This leads to the Schrödinger eqn

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + U\Psi(x,t) + i\hbar\frac{\partial \Psi(x,t)}{\partial t}$$

Put in Potential and solve for $\Psi(x,t)$

$\Psi\Psi \approx$ probability of finding particle in a certain place

etc beyond scope of this course

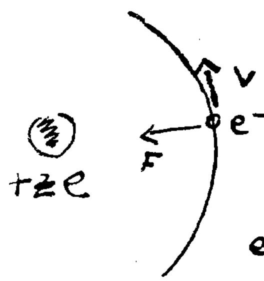
We will NOT be doing formal QM problems

Bohr Model of ~~H~~ atom

~~1926~~ 1913 - pre Quantum Mechanics

Extended work of Einstein, Plank

Classical view



velocity

$$F = \frac{kZe^2}{r^2} = \frac{mv^2}{r}$$

e^- moves in circular orbit

what is potential of Nucleus at radius = r

Potential

$$V(r) = \frac{+kZe}{r}$$

recall $V = \frac{W}{q}$ so PE e^- at $r \equiv U = \cancel{\frac{kZe^2}{r^2}}\ -\frac{Ze^2}{r^2}$

" - " from q of electron

$KE = \frac{1}{2}mv^2$ (velocity) $\qquad v^2$ (from above) $= \frac{kZe^2}{mr}$

$\therefore\ KE = \frac{1}{2}\frac{kZe^2}{r}$

$$\text{Total Energy} = PE + KE = -\frac{1}{2}\frac{kZe^2}{r}$$

Such an atom is unstable because

Charge is constantly accelerated

Should radiate energy + spiral inward

Mechanically stable but NOT Electromagnetically stable !

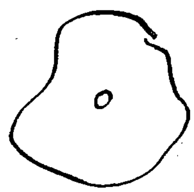
Bohr

⇒ e^- can occupy certain states w/out radiating classically

⇒ e^- can make transitions between these STATES by emitting / Absorbing photons ~~of freq~~

$$E_\gamma = h\nu = E_i - E_f$$

⇒



Assumed stable orbits happen when Circumference = integral # of λ

$$n\lambda = 2\pi r \qquad n = 1, 2, 3 \ldots$$

$$n\frac{h}{p} = 2\pi r$$

quantized Angular Momentum

$$n\hbar = mvr = L$$

quantized velocity

$$\text{or} \quad v = \frac{n\hbar}{mr}$$

So

$$F = \frac{kZe^2}{r^2} = \frac{mv^2}{r}$$

$$\frac{kZe^2}{r^2} = \frac{m}{r}\frac{n^2\hbar^2}{m^2r^2}$$

Bohr radius

$$r_n = \frac{n^2\hbar^2}{mkZe^2} = n^2\frac{a_0}{Z}$$

Assum $Z=1$, $n=1$ — orbit of e^- in gnd STATE of H ATOM

$$r = a_0 = \frac{\hbar^2}{mke^2} = 0.0529 \text{ nm}$$

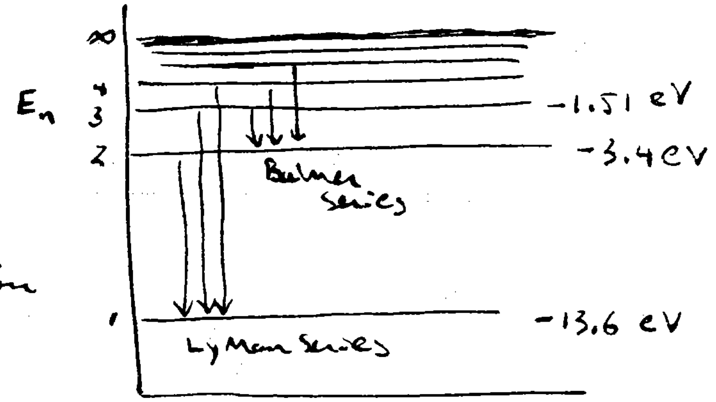
$$E_n = -\frac{1}{2}\frac{kze^2}{r_n} = -\frac{1}{2}\frac{kze^2\, mkZe^2}{n^2\hbar^2}$$

$$E_n = \frac{-mk^2e^4Z^2}{2\hbar^2 n^2} = \frac{-Z^2E_0}{n^2} \qquad n = 1, 2, 3 \cdots$$

Allowed energy levels in the Bohr atom

for H ATOM $Z=1$

Predicts discrete Spectral emission/Absorption lines



⇒ Agrees w/ experiment!

Big push for Quantum Mechanics!

Examples

(I) What is λ of γ emitted as e^- drops (makes a transition) from the $N=2$ level of hydrogen atom to the $N=1$ level?

$$\Delta E = E_2 - E_1 = -E_0\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = \left(\frac{3}{4}\right)\frac{m_e^2k^2e^4Z^2}{2\hbar^2} = 10.2\text{eV}$$

$E = h\nu \qquad 10.2\text{eV} = 4.1\times10^{-15}\text{eV}\cdot\text{s}\ \nu\left(\frac{1}{s}\right) \qquad \nu = 2.49\times10^{15}\frac{1}{s}$ or Hz

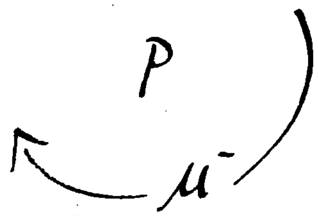
$c = \lambda\nu \qquad 3\times10^8 \text{m/s} = (\lambda \text{ m})(2.49\times10^{15}\text{ Hz})$

$\lambda = 1.2\times10^{-7}\text{ m}$

II How Much Energy does it take to remove e^- from H atom
(Minimum Energy)
if it is in $n=2$ energy level

$$\Delta E = E_\infty - E_2 = -E_0\left(\underbrace{\frac{1}{2^2} - \overset{0}{\frac{1}{\infty^2}}}_{\frac{1}{n_f^2} - \frac{1}{n_i^2}}\right) = E_2 = -3.4\text{ eV}$$

III Suppose one could make "Hydrogen" out of a proton and a Muon (μ^-)



~~[illegible]~~ Muonic Hydrogen

What is the ~~ground state Energy~~ radius of groundstate atom of ~~Muonium~~ Muonic Hydrogen?

Use Bohr Model – Modify as needed

Bohr's Postulates stay the same

$$n\lambda = 2\pi r \longrightarrow n\hbar = mvr$$

$$v = n\hbar / mr \qquad n = 1, 2, 3 \ldots$$

$$F = \frac{kZe^2}{r^2} = \frac{mv^2}{r} \longrightarrow r = \frac{n^2\hbar^2}{m_\mu k Z e^2}$$

e is charge e, μ stays the same!

Z is charge on Nucleus – P, Z=1 same

$m_\mu \simeq 2000\, m_e$

for $n=0$ gnd State

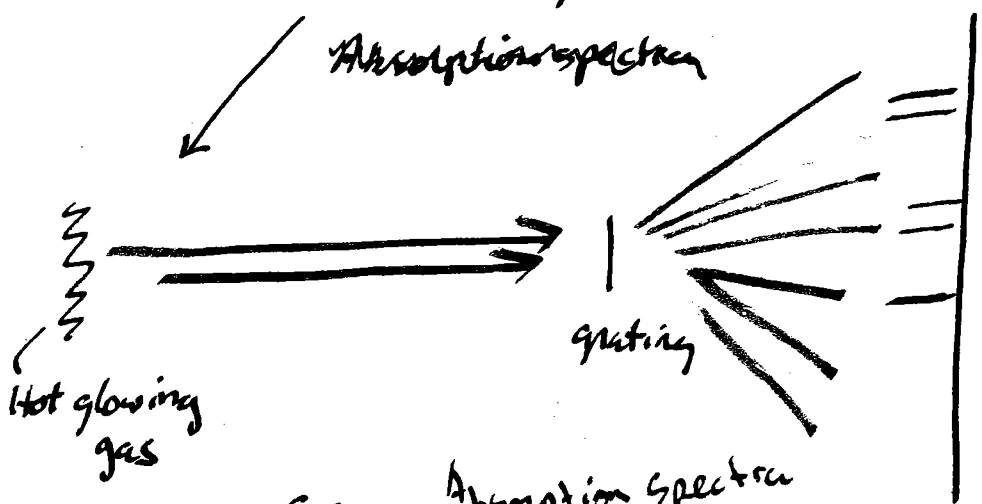
$$r = \frac{n^2\hbar^2}{m_\mu k Z e^2} = \frac{n^2\hbar^2}{2000\, m_e k Z e^2} = \frac{1}{2000} r_0$$

Bohr radius

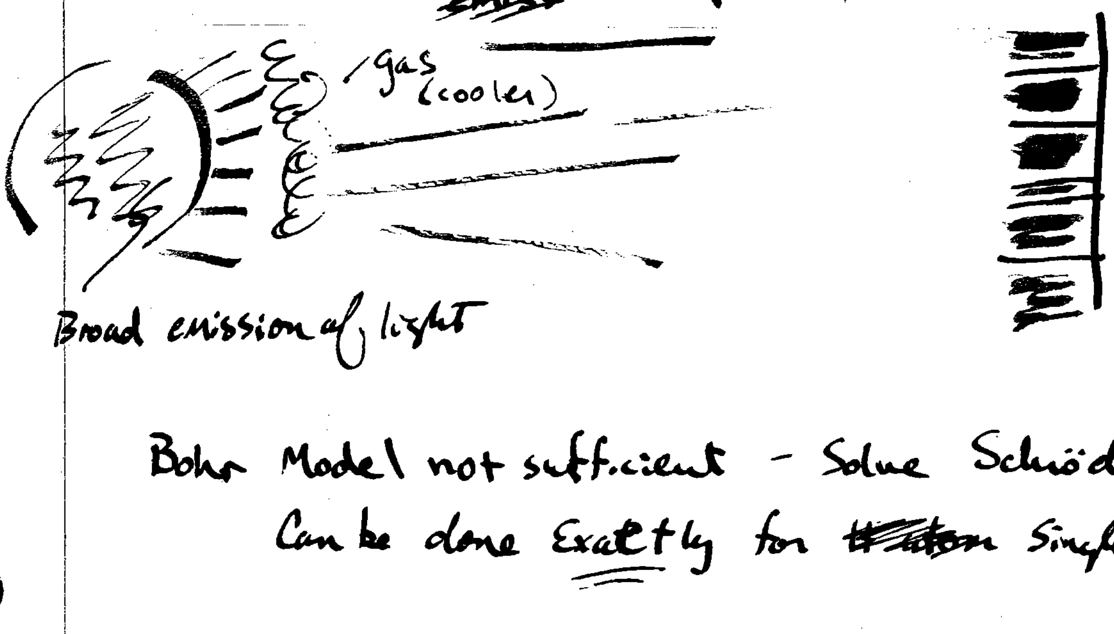
Radius of ~~[illegible]~~ is $\frac{1}{2000}$ that of Hydrogen
Muonic hydrogen

Bohr Model Hugely successful in helping us understand
~~emission~~ emission spectra

~~Absorption spectrum~~



~~Emiss~~ Absorption Spectra



Bohr Model not sufficient - Solve Schrödinger's Eqn
Can be done Exactly for ~~[illegible]~~ Single e^- Atom