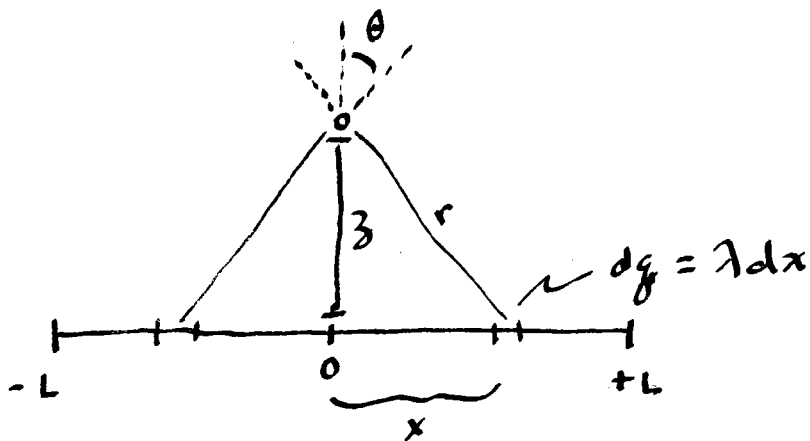


Example

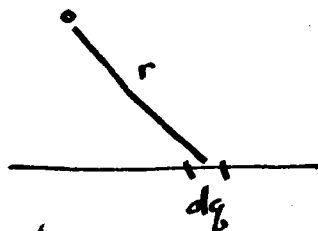
Find \vec{E} at a distance z above the midpoint of a straight line segment of length $2L$ which carries a uniform line charge of $+\lambda$.



What is the symmetry here?

What direction is \vec{E} ?

$$d\vec{E} = \frac{k dq}{r^2} \hat{r}$$



r varies w/ location of dq

$$\vec{dE} = \int_{-L}^{+L} \frac{k dq(x)}{r^2} \hat{r}$$

but notice that x component of E $-L \rightarrow 0$

will cancel w/ x component of E from $0 \rightarrow L$

- And -

① z component of E $-L \rightarrow 0 = z$ comp of E $0 \rightarrow L$

$$E_z = |E| \cos \theta$$

$$d\vec{E} = \frac{1}{2} dE_z \hat{z} = dE \cos \theta \hat{z}$$

$$\vec{E} = \int_{-L}^L k \frac{dq(x)}{r^2} \cos\theta \hat{z} = \int_{-L}^L k \frac{\lambda dx}{r^2} \cos\theta \hat{z}$$

$$= (2) \int_0^L \frac{k \lambda dx}{r^2} \cos\theta \hat{z} \quad \begin{array}{l} \text{limits change +} \\ \text{mult by 2} \\ \text{due to } \textcircled{I} \end{array}$$

$$r^2 = x^2 + z^2 \quad \cos\theta = \frac{z}{r} = \frac{z}{\sqrt{x^2 + z^2}}$$

$$\vec{E} = (2) \int_0^L \frac{k \lambda}{(x^2 + z^2)} \frac{z}{\sqrt{x^2 + z^2}} \hat{z}$$

z fixed by problem, x is variable we integrate over

$$\vec{E} = \cancel{(2)} 2k\lambda z \int_0^L \frac{dx}{(x^2 + z^2)^{3/2}} \hat{z}$$

look up
+
Evaluate
integral

$$= 2k\lambda z \left[\frac{x}{z^2(x^2 + z^2)^{1/2}} \right]_0^L \hat{z}$$

$$\vec{E} = 2k\lambda z \frac{L}{z^2(L^2 + z^2)^{1/2}} \hat{z}$$

limits

$z \gg L$ should look like point charge

$$\vec{E} \rightarrow \frac{2k\lambda L}{z^2} \sim \frac{kQ}{z^2}$$

$L \gg \infty$

$$\frac{dL}{dL} = 1$$

$$\frac{d(zL)}{dL} = z$$

$$\vec{E} \rightarrow \frac{2k\lambda}{z}$$

field around
 ∞ line charge

Is This hard?
^

yes

calculating \vec{E} from a charge distribution directly

Much of the next month will be spent
looking at easier ways to get \vec{E}

NEXT

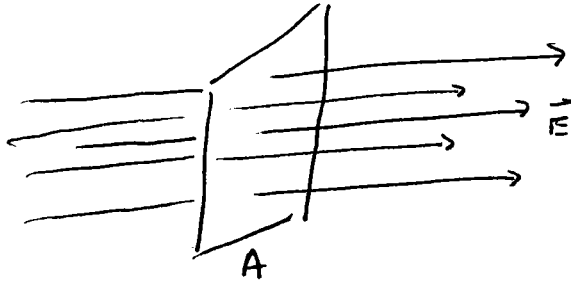
topic \Rightarrow Electric flux

Prelude to



Electric Flux Thru a surface

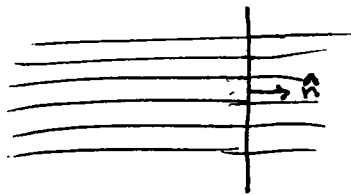
$\equiv \Phi \equiv$ # of \vec{E} field lines crossing the surface.



$\vec{E} \perp$ Surface A

$$\Phi \equiv |\vec{E}| A$$

Use Transparency



vs.



$$d\Phi = |\vec{E}| dA$$

$$\vec{E} \cdot \hat{n} = |\vec{E}|$$

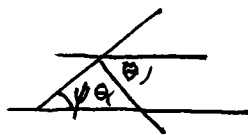
A effectively becomes $(A \sin \theta) E$

height goes from A to $A \sin \theta$

$$\hat{n} \cdot \vec{E} = |\vec{E}| \cos \theta$$



$$\hat{n} \cdot \vec{E} = |\vec{E}| \cos \theta$$



$$\sin \psi = \cos \theta$$

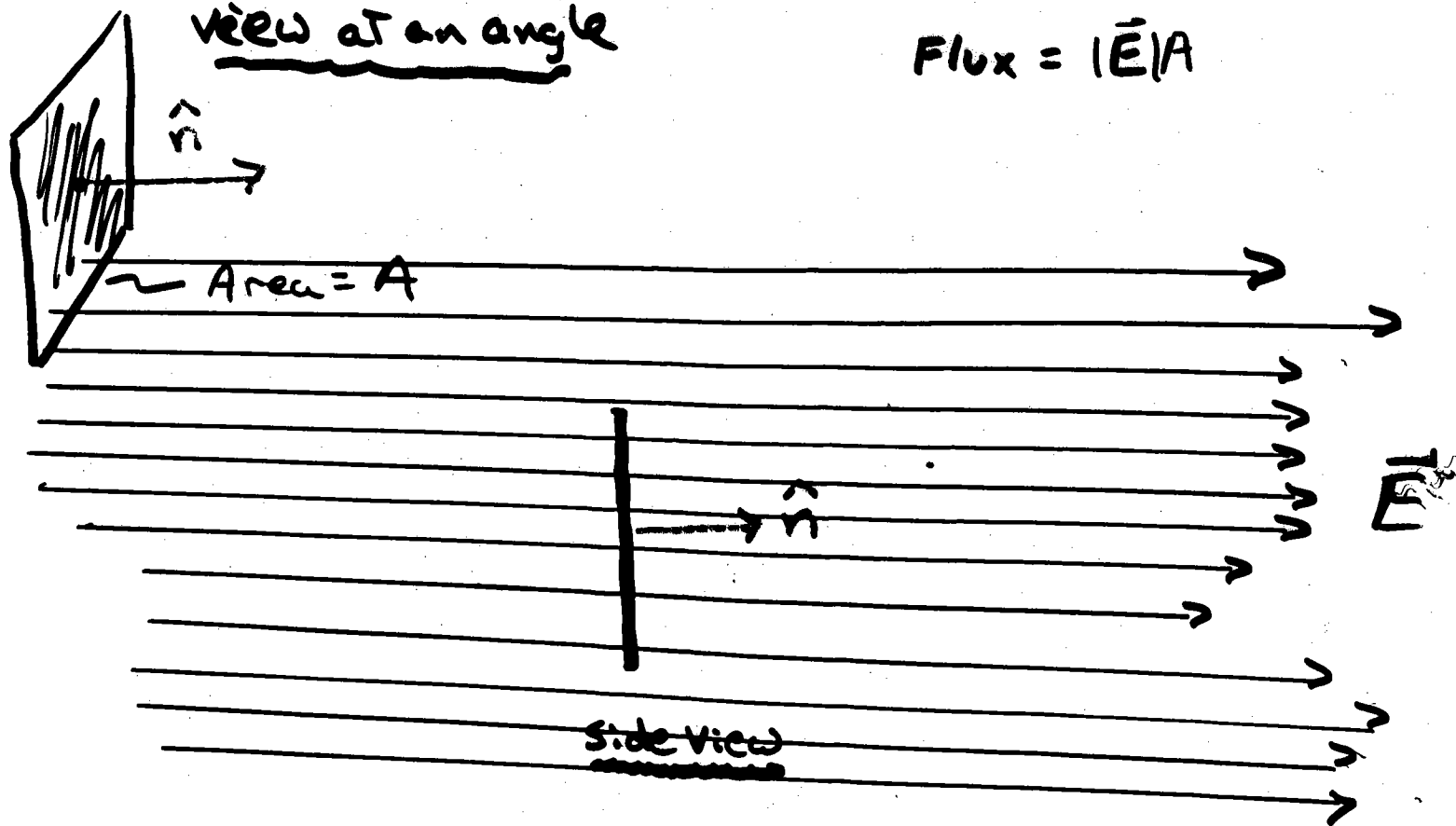
$$d\Phi = |\vec{E}| dA \cos \theta$$

$$d\Phi = \vec{E} \cdot d\vec{A} = E dA \cos \theta = \vec{E} \cdot \hat{n} dA$$

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42-381 50 SHEETS, EYE-EASER, 5 SQUARE
42-382 100 SHEETS, EYE-EASER, 5 SQUARE
42-383 100 SHEETS, EYE-EASER, 5 SQUARE
42-384 100 RECYCLED, WHITE, 5 SQUARE
42-385 200 RECYCLED, WHITE, 5 SQUARE
42-386 200 RECYCLED, WHITE, 5 SQUARE
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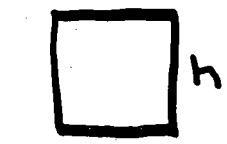


$$\vec{E} \cdot \hat{n} = |\vec{E}| |\hat{n}| \cos \theta = |\vec{E}|$$

$$\theta = 0$$

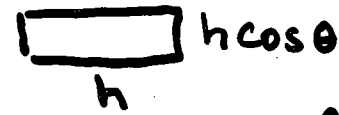
$$|\hat{n}| = 1$$

\vec{E} view



$$A = h^2$$

\Rightarrow



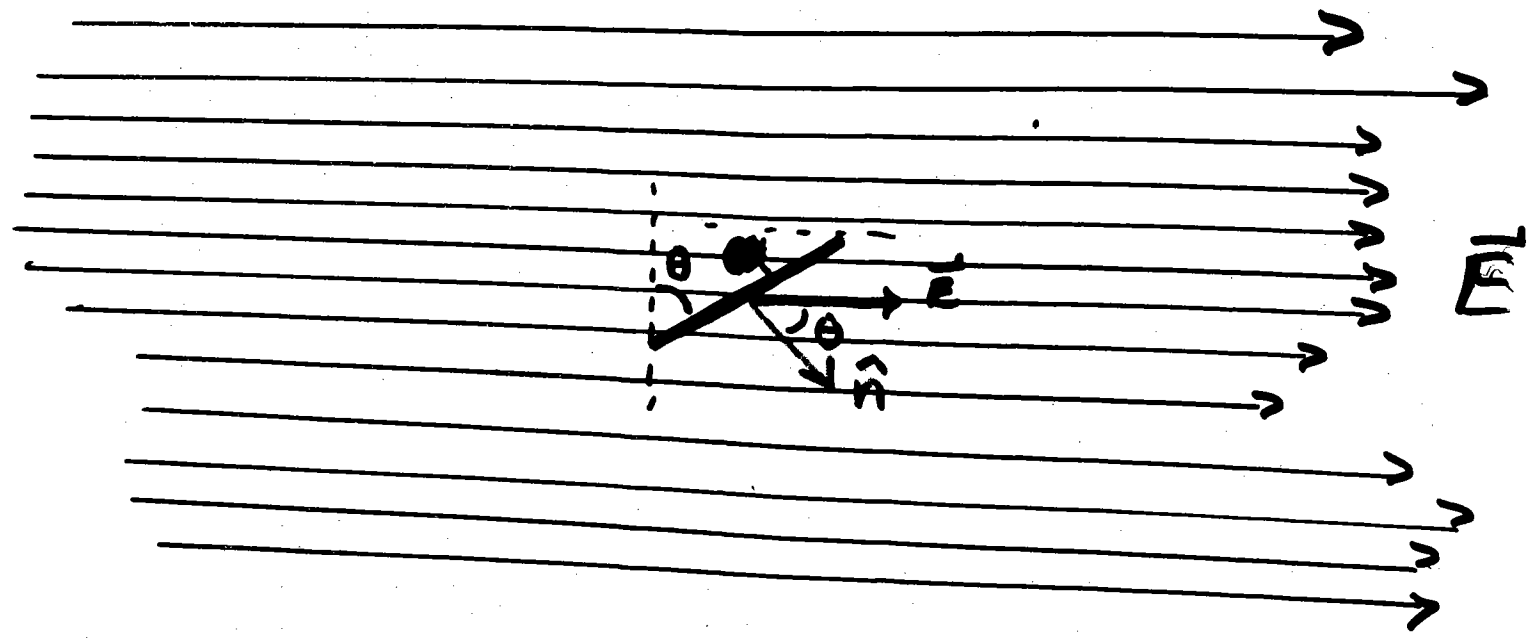
$$A = h^2 \cos \theta$$

effective

$$\text{Flux} = |\vec{E}| A_{\text{effective}} = E A \cos \theta$$

General Expression

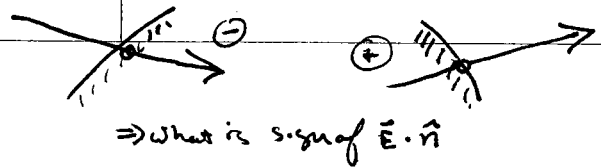
$$\text{Flux} = \vec{E} \cdot \hat{n} A \quad \text{or} \quad d\phi = \vec{E} \cdot \hat{n} dA$$



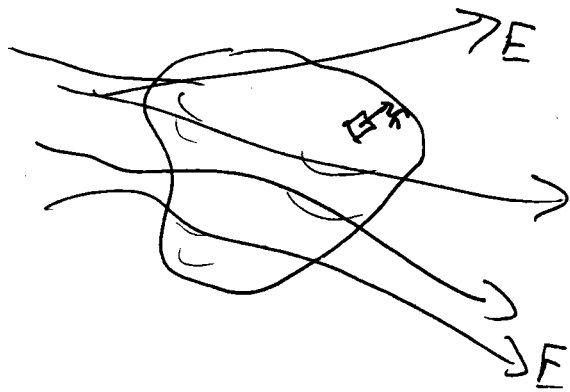
$$\vec{E} \cdot \hat{n} = |\vec{E}| |\hat{n}| \cos \theta$$

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 42-383 100 SHEETS EYE EASE™ 11 SQUARE
 42-384 100 SHEETS EYE EASE™ 14 SQUARE
 42-385 200 RECYCLED WHITE 8 1/2 SQUARE
 42-386 200 RECYCLED WHITE 11 SQUARE
 42-387 200 RECYCLED WHITE 14 SQUARE
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Arbitrary Surface



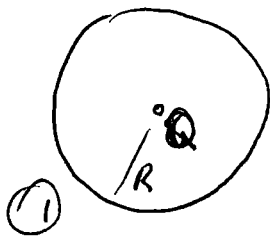
⇒ what is sign of $\vec{E} \cdot \hat{n}$



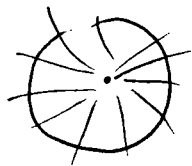
Flux $\equiv \Phi = \int_{\text{Surface}} \vec{E} \cdot \hat{n} \, dA$

$$\Phi = \int_S \vec{E} \cdot d\vec{A}$$

Gauss's Law



consider spherical surface surrounding pt charge Q at origin



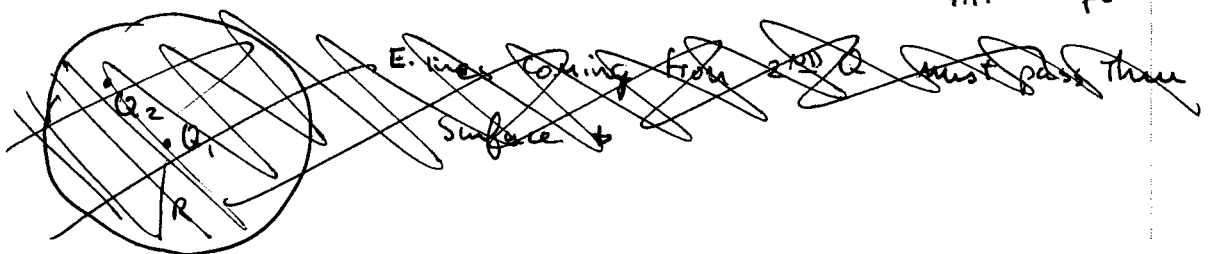
$$\Phi = \int \vec{E} \cdot d\vec{A} = |\vec{E}| \int dA = |\vec{E}| 4\pi R^2$$

$$= |\vec{E}| A = \frac{kQ}{r^2} A$$

$$\Phi = kQ 4\pi = \frac{Q}{\epsilon_0}$$

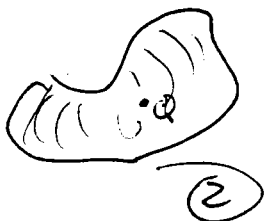
TOTAL flux $\propto Q$

Also $\vec{E} = \frac{kQ 4\pi}{A} = \frac{kQ 4\pi}{4\pi r^2} = \frac{kQ}{r^2} \hat{r}$



consider a different surface

⇒ The total # of lines is the same



∴ $\Phi_{(1)} = \Phi_{(2)}$

∴ $\int \vec{E} \cdot d\vec{A} = kQ 4\pi$
inside

Transparency 6
Figure 23-15, page 698
Spherical surface enclosing a point charge

Tipler: Physics for Scientists and Engineers 4/e
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