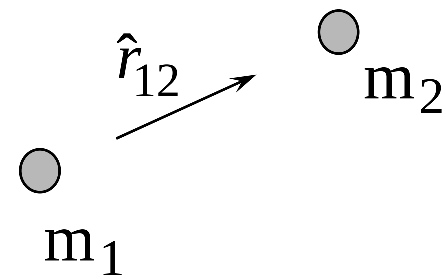
Exeunt Welcome
Enter Electromagnetism

The World According to Physics 121



(Apologies to John Irving)

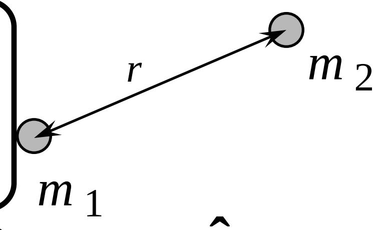
- Things, interactions and trajectories $\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$
  - Specified by geometry, mass and forces
- Forces
  - Gravity: $\vec{F}_{12} = -G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12}$

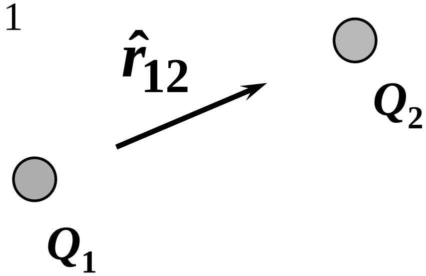


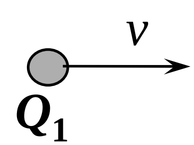
  - Others: Tension, Normal, Friction
- Space and Time
  - Euclidean with Galilean Invariance
    - “ordinary” 3D space; “slow” velocities

Moving Beyond 121…

- Things -- Bodies and Fields
  (e.g., gravitational)
  - Specified by geometry and mass and charge
- Forces
  - Gravity: $$\vec{F}_{12} = -G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12}$$
  - Electric: $$\vec{F}_{12} = k\frac{Q_1 Q_2}{r_{12}^2}\hat{r}_{12}$$
  - Magnetic: $$\vec{F}_{1mag} = Q_1\vec{v}\times\vec{B}$$







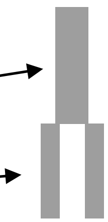
Hmm… gravitational and electric forces look awfully similar…

Static Charges & Electroscopes

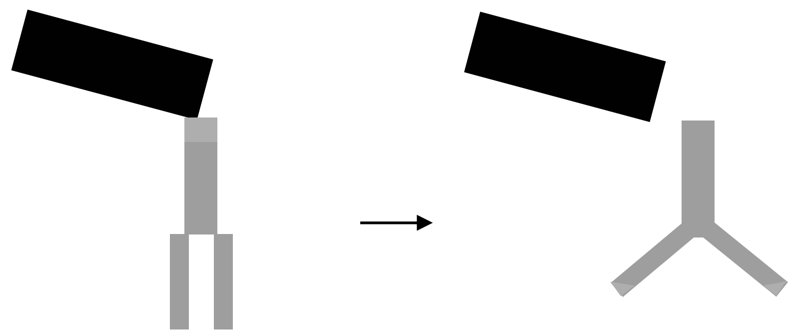
- **The Phenomena**
  - **Charge electroscope with rubber rod which has been rubbed with fur. Gold leaves separate.**
    - » **Bring same rubber rod close to top of electroscope. observe leaves separate further.**
    - » **Bring lucite rod (rubbed with silk) close to top of electroscope. observe leaves approach each other.**

More about charging electroscopes

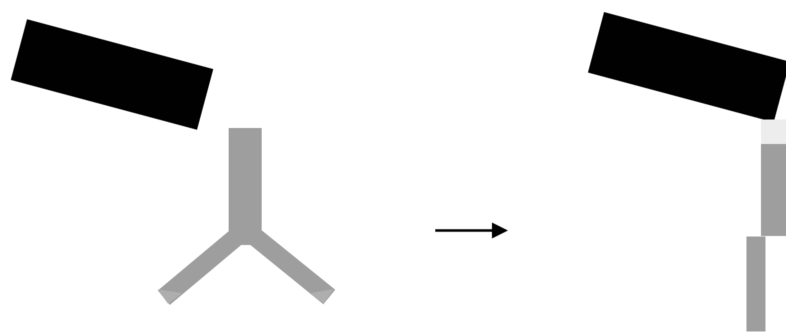
- The electroscope is made out of conductors
  – conducting main electrode
  – 2 conducting gold leaves



- Add some charge (rubber)



- Then add a different (?) charge (lucite)

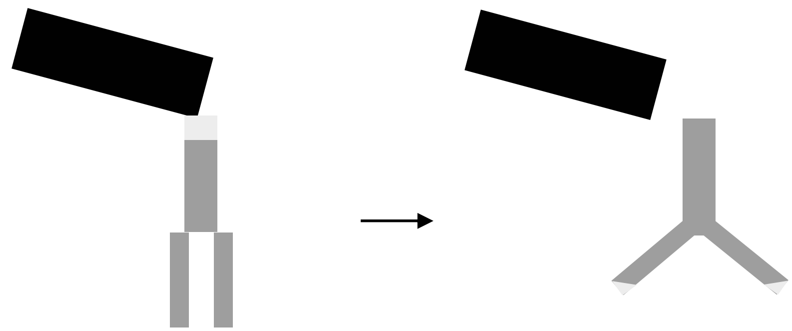


Static Charges & Electroscopes

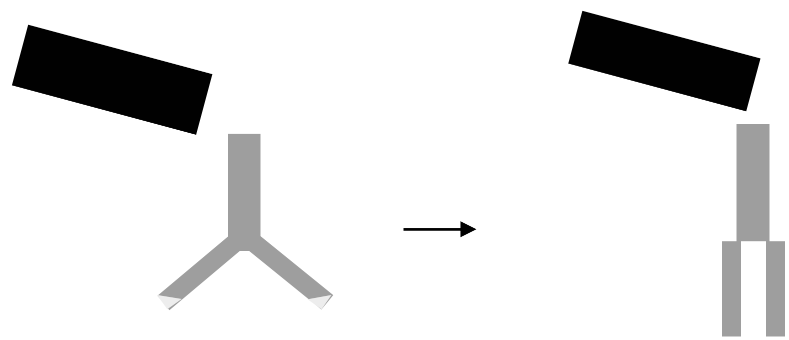
- The Phenomena
  - Charge electroscope with rubber rod which has been rubbed with fur. Gold leaves separate.
    - » Bring same rubber rod close to top of electroscope. observe leaves separate further.
    - » Bring lucite rod (rubbed with silk) close to top of electroscope. observe leaves approach each other.
  - Now repeat experiment, but charge with lucite rod. Gold leaves still separate.
    - » Now rubber rod causes leaves to approach each other.
    - » Lucite rod causes leaves to separate.

More about charging electroscopes (continued)

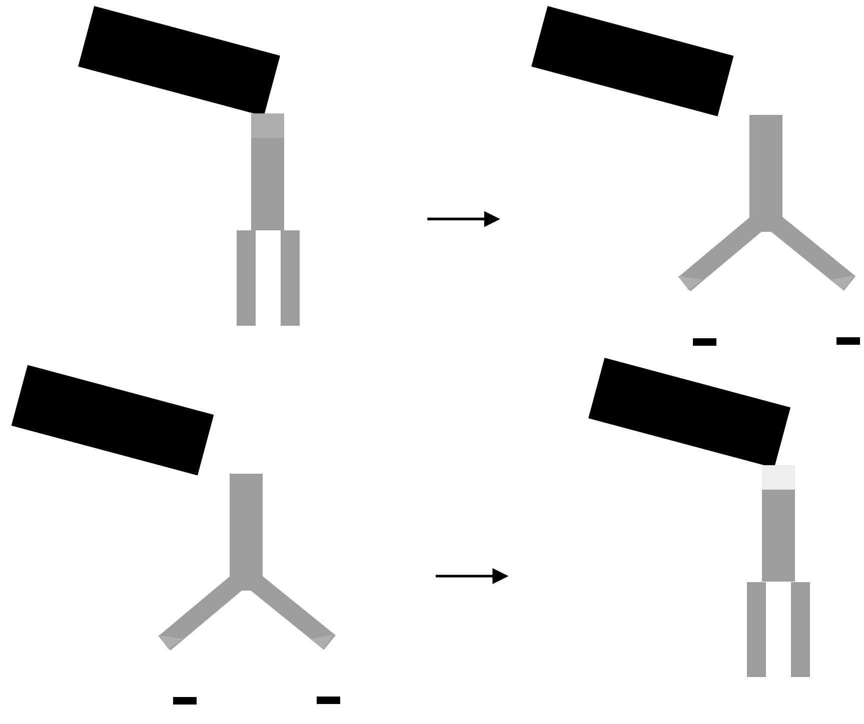
- **Add some charge (lucite)**



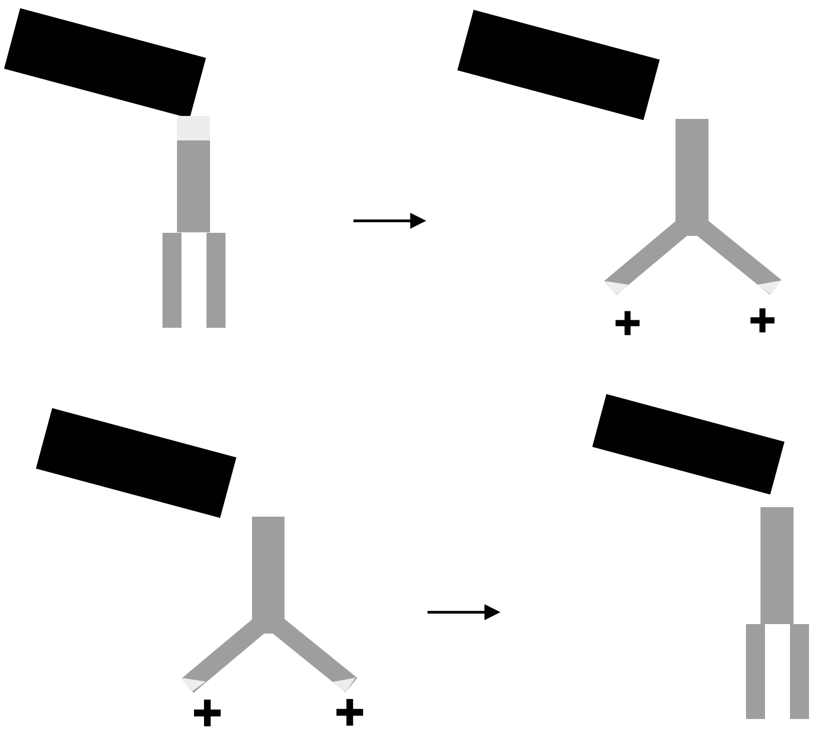
- **Then add charge from rubber to lucite charged leaves**



Electroscope Conundrum



- Rubber charge
- Lucite charge
  - Opposite of rubber!



- And start with lucite charge…
- Which gets cancelled by rubber charge!
- Hey, we could call these plus and minus!

Static Charges & Electroscopes

- The Phenomena
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    - » Bring same rubber rod close to top of electroscope. observe leaves separate further.
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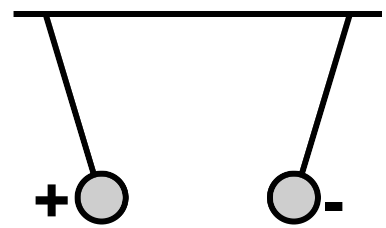
  - Now repeat experiment, but charge with lucite rod. Gold leaves still separate.
    - » Now rubber rod causes leaves to approach each other.
    - » Lucite rod causes leaves to separate.

- What have we learned?
  - Similar charges repel (leaves separate)
  - There exist two kinds of charge (lucite negates rubber and vice versa)

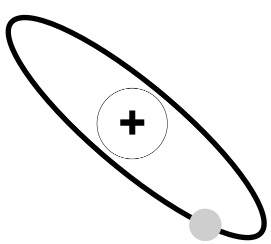
Text Reference: Chapter 21.1 $\longrightarrow$ 21.5

What it this “charge”?

- The charge we see in these experiments comes from atoms: electrons and ions
  - Friction (rubbing) separates them
  - Two types: positive and negative (Ben Franklin!)
    - Positive is charge on rubbed lucite
    - Negative is charge we see on plastic or rubber
- As we saw, like charges repel
  - Opposite charges attract! <u>See pith balls</u>



- Atomic charges are huge! (N_A~6x10^{23})
  - Electron charge is -1.6x10^{-19} Coulombs
    - Implies that ionized atoms are positive



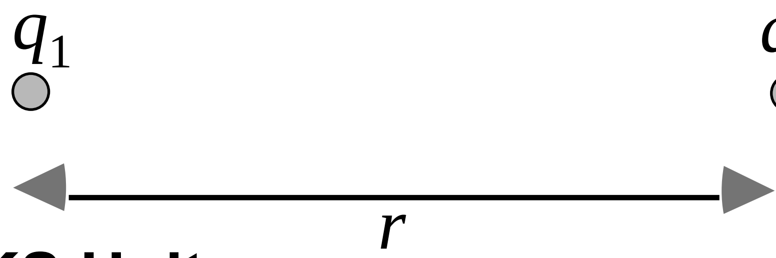
Law of Electrical Force
Charles-Augustin Coulomb
1785

- "Construction and use of an electric balance - based on the properties of metallic wires of having a force of reaction of torsion proportional to the angle of torsion."

- " The repulsive force between two small spheres charged with the same sort of electricity is in the inverse ratio of the squares of the distances between the centers of the spheres"

What We Call Coulomb's Law

The force from 1 acting on 2



$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

MKS Units:

- ***r*** **in meters**
- ***q*** **in Coulombs**
- $\vec{F}$ **in Newtons**

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 \text{ N-m}^2/\text{C}^2 = k$$

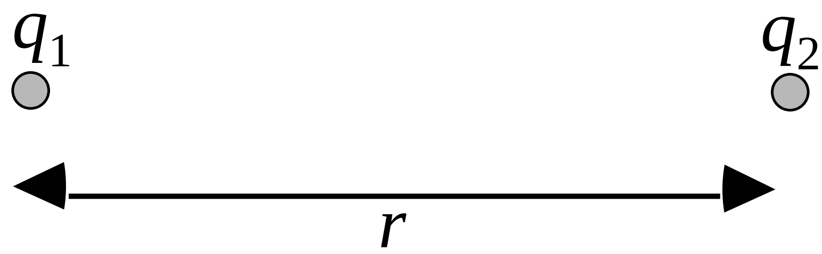
We call this group of constants "*k*" as in: $\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$

This force has same spatial dependence as the gravitational force, BUT there is NO mention of mass here!!

The strength of the FORCE between two objects is determined by the charge of the two objects, and the separation between them.

The direction of the FORCE is along the line of separation. "Central force"

Coulomb Law Qualitative



The force from 1 acting on 2

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

- **What happens if q_1 increases?**

  F (magnitude) increases

- **What happens if q_1 changes sign (+ $\longrightarrow$ -)?**

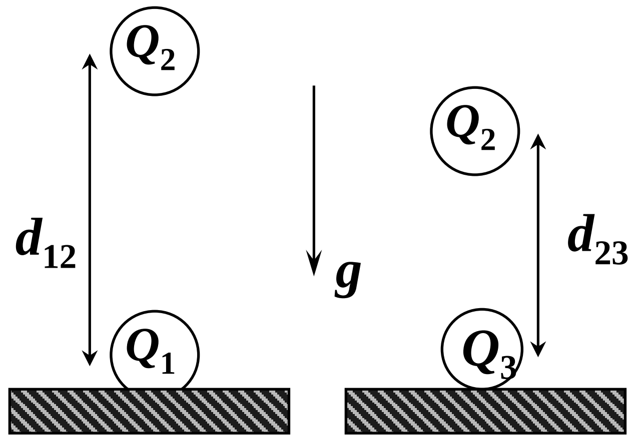
  The direction of $\vec{F}$ is reversed

- **What happens if r increases?**

  F (magnitude) decreases

Lecture 1, Concept 1

- A <u>charged</u> ball Q_1 is fixed to a horizontal surface as shown. When another <u>massive charged</u> ball Q_2 is brought near, it achieves an equilibrium position at a distance d_{12} directly above Q_1.
- When Q_1 is replaced by a different <u>charged</u> ball Q_3, Q_2 achieves an equilibrium position at distance d_{23} (< d_{12}) directly above Q_3.



1a: A) The charge of Q_3 has the same sign of the charge of Q_1
B) The charge of Q_3 has the opposite sign as the charge of Q_1
C) Cannot determine the relative signs of the charges of Q_3 & Q_1

1b: A) The magnitude of charge Q_3 < the magnitude of charge Q_1
B) The magnitude of charge Q_3 > the magnitude of charge Q_1
C) Cannot determine relative magnitudes of charges of Q_3 & Q_1

Lecture 1, Concept 1

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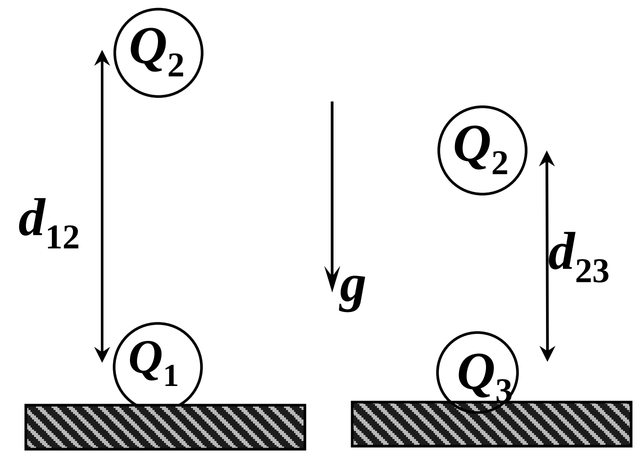

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B) The charge of Q_3 has the opposite sign as the charge of Q_1
C) Cannot determine the relative signs of the charges of Q_3 & Q_1

- To be in equilibrium, the total force on Q_2 must be zero.
- The only known (from 121) force acting on Q_2 is its weight.
- Therefore, in both cases, the electrical force on Q_2 must be directed upward to cancel its weight.
- Therefore, the sign of Q_3 must be the SAME as the sign of Q_1

Lecture 1, Concept 1

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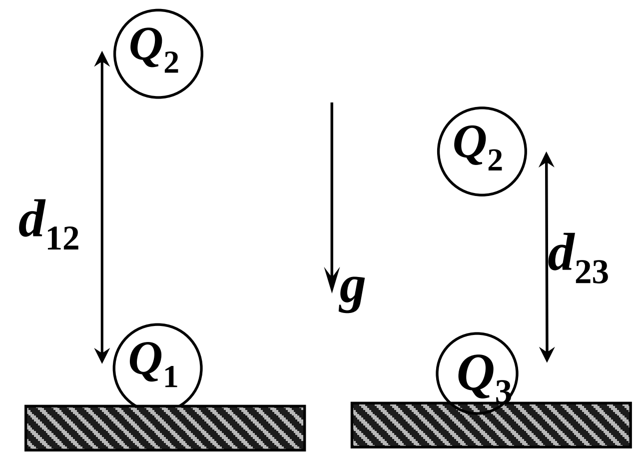
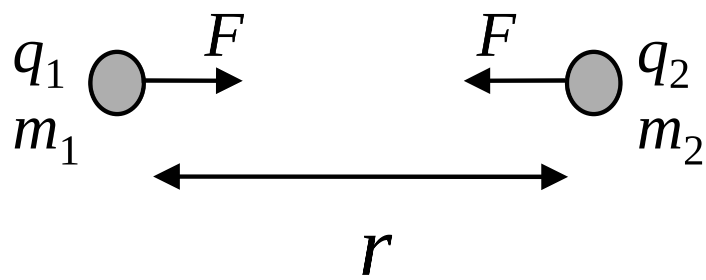


1b: A) The magnitude of charge Q_3 < the magnitude of charge Q_1
B) The magnitude of charge Q_3 > the magnitude of charge Q_1
C) Cannot determine relative magnitudes of charges of Q_3 & Q_1

- The electrical force on Q_2 must be the same in both cases … it just cancels the weight of Q_2 .
- Since d_{23} < d_{12} , the charge of Q_3 must be SMALLER than the charge of Q_1 so that the total electrical force can be the same!!

Gravitational vs. Electrical Force



$$F_{elec} = k\,\frac{q_1 q_2}{r^2}$$

$$F_{grav} = G\,\frac{m_1 m_2}{r^2}$$

$$\rightarrow \quad \frac{F_{elec}}{F_{grav}} = \frac{q_1 q_2}{m_1 m_2}\,\frac{k}{G}$$

For a proton:

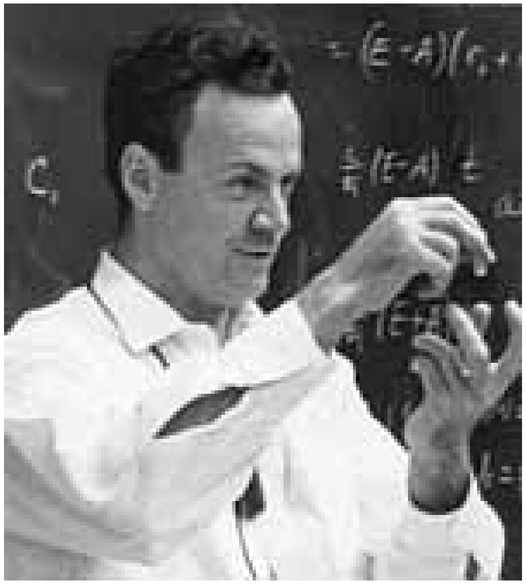
* $q = 1.6 \times 10^{-19}$ C
$m = 1.67 \times 10^{-27}$ kg

$$\rightarrow \quad \frac{F_{elec}}{F_{grav}} = 1.23 \times 10^{+36}$$

* Proton charge is smallest charge seen (alone) in nature!

How Strong is the Electrical Force? Really?

(see Appendix for justification)



Richard Feynman (1918-1988)

- **Nobel Prize for QED**
- **Explains Challenger disaster**
- **Educator Extraordinaire**

Richard Feynman, *The Feynman Lectures*:

"If you were standing at arm's length from someone and each of you had *one percent* more electrons than protons, the repelling force would be incredible. How great? Enough to lift the Empire State Building? No! To lift Mount Everest? No! The repulsion would be enough to lift a 'weight' equal to that of the entire earth!"

Notation For Vectors and Scalars

Vector quantities are written like this : $\vec{F}$, $\vec{E}$, $\hat{x}$, $\hat{r}$

To completely specify a vector, the magnitude (length) and direction must be known

For example, the following equation shows $\vec{F}$ **specified in terms of** $\hat{r}$, q_1, q_2, **and** r

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$$

The magnitude of $\vec{F}$ **is** $\left|\vec{F}\right| = F = k\frac{q_1 q_2}{r^2}$

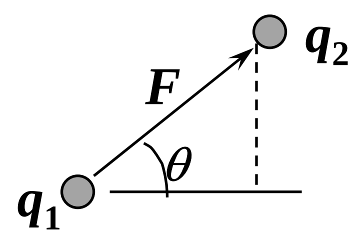
F **(the magnitude of** $\vec{F}$ **) is a scalar quantity.**

The vector $\vec{F}$ **can be broken down into** x, y, **and** z **components,**

$$\vec{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z}$$

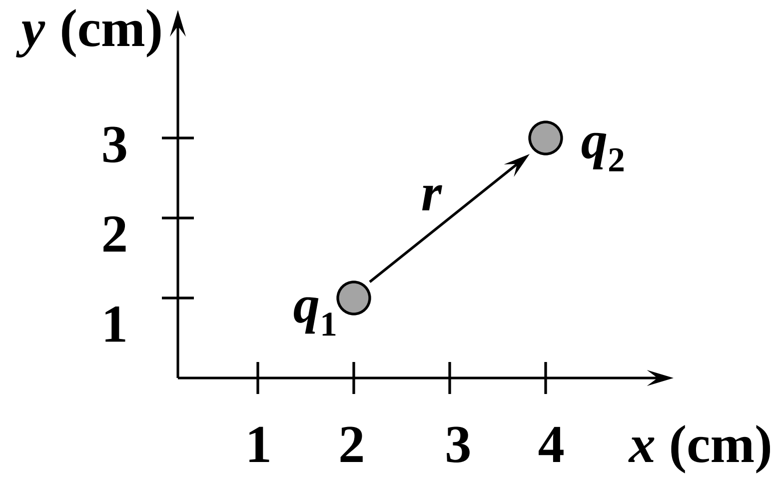
where F_x, F_y, **and** F_z **(the** x, y, **and** z **components of** F **) are scalars.**

Components of $\vec{F}$ **can be found by *projecting* vector onto an axis**



Vectors and Coulomb's Law: an Example

q_1 and q_2 are point charges, q_1 = +2μC and q_2 = +3 μC. q_1 is located at $\vec{r}_1 = (2\text{cm},1\text{cm})$ and q_2 is located at $\vec{r}_2 = (4\text{cm},3\text{cm})$



Find F_{12} (the magnitude of the force of q_1 on q_2). How?

To do this, use Coulomb's Law: $$F_{12} = k\frac{q_1 q_2}{r^2}$$

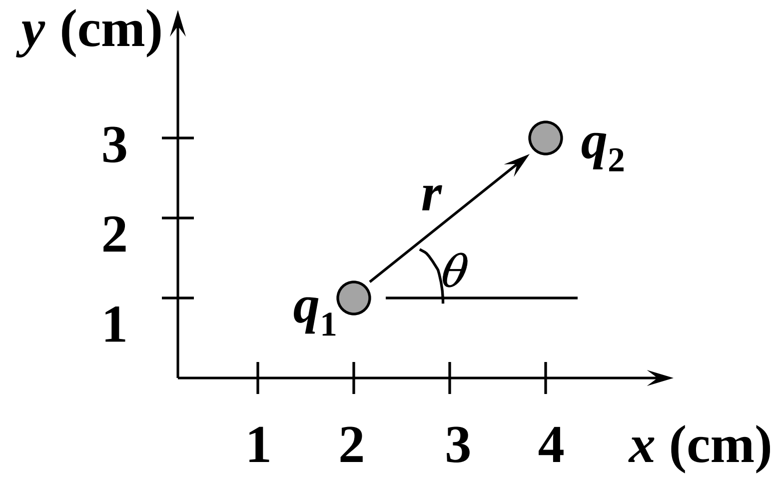
where $r = |\vec{r}_2 - \vec{r}_1| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Now, calculate r... How? $$r = \sqrt{(4-2)^2 + (3-1)^2} = \sqrt{8}\text{ cm} = 2.828\text{ cm}$$

$$F_{12} = k\frac{q_1 q_2}{r^2} = (8.98\times 10^9\,\text{Nm}^2/\text{C}^2)\frac{(2\times 10^{-6}\,\text{C})(3\times 10^{-6}\,\text{C})}{(0.028)^2\,\text{m}^2} = 68.7\,\text{N}$$

Vectors: an Example continued

Now, find F_x and F_y, the x and y components of the force of q_1 on q_2



Direction of force?
Two positive charges repel!

Project Components of Vectors

$$F_x = F_{12}\frac{x_2 - x_1}{r}$$

Span in x ($x_2 - x_1$)

Total length (r)

$$F_y = F_{12}\frac{y_2 - y_1}{r}$$

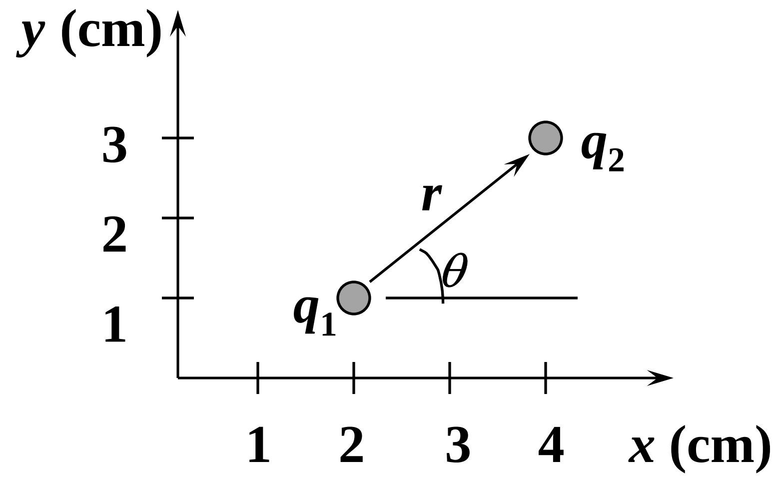
Got Trigonometry?

$$\cos\theta = \frac{x_2 - x_1}{r}$$

$$\sin\theta = \frac{y_2 - y_1}{r}$$

Vectors: an Example continued

Now, find F_x and F_y, the x and y components of the force of q_1 on q_2



Components of Vectors

$$F_x = F_{12}\,\frac{x_2 - x_1}{r}$$

Span in x ← $x_2 - x_1$

Total length ← r

$$F_y = F_{12}\,\frac{y_2 - y_1}{r}$$

What are those factors numerically?

$$\frac{x_2 - x_1}{r} = \frac{4-2}{\sqrt{8}} = .707$$

F_x **= 47.74** N

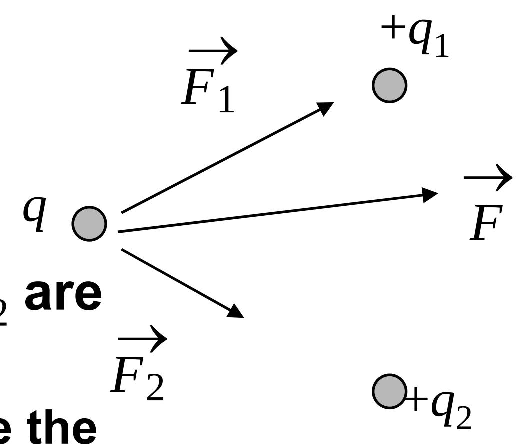
$$F_{12} = k\,\frac{q_1 q_2}{r^2} = 68.7\,\mathrm{N}$$

$$\frac{y_2 - y_1}{r} = \frac{3-1}{\sqrt{8}} = .707$$

F_y **= 47.74** N

What happens when you consider more than two charges?

- **If q_1 were the only other charge, we would know the force on q due to q_1.**
- **If q_2 were the only other charge, we would know the force on q due to q_2.**

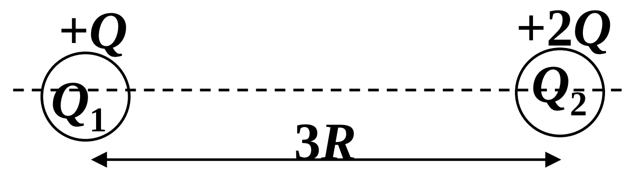


- **What is the force on q when both q_1 and q_2 are present??**
  - **The answer: just as in mechanics, we have the Law of Superposition:**
    - **The TOTAL FORCE on the object is just the VECTOR SUM of the individual forces.**

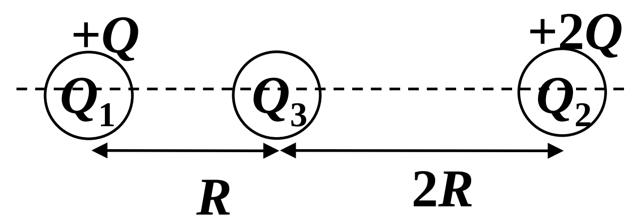
$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

Lecture 1, Concept 2

- Two balls, one with charge $Q_1 = +Q$ and the other with charge $Q_2 = +2Q$, are held fixed at a separation $d = 3R$ as shown.



- Another ball with (non-zero) charge Q_3 is introduced in between Q_1 and Q_2 at a distance = R from Q_1 .
- Which of the following statements is true?



(a) The net force on Q_3 can be zero if Q_3 is positive.

(b) The net force on Q_3 can be zero if Q_3 is negative.

(c) The net force on Q_3 can never be zero, no matter what the (non-zero!) charge Q_3 is.

Lecture 1, Concept 2

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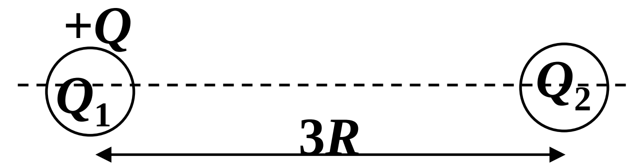

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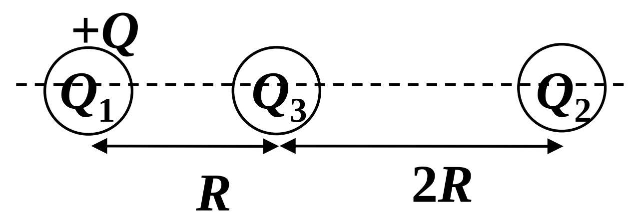


(a) The force on Q_3 can be zero if Q_3 is positive.

(b) The force on Q_3 can be zero if Q_3 is negative.

(c) The force on Q_3 can never be zero, no matter what the (non-zero) charge Q_3 is.

The magnitude of the force on Q_3 due to Q_2 is proportional to $(2Q\,Q_3/(2R)^2)$

The magnitude of the force on Q_3 due to Q_1 is proportional to $(Q\,Q_3/R^2)$

These forces can never cancel, because the force Q_2 exerts on Q_3 will always be 1/2 of the force Q_1 exerts on Q_3!!

Today's Summary

- **Charges come in two varieties**
  - negative and positive
  - like charges repel but opposites attract

- **Coulomb Force**
  - linear in both charges
  - inversely proportional to square of separation
  - central force

$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{12}$$

- **Law of Superposition**

$$\vec{F} = \vec{F_1} + \vec{F_2}$$

Reading assignment: (21.1-5 review) 21.6, 21.8-9, 21.11
study examples 21.2, 21.3, 21.6, 21.7, 21.16

Appendix B: Should we believe Feynman?

- **How many electrons in a person?**
  - **What do we assume is the chemical composition of a person?**

    Simplify: assume water (molecular weight = 18)
  - **What then is the number of electrons/gram in a person?**

$$\frac{6 \times 10^{23} \text{ molecules/mole}}{18 \text{ g/mole}} \times 10 \text{ e}^{-}\text{/molecule} = 3.3 \times 10^{23} \text{ e}^{-}\text{/g}$$

  - So, how many electrons in a person?

Assume weight = 80 kg ⇨ $3.3 \times 10^{23} \text{ e}^{-}\text{/g} \times 80 \text{ kg} = 2.6 \times 10^{28} \text{ e}^{-}$

- How much charge is 1% of electrons in a person?

$$1\% \times 2.6 \times 10^{28} \text{ e}^{-} \times 1.6 \times 10^{-19} \text{ C/e}^{-} = 4.2 \times 10^{7} \text{ C}$$

Should we believe?

- What is the force between 2 people an arm's length apart if they each had an excess of 1% electrons?

$$F = (9 \times 10^9 \text{ N-m}^2/\text{C}^2) \times \left(\frac{4.2 \times 10^7 \text{ C}}{0.75 \text{ m}} \right)^2$$

$$F = 2.8 \times 10^{25} \text{ N}$$

- What is the "weight" of the earth?

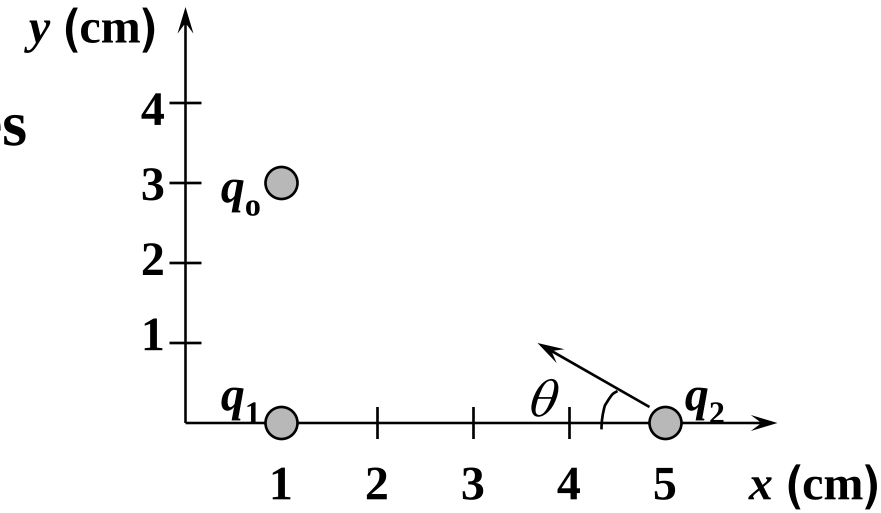
$$W_{earth} = 6 \times 10^{24} \text{ kg} \times 9.8 \text{ m/s}^2$$

$$W_{earth} = 5.9 \times 10^{25} \text{ N}$$

- Yes, that's INCREDIBLE!!

Appendix C: A Longer Example

q_o, q_1, and q_2 are all point charges where q_o = -1μC, q_1 = 3μC, and q_2 = 4μC. Their locations are shown in the diagram. What is the force acting on q_0? ($\vec{F}_0$)



Superposition:

Find $\vec{F}_{10}$ and $\vec{F}_{20}$

$$F_{10} = \left| k \frac{q_0 q_1}{r_{10}^2} \right| \qquad F_{20} = \left| k \frac{q_0 q_2}{r_{20}^2} \right|$$

Direction?

$$\vec{F}_{10} = -F_{10}\hat{y} \quad \vec{F}_{20} = -F_{20}\hat{r}_{20}$$

Decompose $\vec{F}_{20}$ into its x and y components

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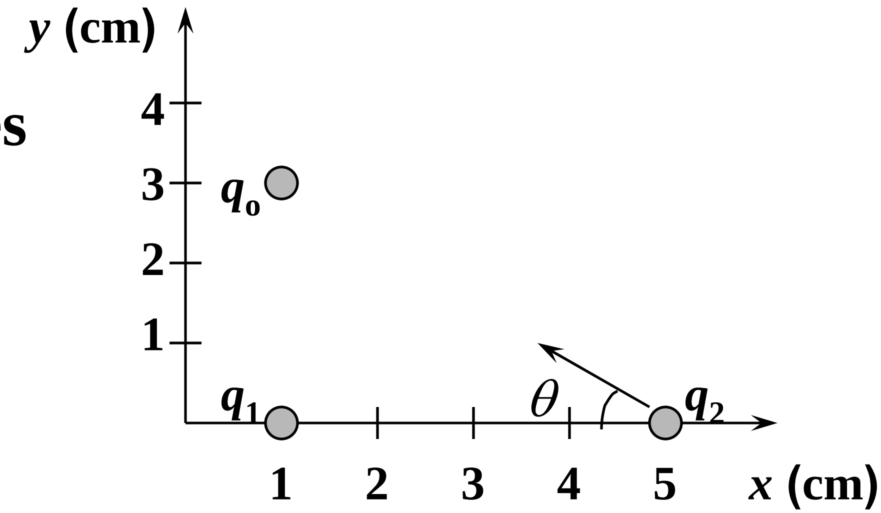

Decompose $\vec{F}_{20}$ into its x and y components

Span in x

$$\vec{F}_{20} = F_{20}\,\frac{x_2 - x_0}{r_{20}}\,\hat{x} + F_{20}\,\frac{y_2 - y_0}{r_{20}}\,\hat{y}$$

Total length

Superimposing forces…

$$\vec{F}_0 = \vec{F}_{10} + \vec{F}_{20} = F_{20}\,\frac{x_2 - x_0}{r_{20}}\,\hat{x} + \left(F_{20}\,\frac{y_2 - y_0}{r_{20}} - F_{10} \right)\hat{y}$$

A Longer Example continued

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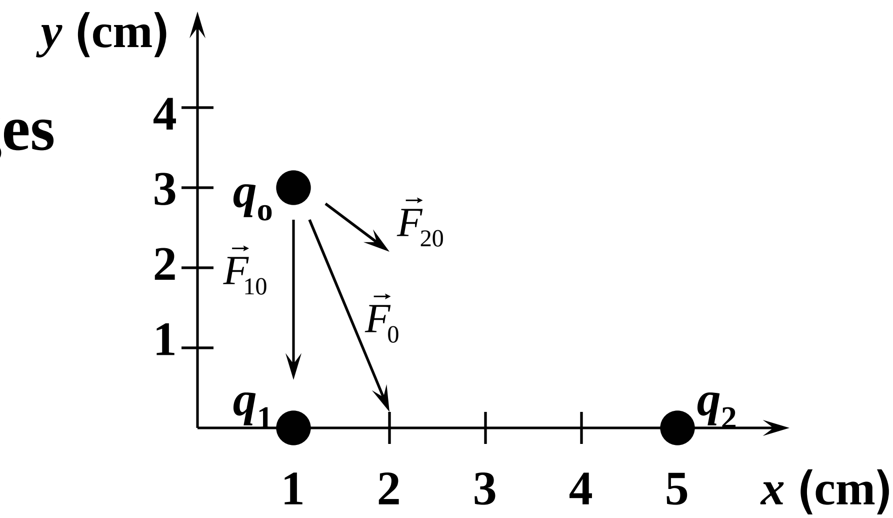


Let's put in the numbers . . .

$$r_{10} = 3\text{cm}$$

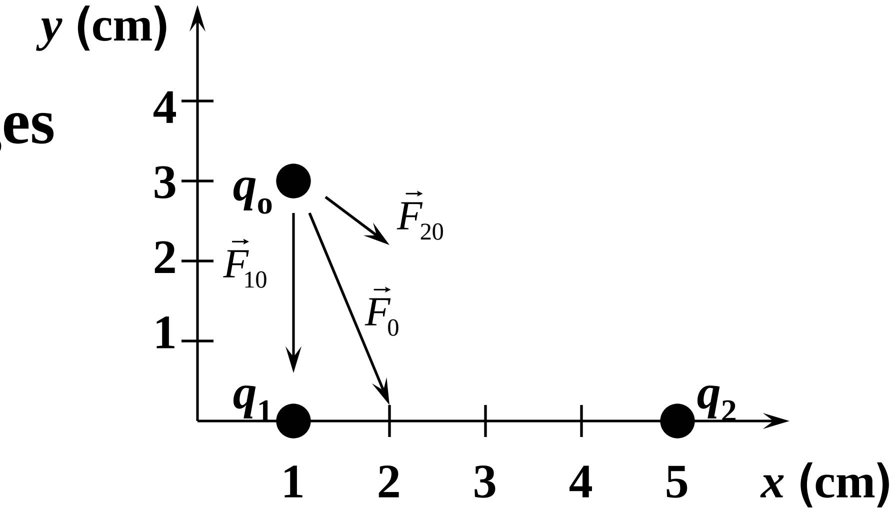
$$r_{20} = \sqrt{(x_2 - x_0)^2 + (y_2 - y_0)^2} = \sqrt{4^2 + (-3)^2}\,\text{cm} = 5\text{cm}$$

$$F_{10} = k\frac{|q_0 q_1|}{r^2} = (8.98\times10^9\,\text{Nm}^2/\text{C}^2)\frac{(1\times10^{-6}\,\text{C})(3\times10^{-6}\,\text{C})}{(0.03)^2\,\text{m}^2} = 29.9\text{N}$$

$$F_{20} = k\frac{|q_0 q_2|}{{r_{20}}^2} = (8.98\times10^9\,\text{Nm}^2/\text{C}^2)\frac{(1\times10^{-6}\,\text{C})(4\times10^{-6}\,\text{C})}{(0.05)^2\,\text{m}^2} = 14.4\text{N}$$

A Longer Example continued

q_o, q_1, and q_2 are all point charges where q_o = -1μC, q_1 = 3μC, and q_2 = 4μC. Their locations are shown in the diagram. What is the force acting on q_0? ($\vec{F}_0$)



Let's put in the numbers . . .

$$r_{10} = 3\text{cm} \qquad F_{10} = 29.9\text{N}$$
$$r_{20} = 5\text{cm} \qquad F_{20} = 14.4\text{N}$$

$$\vec{F}_0 = F_{20}\frac{x_2 - x_0}{r_{20}}\hat{x} + \left(F_{20}\frac{y_2 - y_0}{r_{20}} - F_{10}\right)\hat{y}$$
$$= F_{20}\frac{(5-1)\text{cm}}{5\,\text{cm}}\hat{x} + \left(F_{20}\frac{(0-3)\text{cm}}{5\,\text{cm}} - F_{10}\right)\hat{y}$$
$$= (11.5\,\hat{x} - 38.5\,\hat{y})\,\text{N}$$