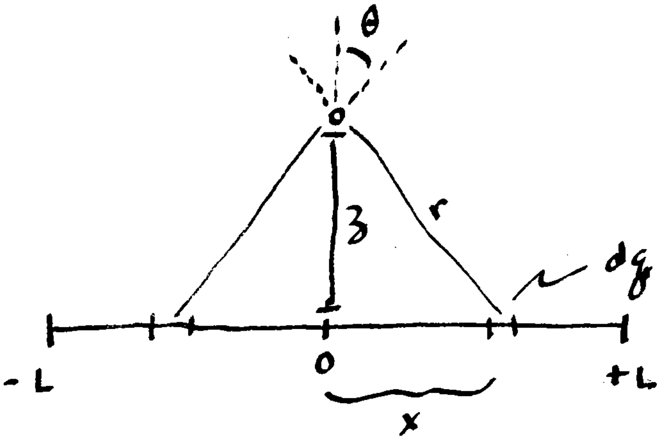
Example

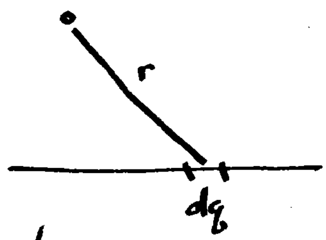
Find E at a distance z above the midpoint of a straight line segment of length $2L$ which carries a uniform line charge of $+\lambda$.



What is the symmetry here?

What direction is $\vec{E}$?

$$d\vec{E} = \frac{k\,dq}{r^2}\,\hat{r}$$



r varies w/ location of dq

$$\vec{dE} = \int_{-L}^{+L} \frac{k\,dq(x)}{r^2}\,\hat{r}$$

$E_x = |E|\sin\theta$

but Notice that x component of E $-L \to 0$ will cancel w/ x component of E from $0 \to L$

- And -

(I) z component of E $-L \to 0$ = z comp of E $0 \to L$

$$E_z = |E|\cos\theta$$

$$d\vec{E} = \cancel{z}\, dE_z\,\hat{z} = dE\cos\theta\,\hat{z}$$

$$\vec{E} = \int_{-L}^{L} \frac{k\, dq(x)}{r^2} \cos\theta\, \hat{z} = \int_{-L}^{L} \frac{k\, \lambda\, dx}{r^2} \cos\theta\, \hat{z}$$

$$= (2) \int_{0}^{L} \frac{k\, \lambda\, dx}{r^2} \cos\theta\, \hat{z}$$

limits change + mult by 2 due to I

$$r^2 = x^2 + z^2 \qquad \cos\theta = \frac{z}{r} = \frac{z}{\sqrt{x^2+z^2}}$$

$$\vec{E} = (2) \int_{0}^{L} \frac{k\lambda}{(x^2+z^2)} \frac{z}{\sqrt{x^2+r^2}}\, \hat{z}$$

z fixed by problem, x is variable we integrate over

$$\vec{E} = 2k\lambda z \int_{0}^{L} \frac{dx}{(x^2+z^2)^{3/2}}\, \hat{z}$$

Look up + Evaluate integral

$$= 2k\lambda z \left[\frac{x}{z^2 (x^2+z^2)^{1/2}} \right]_0^L \hat{z}$$

$$\vec{E} = 2k\lambda \cancel{z} \frac{L}{z^2 (L^2+z^2)^{1/2}}\, \hat{z}$$

limits

$z \gg L$ should look like point charge

$$\vec{E} \to \frac{2k\lambda L}{z^2} \sim \frac{kQ}{z^2} \;!$$

$L \gg \infty$

$$\frac{dL}{dL} = 1 \qquad \frac{d\, zL}{dL} = z$$

$$\vec{E} \to \frac{2k\lambda}{z}$$

field ~~around~~ around ∞ line charge

Is This hard ?   Yes

Calculating $\vec{E}$ from a charge distribution directly

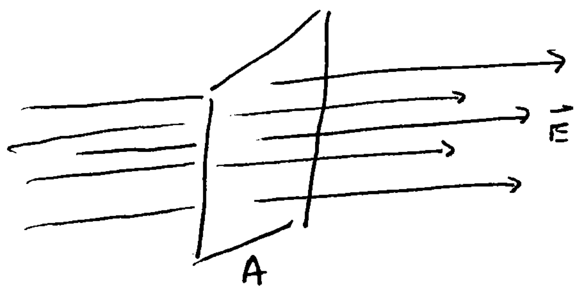
Much of The next Month will be spent looking at easier ways to get $\vec{E}$

NEXT topic $\Rightarrow$ Electric flux

Prelude to

Electric Flux Thru a surface

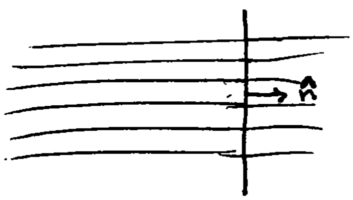
$\equiv \phi \equiv$ # of $\vec{E}$ field lines crossing the surface.



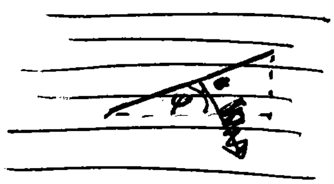
$\vec{E} \perp$ ~~A~~ surface A

$$\phi \equiv |\vec{E}| A$$

Use Transparency



vs.



Presentation Better w/ Transparencies

$d\phi = |\vec{E}| dA$

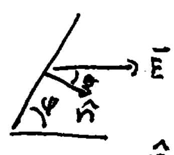
$\vec{E} \cdot \hat{n} = |\vec{E}|$

A effectively becomes

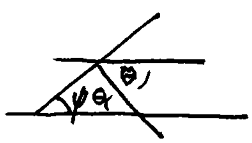
$(A \sin \varphi) E$

height goes from A to $A \sin\theta$

$| \rightarrow \hat{n} \cdot \vec{E} = \vec{E}$



$\hat{n} \cdot \vec{E} = \hat{n} \cos\theta$

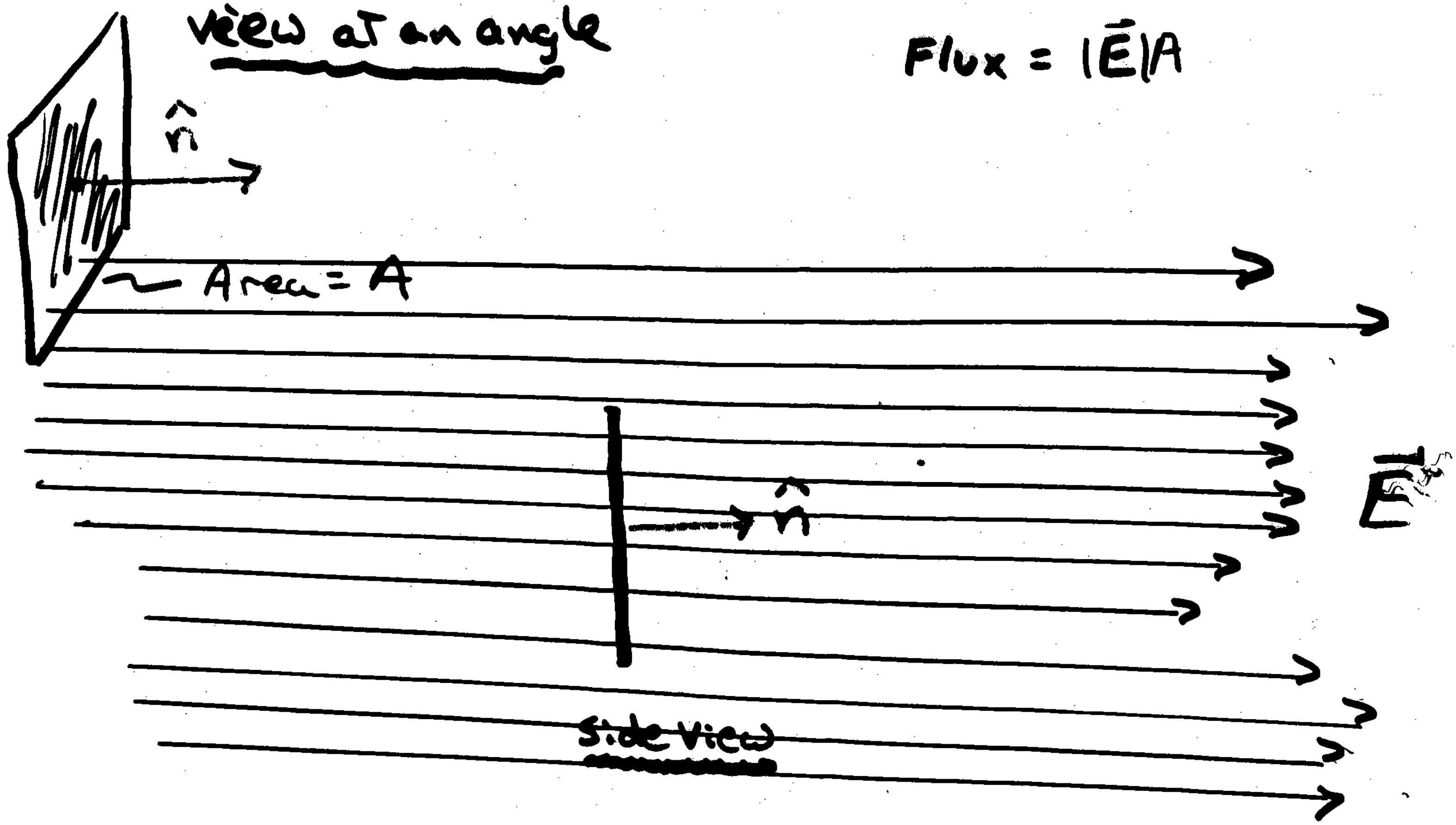


$\sin\varphi = \cos\theta$

$d\phi = |\vec{E}| dA \cos\theta$

$d\phi = \vec{E} \cdot d\vec{A} = \cancel{E} dA \quad \vec{E} \cdot \hat{n}\, dA$

$\text{Flux} = |\vec{E}|A$

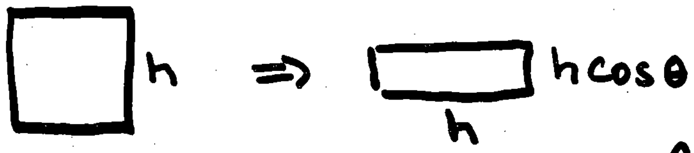


$$\vec{E}\cdot\hat{n} = |\vec{E}||\hat{n}|\cos\theta = |\vec{E}|$$

$\theta = 0$

$|\hat{n}| = 1$

$\vec{E}$ view



$A = hh$

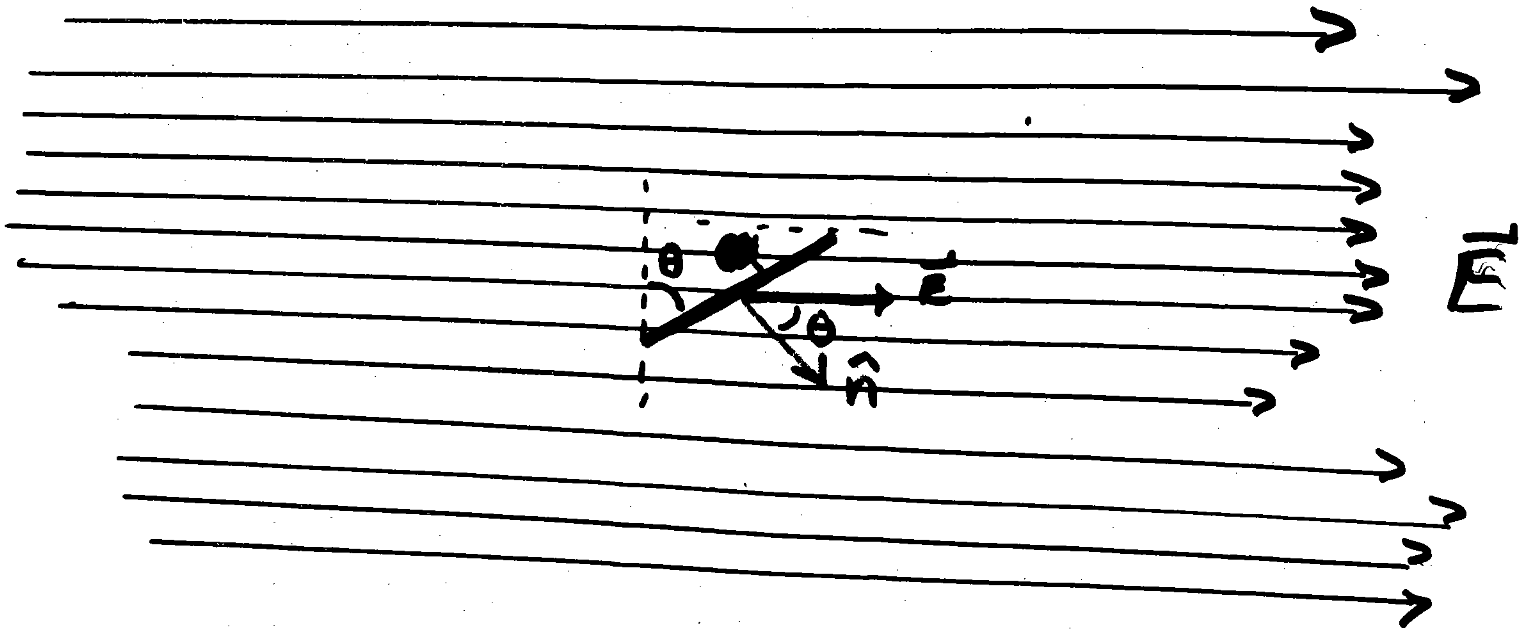
$A_{effective} = h h \cos\theta$

Flux $= |\vec{E}| A_{effective} = E A \cos\theta$

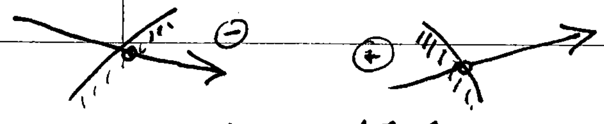
General Expression

Flux $= \vec{E}\cdot\hat{n}\, A$ or

$d\phi = \vec{E}\cdot\hat{n}\, d$

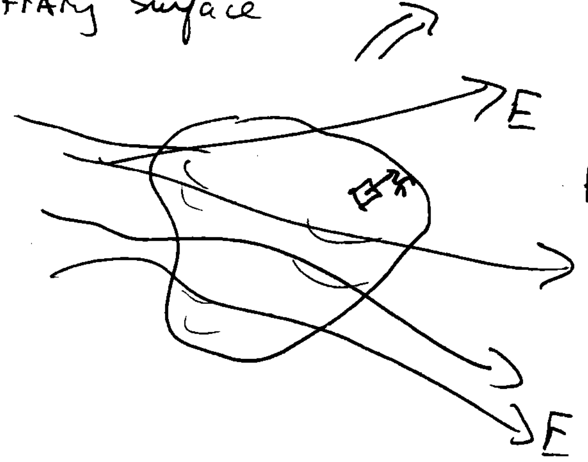


$\vec{E}\cdot\hat{n} = |\vec{E}||\hat{n}|\cos\theta$



$\Rightarrow$ what is sign of $\vec{E}\cdot\hat{n}$

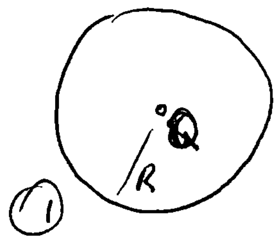
Arbitrary Surface



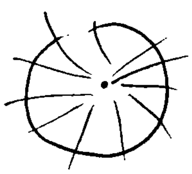
$$\text{flux} \equiv \phi = \int_{\text{Surface}} \vec{E}\cdot\hat{n}\, dA$$

$$\phi = \int_S \vec{E}\cdot d\vec{A}$$

Gauss's Law



consider spherical surface surrounding pt charge Q at origin



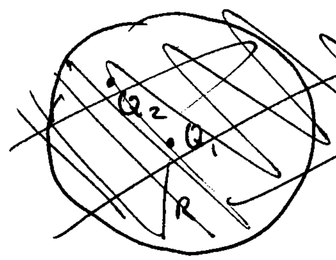
$$\phi = \int \vec{E}\cdot d\vec{A} = |\vec{E}|\int dA = |\vec{E}|\, 4\pi R^2$$

$$= |\vec{E}|\, A = \frac{kQ}{r^2} A$$

$$\phi = kQ\,4\pi = \frac{Q}{\epsilon_0}$$

TOTAL flux $\propto Q$

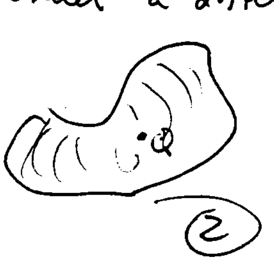
Also $\vec{E} = \frac{kQ4\pi}{A} = \frac{kQ4\pi}{4\pi r^2} = \frac{kQ}{r^2}\hat{r}$



~~E. lines coming from 2nd Q must pass thru surface~~

consider a different surface $\rightsquigarrow$ The total # of lines is the same

$\therefore \quad \phi_{①} = \phi_{②}$



$$\therefore \int \vec{E}\cdot d\vec{A} = kQ_{\text{inside}}\,4\pi$$

Transparency 6
Figure 23-15, page 698
Spherical surface enclosing a point charge