Let charge move between points $I$ (initial) and $F$ (final). The difference in the potential energy of the charge is equal to the work done against the $E$ field to move the charge:

\[
W = \int_{I}^{F} \mathbf{F} \cdot d\mathbf{s} = -\int_{I}^{F} \mathbf{E} \cdot d\mathbf{s} \quad \text{(line integral)}
\]

\[
\Delta PE = \Delta U = -\int_{I}^{F} \mathbf{E} \cdot d\mathbf{s}
\]

\[
\frac{W}{q_0} = \frac{\Delta PE}{q_0} = \text{Potential difference between } V_F - V_I \quad \text{(Joules/Coulomb)}
\]

If we define a "zero" of potential energy we can calculate absolute potential energy and absolute potential.

Completely analogous to gravitational potential and potential energy.

What is real are potential differences.

Absolute value of potential at a point depends on definition of "zero" of potential.
Work required to bring a test charge to a point a distance $r$ from a point charge.
\[ W = \int_A^B F \cdot ds = \int_A^B \frac{q_0 E ds}{4\pi \epsilon_0} \]

\[ = \int_A^B \left( -\frac{q \cdot r}{r^2} \right) dr = \frac{1}{4\pi \epsilon_0} \left[ \frac{q q_0}{r} \right]_A^B \]

\[ = \frac{1}{4\pi \epsilon_0} \left( \frac{1}{r_B} - \frac{1}{r_A} \right) \]

\[ r_A > r_B \quad \Rightarrow \quad W > 0 \]

I have done work on system

I have increased the potential energy of the system

\[ W = \Delta PE \quad \text{of system} \]

\[ \frac{W}{q_0} = \frac{\Delta PE}{q_0} \quad \text{work per unit charge in moving from } A \to B \]

\[ \Delta V = \text{Potential Difference, } \Delta V \]

\[ V_B - V_A \]
Completely analogous to gravitational potential energy and gravitational potential

Suppose I define "zero" of electrostatic potential energy of Q-8o system to be when 8o is at \( r_A = \infty \)

Then

\[ \Delta PE = \frac{1}{4\pi \epsilon_0} \frac{Q8o}{r_8} \]

\[ \frac{\Delta PE}{8o} = V_B - V_8o = V_8 = \frac{1}{4\pi \epsilon_0} \frac{Q}{r_8} \]

This is the absolute potential at the point \( r_8 \)!
Suppose I have

\[ q_1, q_2, \ldots, q_n \]

\[ q_i \] instead of \( q \)

\[ \vec{A} \]

What is potential at \( A \) due to \( q_1 - q_n \)

\[ V_A = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} \frac{kq_i}{r_i} \]

If charge distribution is continuous

\[ V_A = \int_{V} dV = \int_{V} \frac{k \rho}{r} \]

This is sometimes easier to calculate than \( \vec{E} \)

1) Scalar
2) \( \frac{1}{r} \) not \( \frac{1}{r^2} \)

This is important because \( V \) can be used to calculate \( \vec{E} \) (Always want \( \vec{E} \) to find force on charges)
\[ E = \frac{|E| \hat{\imath}}{g_0} \quad \text{or} \quad \hat{x} \quad \text{if you prefer} \]

\[ \mathbf{v} = \frac{w}{g_0} \quad \Rightarrow \quad d\mathbf{v} = \frac{d\omega}{g_0} = -E \cdot ds \]

\[ = -E_s \, ds \]

"negative of the"

\[ E_s = -\frac{d\mathbf{v}}{ds} \]

The rate of change of the potential in any direction is equal to the component of the electric field in that direction.

\[ E_x = -\frac{\partial v}{\partial x} \quad E_y = -\frac{\partial v}{\partial y} \quad E_z = -\frac{\partial v}{\partial z} \]

\[ \Rightarrow \text{what are these funny symbols?} \]

Know this - you'll see a 1-d problem on this, perhaps single 2-d questions.

Really using this is beyond the scope of this course.

Define a vector operator, \( \nabla \) or \( \text{grad} \), called the gradient.

\[ \nabla \text{ or grad } = \left( \frac{\partial}{\partial x} \hat{\imath}, \frac{\partial}{\partial y} \hat{\jmath}, \frac{\partial}{\partial z} \hat{k} \right) \]

So

\[ \mathbf{E} = -\nabla v \quad \text{or} \quad \mathbf{E} = -\nabla \cdot \mathbf{v} \]
In one dimensional problems

\[ E_x = -\frac{dV(x)}{dx}, \quad \text{if } V = V(x) \]

\[ E_y = -\frac{dV(y)}{dy}, \quad \text{if } V = V(y) \]

\[ E_z = -\frac{dV(z)}{dz}, \quad \text{if } V = V(z) \]

\[ E_r = -\frac{dV(r)}{dr}, \quad \text{if } V = V(r) \]

2 dimensional example -

\[ V(x, y) = 3 + 2x - 6y \]

\[ E_x = \frac{\partial V}{\partial x} = -\frac{dV}{dx}\bigg|_{y=\text{const}} = -2 \text{ Volts} \]

\[ \Rightarrow \text{How do I know this is Joules/Coulomb?} \]

\[ E_y = \frac{\partial V}{\partial y} = -\frac{dV}{dy}\bigg|_{x=\text{const}} = 6 \text{ Volts} \]

\[ \Rightarrow \text{How do I know this is N/Coulomb?} \]

another example - electric charge

What is \( \mathbb{E} \) for a point charge?

\[ V(r) = \frac{kQ}{r} \]

\[ \mathbb{E}(r) = -\frac{dV(r)}{dr} = +\frac{kQ}{r^2} \]

Equi-potential Surface

Surface where \( V \) is constant

\[ dV = 0 \]

\[ \text{Since } \frac{dV}{ds} = -\mathbb{E} \cdot ds \]

\[ \Rightarrow \text{implies } \mathbb{E} = 0 \text{ or } \mathbb{E} \perp \text{to } ds \]

\[ \text{More general!} \]
Another example:

This time use $E$ to get $V$

\[
\int E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}
\]

$Q_{enc} = \Sigma A$

2 x ends

\[
2E_1 \cdot dA = \frac{\Sigma A}{\varepsilon_0}
\]

\[
1E_1 = \frac{\sigma}{2\varepsilon_0}
\]

Now for problem:

\[
E_{between} = \frac{\sigma}{\varepsilon_0}
\]

2 2o conducting planes \parallel to each other

each of $V$

Put 0 plane at $x=0$ - plane at $x=d$

\[
dV = -E \cdot ds = -1E_1 \, dx
\]

\[
V_d - V_0 = \int_0^d 1E_1 \, dx = 1E_1 d
\]