Consider \( Q \) outside

\[
\phi = \frac{Q_{\text{inside}}}{\varepsilon_0} = \int \vec{E} \cdot d\vec{A} = \vec{E}_0 \int d\vec{A} = \vec{E}_0 4\pi R^2
\]

\[
\vec{E}_0 = \frac{Q_{\text{inside}}}{4\pi \varepsilon_0 R^2}
\]

\[
Q_{\text{inside}} = \frac{4}{3} \pi R^3 \rho
\]

\[
\vec{E}_0 \cdot \hat{r} = \frac{4}{3} \frac{\pi R^3 \rho}{4\pi \varepsilon_0 R^2} = \frac{R^3 \rho}{3\varepsilon_0 R^2} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{R^2}
\]

by symmetry \( \vec{E} \) is in \( \hat{r} \) direction.

 Gauss's Law

\[
\int \vec{E} \cdot d\vec{A} = \frac{1}{4\pi \varepsilon_0} 4\pi Q_{\text{inside}} = \frac{1}{\varepsilon_0} Q_{\text{inside}}
\]

Also one of Maxwell's equations

Why do you care?

One can use Gauss's law + Symmetry to solve for \( \vec{E} \)

Almost Trivially

recall our awful example of field outside sphere of uniform charge density

\[ r > R \]

Consider \( r > R \)
Gaussian surface outside a spherical shell (left) and inside a spherical shell (right).
\[ E_{r>R} = \frac{9R^3}{3\varepsilon_0 r^2} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2}, \quad r \geq R \]

Suppose \( r \leq R \)

\[ (E) = \frac{Q_{\text{inside}}}{4\pi \varepsilon_0 r^2} \]

\[ Q_{\text{inside}} = \frac{9}{3} \frac{4}{3} \pi r^3 \]

\[ E = \frac{9}{4\pi \varepsilon_0} \pi r^3 = \frac{9}{3\varepsilon_0} \frac{r}{3} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2}, \quad r \leq R \]

\[ E(r) \propto r \]

\[ F(r) \propto r^2 \]

do Telegram
Figure 23-21, page 702; Figure 23-24, page 705

$E$ due to a spherical shell of charge (left) and due to a solid sphere of charge (right)

\[ E_r = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \]

\[ E_r = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^3}, \ r \leq R \]

\[ E_r = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}, \ r \geq R \]
\textbf{P1/4 Telegram:}

\[ \vec{E} \text{ can be calculated directly from } \phi (\text{or } \rho, \text{or } q) \]

\[ \vec{E} = \int \frac{k \phi}{r^2} \]

often This is hard

Use Gauss's law when symmetry allows \( I \vec{E} \) to be moved out of integral

\[ \int \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} \rightarrow I \vec{E} \int d\vec{A} = \frac{Q}{\varepsilon_0} \]

Symmetries often seen

\[ \text{spherical} \]
\[ \text{spherical surface} \]

\[ \text{line} \rightarrow \text{cylindrical surface} \]

\[ \text{Plane - pillbox} \]
\[ \text{Cylindrical Surface} \]
$E$ calculated directly from charge distribution

$$E = \int \frac{k \rho \, dV}{r^2}$$

Volume

$$E = \int \frac{k \rho \, dV}{r^2}$$

Volume

$$\begin{array}{c}
\frac{dV}{r} \\
\rightarrow 0
\end{array}$$

Volume

$$\begin{array}{c}
\frac{dl}{r} \\
\rightarrow 0
\end{array}$$

Line

$$E = \int \frac{k \rho \, dl}{r^2}$$

Surface ... etc

This is hard!!

Gauss's law allows one to calculate $E$ in a much easier way if symmetry allows and you choose the right Gaussian surface

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

Gaussian Surf
Look at symmetry of charge distribution and choose Gaussian surface so that $E \cdot dA$ is easy to evaluate.

Examples

Point

Line

Sphere

Plane
What is \( \vec{E} \) for a line charge?

\[ \int \vec{E} \cdot dA = \int \vec{E}_1 \cdot dA + \int \vec{E}_2 \cdot dA + \int \vec{E}_3 \cdot dA \]

- \( \vec{E}_1 \cdot dA = 0 \) because \( \vec{E} \) is radial by symmetry.
- \( \vec{E}_2 \cdot dA = 0 \) because \( \vec{E} \) is constant at a fixed radius.
- \( \vec{E}_3 \cdot dA \) is due to the line charge.

\[ \int \vec{E}_3 \cdot dA = (E) \int dA = E \times \text{cylinder} = E \times 2\pi r l = \frac{Q_{\text{enc}}}{\varepsilon_0} \]

By Gauss's law, \( Q_{\text{enc}} = \pi l \)

\[ E = \frac{\pi l}{2\pi r \varepsilon_0} = \frac{\lambda}{2\pi \varepsilon_0} \] radially outward.
Conductor - charges can move freely (in response to $\vec{E}$)

Insulator - charges cannot move freely

under normal circumstances

Remember: One Demo

$\Rightarrow$ Do Demo insulators can break down

Semi-conductor - conductivity depends on environment (size of electric field)

Do Shielded Electric scope case Demo

$\Rightarrow$ Why does this happen?

Solid conductor

Place charge instantaneously in arbitrary spots inside

$\Rightarrow$ What happens

Charges rearrange until no $\vec{E}$ inside

if $\vec{E} -$ charges flow until no $\vec{E}$
\[ \mathbf{E} = 0 \text{ inside a conductor} \]

Gauss's Law

\[
\int \mathbf{E} \cdot d\mathbf{a} = \frac{Q_{\text{inside}}}{\varepsilon_0} \implies \mathbf{E} = 0 \text{ gives} \quad Q_{\text{inside}} = 0
\]

\[ \text{conductor surface} \]

\[ \text{Gaussian surface} \]

- **All charge = No charge inside**

Excess charge on an isolated conductor rests entirely on the outside surface of that conductor.

\[ \implies \text{Now why does electroscope in demo not respond to charged rod?} \]

Electro Static \[ \implies \] Electro Magnetic

Shielding

Micro wave doors

Metal cabinets on computers

etc.

Little in life is static ... but we will learn concept of light as induced fields

You are conducting an important exp't. for Medical research

Discover Patient's brainwaves go nuts in anticipation of coffee from coffee man.

Actually "Noise" from Coffee Man's truck
electromag.

Exp't noise problems in morning and at evening