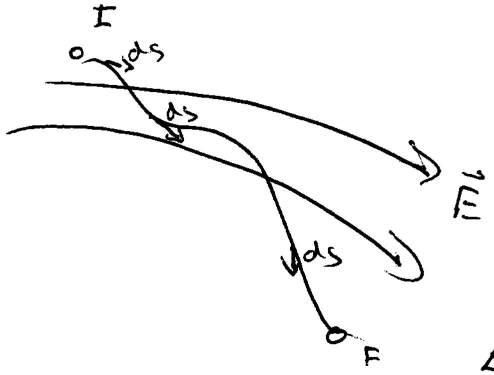
Let charge Move between points I (initial) and F (Final). The difference in the Potential Energy of the charge is equal to the work done against the $\vec{E}$ field to Move the charge



$$W = \int_I^F \vec{F} \cdot d\vec{s} = -\int_I^F q_0 \vec{E} \cdot d\vec{s}$$

Scalar — line integral — dot product gives scalar

$$\Delta PE \equiv \Delta U = -\int_I^F q_0 \vec{E} \cdot d\vec{s}$$

$$\frac{W}{q_0} = \frac{\Delta PE}{q_0} \equiv \text{Potential difference between pts I and F} = V_F - V_I$$

⇒ what are units Joules/coulomb

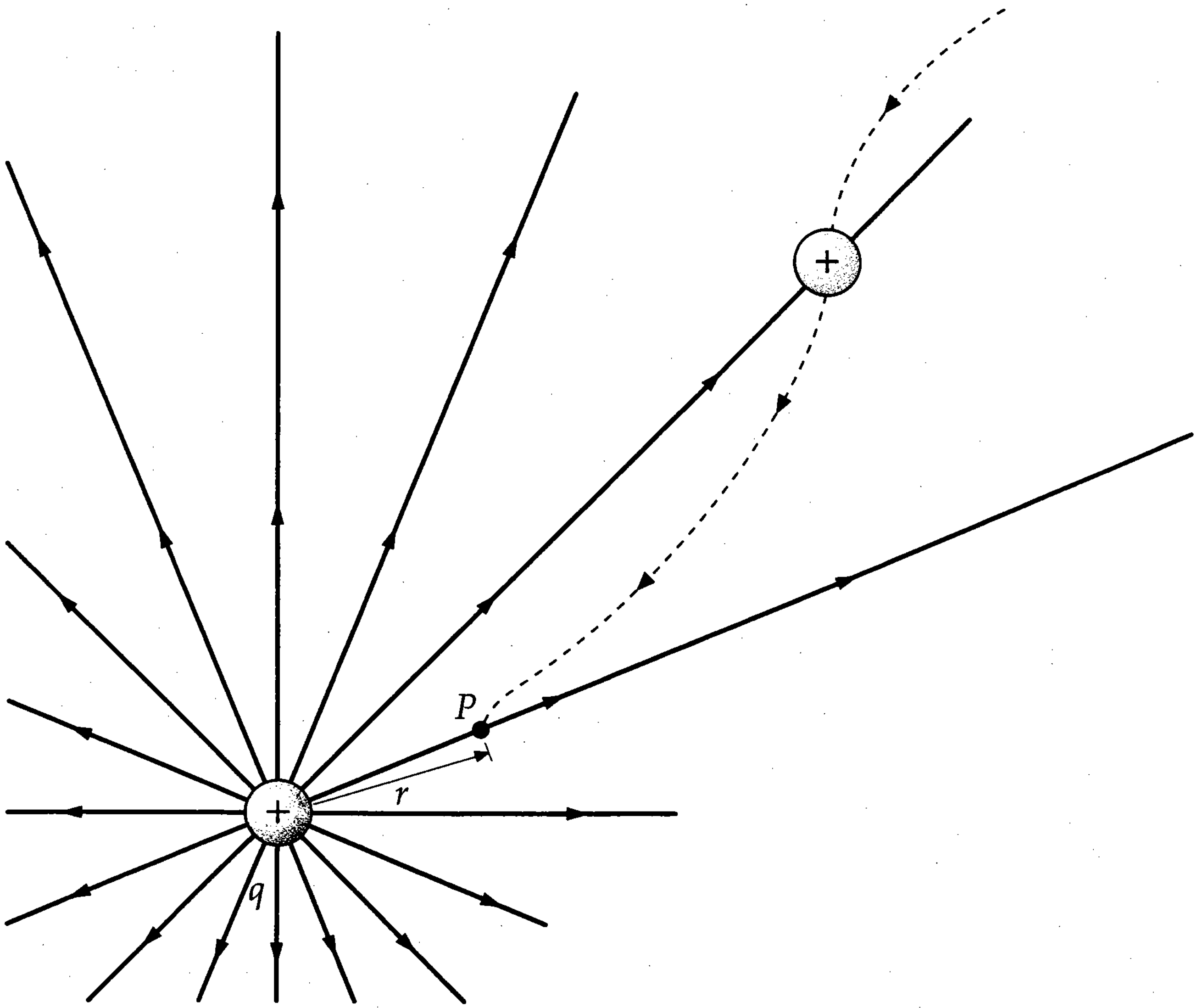
If we define a "zero" of potential energy we can calculate Absolute Potential energy and Absolute Potential

Completely Analagous to gravitational Potential and potential energy

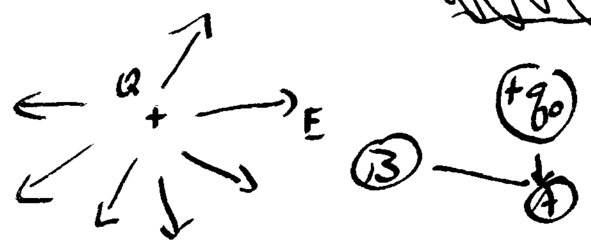
What is <u>real</u> are Potential differences

Absolute value of Potential at a point depends on Definition of "zero" of potential

Transparency 13
Figure 24-3, page 724
Work required to bring a test charge to a point a distance *r* from a point charge



~~Last class I blew a subtle sign~~



Charge Q

Move charge q_0 from A to B

ds

F on charge due to other charge

F I put in

$$W = \int_A^B \vec{F}\cdot d\vec{s} = \cancel{\int_A^B -q_0 E\,ds}$$

$$= \int_A^B |F|\,ds = -\int_A^B q_0 E\,dr$$

$$= -\int_A^B q_0 \frac{Q}{r^2}\frac{1}{4\pi\epsilon_0}\,dr = \frac{1}{4\pi\epsilon_0}\left[\frac{Q q_0}{r}\right]_A^B$$

$$= \frac{1}{4\pi\epsilon_0} Q q_0 \left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$r_A > r_B \quad \therefore$ W is (+)

I have done work on system

I have increased the potential Energy of the system

$$W_{\text{Put into system}} = \Delta PE_{\text{of system}}$$

$$\frac{W}{q_0} = \frac{\Delta PE}{q_0}$$

work per unit charge in moving from A → B

or ΔPE per unit charge moving from A → B

$\equiv$ Potential Difference, ΔV

$$V_B - V_A$$

Completely analogous to gravitational potential energy and gravitational potential

Suppose I define "zero" of electrostatic potential energy of Q-q_0 system to be when q_0 is at ∞

$$r_A = \infty$$

Then
ΔPE as go from ∞ to r_B

$$\Delta PE = \frac{1}{4\pi\epsilon} \frac{Q q_0}{r_B}$$

$$\frac{\Delta PE}{q_0} = V_B - V_\infty = V_B = \frac{1}{4\pi\epsilon_0} \frac{Q}{r_B}$$

This is the absolute potential at the point r_B!

Suppose I have

q_1
q_2
q_3 q_5 q_4

instead of Q

A

What is potential at A due to $q_1 - q_N$

$$V_A = \sum_n V_i = \sum_n \frac{kq_i}{r_i}$$

If charge distribution is continuous

$$V_A = \int dV = \int k\frac{dq}{r}$$

Not volume

Continuous Charge Dist.

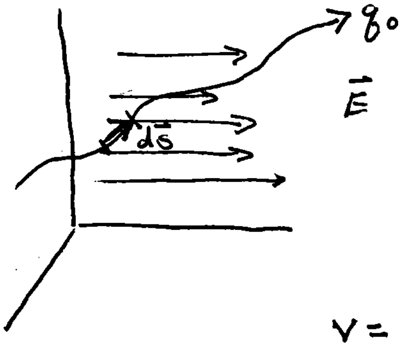
sometimes harder: vector parts sometimes cancel

$1/r^2$ sometimes easier

This is sometimes easier to calculate ~~than~~ $\vec{E}$

1) scalar
2) $1/r$ not $1/r^2$

This is impt because V CAN be used to calculate $\vec{E}$ (Always want $\vec{E}$ to find force on charges)



$\vec{E} = |\vec{E}|\,\hat{i}$ ~ or $\hat{x}$ if you prefer

$$V = \frac{W}{q_0} \Rightarrow dV = \frac{dW}{q_0} = -\vec{E}\cdot\vec{ds}$$

$$= -E_s\, ds$$

$$\therefore \quad E_s = -\frac{dV}{ds}$$

The "negative of the" rate of change of the potential in any direction is equal to the component of the electric field in that direction

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

⇒ what are these funny symbols ? ∂

Know This - You'll see a 1-d problem on this, maybe simple 2-d

vector, vector, vector > must add w/ vector addition

$$\vec{E} = -\left(\overbrace{\frac{\partial V}{\partial x}\hat{i}}^{\text{vector}} + \overbrace{\frac{\partial V}{\partial y}\hat{j}}^{\text{vector}} + \overbrace{\frac{\partial V}{\partial z}\hat{k}}^{\text{vector}}\right)$$

Really using this is beyond scope of this course

define a Vector operator, $\vec{\nabla}$ or grad, called the gradient

$$\vec{\nabla} \text{ or } \vec{\text{grad}} \equiv \left(\frac{\partial}{\partial x}\hat{i}, \frac{\partial}{\partial y}\hat{j}, \frac{\partial}{\partial z}\hat{k}\right)$$

So $\quad \vec{E} = -\vec{\nabla} V \quad$ or $\quad \vec{E} = -\vec{\text{grad}}\ V$

In one dimensional problems

$$E_x = -\frac{dV(x)}{dx} \quad \text{if } V = V(x)$$

$$E_y = -\frac{dV(y)}{dy} \quad \text{if } V = V(y)$$

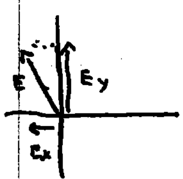
$$E_z = -\frac{dV(z)}{dz} \quad \text{if } V = V(z)$$

$$E_r = -\frac{dV(r)}{dr} \quad \text{if } V = V(r)$$

2 dimensional example –

$$V(x,y) = 3 + 2x - 6y \quad \underbrace{\text{Joules/coulomb}} \text{ or } \equiv \text{VOLTS}$$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{dV}{dx}\bigg|_{y=\text{const}} = -2 \ \text{NT/coul}$$

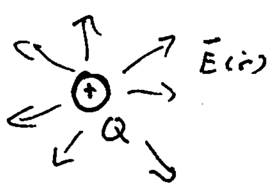


⇒ How do I know this is Joules/coul?

$$E_y = \frac{\partial V}{\partial y} = -\frac{dV}{dy}\bigg|_{x=\text{const}} = +6 \ \text{NT/coul}$$

⇒ How do I know this is NT/coul?

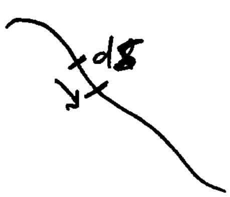
another example – PT charge what is $\vec{E}$ for a PT charge?



Have shown $V(r) = \frac{kQ}{r}$

$$\vec{E}(r) = -\frac{dV(r)}{dr}\hat{r} = +\frac{kQ}{r^2}\hat{r}$$

Equipotential Surface



$\underbrace{dV = 0}$ Surface where V is CONSTANT

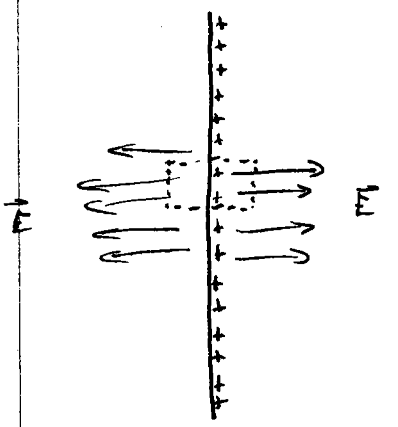
Since $dV = -\vec{E}\cdot\vec{ds}$

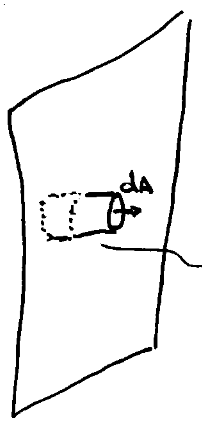
⟶ implies $\vec{E} = 0$ or $\underbrace{\vec{E} \perp \text{to } \vec{ds}}$

└ More general!

Another example

This time use E to get V





stubby cylinder

$$\int \bar{E}\cdot d\bar{A} = |E| \int dA = \frac{Q_{encl}}{\epsilon_0}$$

$$Q_{encl} = \sigma A$$

∞ plane

$dQ = \sigma \, dA$

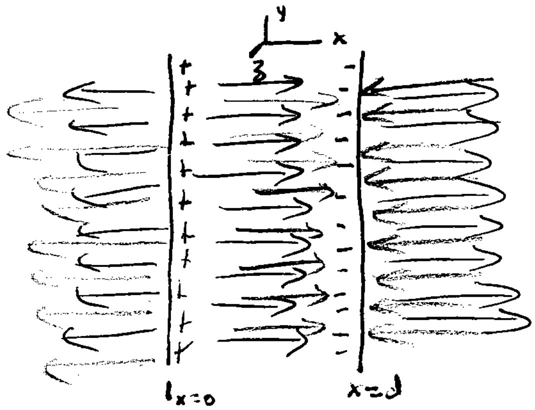
uniform charge density

2 x ends

$$2 |E| A = \frac{\sigma A}{\epsilon_0}$$

$$|E| = \frac{\sigma}{2\epsilon_0}$$

Now for problem



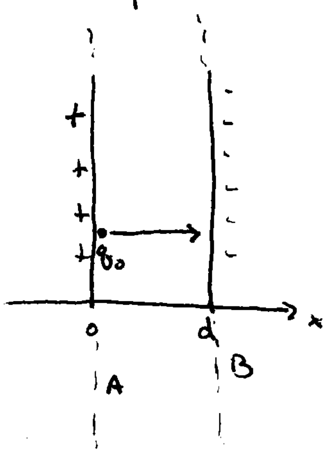
use two TRANSPARENCIES

$$E_{between} = \frac{\sigma}{\epsilon_0} \hat{x}$$

2 ∞ conducting planes // to each other

each w/ σ

Put + plane at $x=0$ - plane at $x = d$



Move q_0 from A to B plane

$$dV = -\bar{E}\cdot d\bar{s} = -|E|\,dx$$

$$V_d - V_0 = \int_0^d |E|\,dx = |E|\,d$$