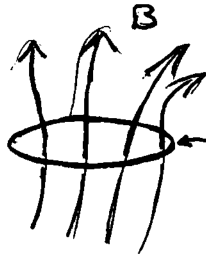
So, Think abt wire loop $\qquad \mathcal{E} = \oint \vec{B}\cdot d\vec{l} = -\frac{d\Phi_M}{dt}$

Δ'ing geometry of loop

Δ'ing B magnitude or Angle will cause Δ in Magnetic flux




changing B flux thru loop induces $\mathcal{E}$

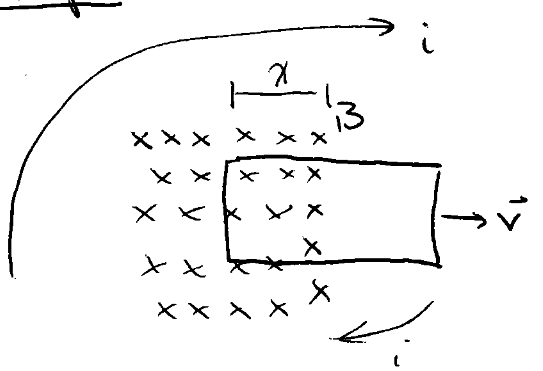
i or current in wire loop

So we have induced $\mathcal{E}$ .... current flows in loop
→ induced current

What direction? ~> NOTE the "−" in Faraday's law

Lenz's Law - An induced current in a closed conducting loop will appear in such a ~~way~~ direction that it opposes the change that produced it!

Examples




Explain $\Phi_M$ — Measured

induced i wants to ↑ increase

goes in what direction?

$\ell$

Wave arms

$\Phi_M = B\ell x$

$$\mathcal{E} = -\frac{d\Phi_M}{dt} = -B\ell\frac{dx}{dt} = -B\ell v$$

Sets up a current $|i| = \frac{B\ell v}{R}$ — loop resistance

Clockwise — why?

3 ways to look at —

① $\Phi_M$ being reduced

induced $i$ creates B that increased $\Phi_M$

B of loop

② Motional EMF

$\uparrow F$ $\times B$ $\bullet \rightarrow v$

q $\quad$ v of

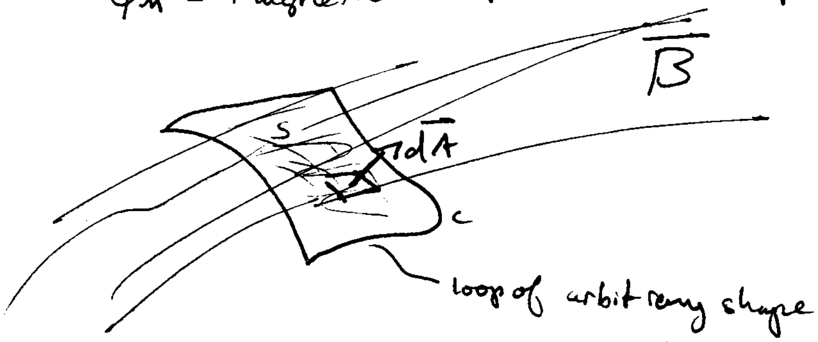
Force on "conductor q" due to V

③ $\leftarrow F$ $\times$ $\uparrow i$ $\times B$

F produced by current works to slow down $\vec{v}$

what is $\Phi_M$

$\Phi_m \equiv$ Magnetic flux thru the loop



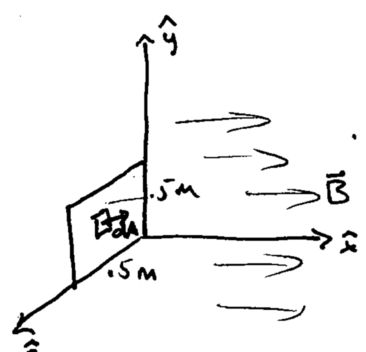

$$\int_S \vec{B}\cdot d\vec{A} = \Phi_M$$

Think of this exactly as you thought of $\int E \cdot dA$ for Gauss's Law

But Surface is NOT closed

and use E instead of B

Simple example of Magnetic flux calculation



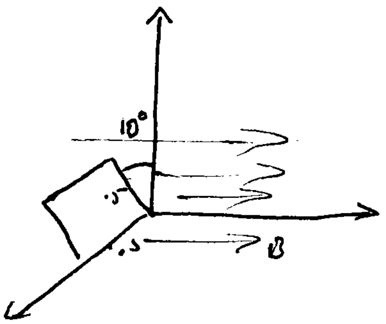

.5 m square loop in y-z plane

$\vec{B}$ uniform - 2 Tesla in $\hat{x}$ direction

$$\vec{B}\cdot\vec{dA} = B\,dA$$

$$\int \vec{B}\cdot\vec{dA} = |B|\int dA = BA = 2T(.5\,m)^2$$

$$= .5\ T\cdot m^2$$

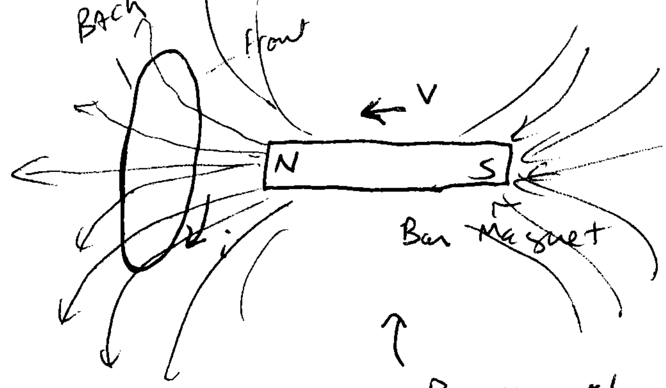


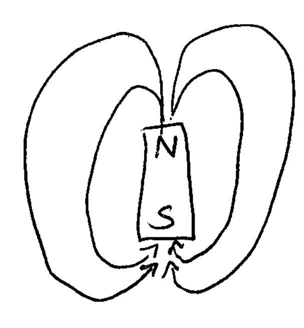

Same problem:

Except loop now rotated


$$\vec{B}\cdot\vec{dA} = |B||dA|\cos\theta$$

$$\int \vec{B}\cdot\vec{dA} = |B|\cos\theta\int dA = (\cos 10^\circ)\ .5\ T\cdot m^2$$




Bar magnet looks like a dipole for electrostatics

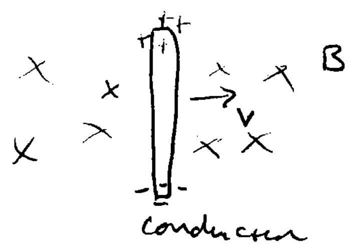
$\phi_m$ increases

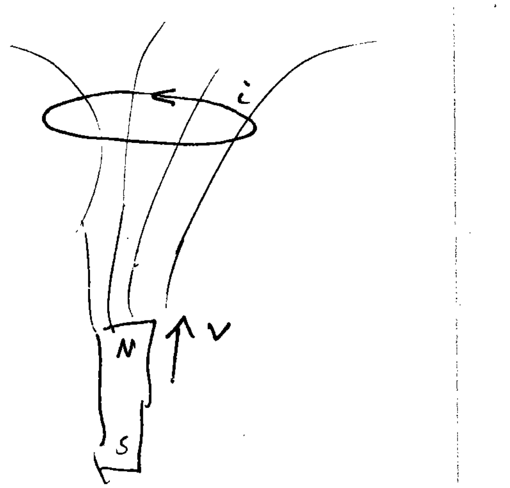
B due induced i decreased B

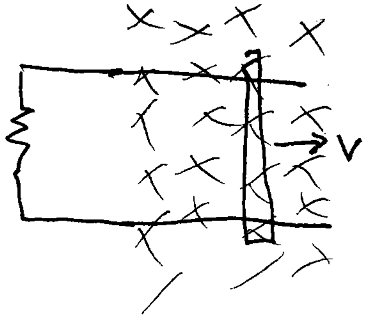
↑B

Note:

Motional EMF vs. EMF induced by changing B field



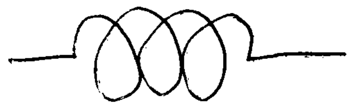




Sometimes Both — can think of either way (NOT both added)

$\mathcal{E}iR$

$$\mathcal{E} = -L\frac{di}{dt}$$



Symbol of inductor in circuit

$$\mathcal{E} = -L\frac{di}{dt} = -L\frac{d^2q}{dt^2}$$

Remember $\mathcal{E}$ 

<u>$\Sigma V$ eqn</u>

$V = iR = \frac{dq}{dt}$ 

$Q/C = V$ 

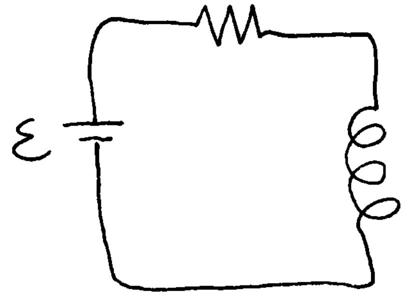
$\mathcal{E} = -L\frac{d^2q}{dt^2}$ 

Basic Circuit Elements

inductors:

- will NOT do RL, RLC circuits
- will have to calculate inductances, L
- " " " " $\mathcal{E}$ ~~using~~ or something using L

# Energy + The Magnetic Field



$$\mathcal{E} = iR + L\frac{di}{dt}$$

$$\mathcal{E}i = i^2R + Li\frac{di}{dt}$$

$P = iV$

Power out of EMF

Power dissipated by Resistor

Power in or out of inductor

rate at which energy stored in the Magnetic field

$$\text{Power} = \frac{dU_B}{dt} = Li\frac{di}{dt}$$

$$dU_B = L\,i\,di$$

$$U_B = \int_0^I L\,i\,di = \frac{1}{2}Li^2$$

Energy stored in an inductor carrying current $i$

(similar to $U = \frac{1}{2}CV^2$ for capacitors !)

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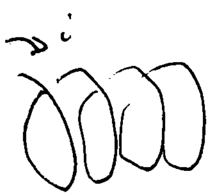
**Example** Consider a solenoid carrying a current $i$



$n$ turns/Length

What is the Energy density of the Magnetic field inside ?

Find in terms of $B$.

$$B_{\text{solenoid}} = \mu_0 n i \quad \text{inside}$$

what direction ?

$$= 0 \quad \text{outside}$$

$$u_B = \text{energy density} = \frac{U_B}{A\ell}$$ (Area)(length)

$$u_B = \frac{\frac{1}{2} L i^2}{A\ell}$$

recall for solenoid

$$\Phi_M \propto i \qquad \Phi_M = L i$$

$$\text{or} \quad L = \frac{\Phi_M}{i} = \frac{(BA)(n\ell)}{i}$$

\# Turns

$$B_{\text{solenoid}} = \mu_0 n i \longrightarrow L = \mu_0 n^2 A \ell$$

Sub into $u_B$

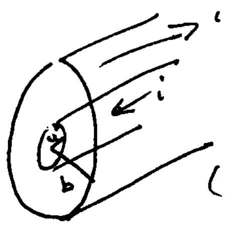
$$u_B = \frac{1}{2} \frac{\mu_0 n^2 A \ell\, i^2}{A\ell} = \frac{1}{2}\mu_0 i^2 n^2 = \frac{B^2}{2\mu_0}$$

$$\boxed{u_B = B^2/2\mu_0}$$

derived w/ Solenoid example
True in general !

Exactly Analogous to $u_E = \frac{1}{2}\epsilon_0 E^2$

<u>Example</u>



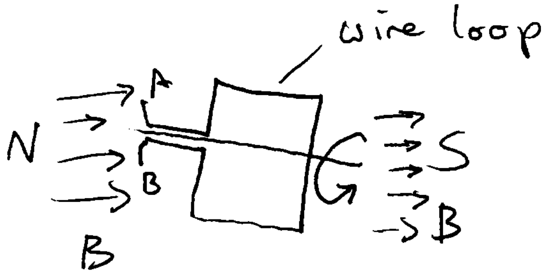

Concentric conducting shells
coAxial

(a) Find B everywhere

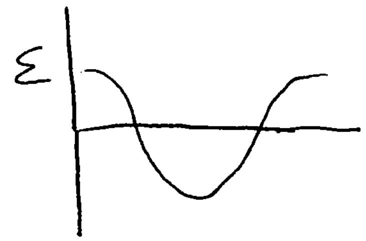
(b) Find total $U/\ell$ for System

# AC circuits

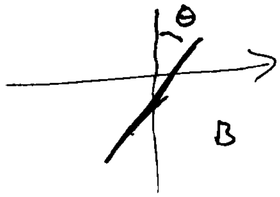
NOT covered in this course but –




Plot $V_{AB}$ induced




Flux depends on Angle




$\phi \propto \cos\theta$

$\theta = \omega t$

$\therefore \frac{d\phi}{dt} \sim \mathcal{E} \sim \sin\theta$

Sinusoidally Varying $\mathcal{E}$

Alternating current

$\Rightarrow$ Time dependence in everything!

$\Rightarrow$ More complex than Direct Current

Know your limitations!

$\Rightarrow$ This is what you get out of the wall

60 Hz Frequency

look for it ~~as~~ ~~a~~ Noise Source.