So, think of a wire loop

\[ E = \oint B \cdot dl = -\frac{d\Phi_m}{dt} \]

Changing flux through loop

\[ \Phi_m \] 

Induced current in wire loop

\[ I \]

So we have induced \( E \) \( \rightarrow \) current flows in loop

\[ \rightarrow \text{induced current} \]

What direction? Note the "-" in Faraday's Law

Lenz's Law - An induced current in a closed conducting loop will appear in such a direction that it opposes the change that produced it.

**Examples**

\[ \Phi_m = Blx \]

\[ E = \frac{d\Phi_m}{dt} = -Bl \frac{dx}{dt} = -BlV \]

Sets up a current \[ I = \frac{BlV}{R} \] - loop resistance

Clockwise - why?

3 ways to look at -

1. \( \Phi_m \) being reduced

2. \( F \) produced by current

3. \( F \) produced by current

induced \( i \) creates \( B \) that increased \( \Phi_m \)

\[ F \equiv \text{motion} \]

\[ \vec{F} \]

\[ x \]

\[ v \]

\[ F \]

\[ F \equiv \text{F produced by current works to slow down} \]

\[ v \]

\[ \text{due to} \]

\[ \text{V} \]
What is \( \Phi_m \)

\[ \Phi_m = \text{Magnetic flux thru the loop} \]

\[ \oint \mathbf{B} \cdot d\mathbf{A} = \Phi_m \]

Think of this exactly as you thought of \( \int \mathbf{E} \cdot d\mathbf{A} \) for Gauss' Law.

But surface is NOT closed and use \( \mathbf{E} \) instead of \( \mathbf{B} \).

Simple example of Magnetic flux calculation

- \( 0.5 \text{ m} \) square loop in \( y-z \) plane
- \( \mathbf{B} \) uniform = \( 2 \text{ Tesla} \) in \( \hat{x} \) direction
- \( \mathbf{B} \cdot d\mathbf{A} = \mathbf{B} d\mathbf{A} \)

\[ \int \mathbf{B} \cdot d\mathbf{A} = |\mathbf{B}| \int d\mathbf{A} = \mathbf{B} \cdot d\mathbf{A} = 2 \times 0.5 \times (0.5 \text{ m})^2 = 0.5 \text{ T} \cdot \text{m}^2 \]

Same problem:
- Except loop now rotated
- \( \mathbf{B} \cdot d\mathbf{A} = |\mathbf{B}| d\mathbf{A} \cos \theta \)

\[ \int \mathbf{B} \cdot d\mathbf{A} = |\mathbf{B}| \cos \theta \int d\mathbf{A} = (\cos 10^\circ) \times 0.5 \text{ T} \cdot \text{m}^2 \]
Bar magnet looks like a dipole for electrostatics.

\( \Phi_m \) increases

B due \( \cdot \) induced \( i \) decreased \( B \)

Note:

Motional EMF \( \text{ vs. } \) EMF induced by changing B field.

Sometimes both - can think of either way (not both added).
\[ E = L \frac{di}{dt} \]

Symbol of inductor in circuit

\[ E = -L \frac{di}{dt} = -L \frac{d^2 q}{dt^2} \]

Remember \[ E \frac{1}{1} \]

\[ v = iR = \frac{dq}{dt} \frac{1}{1} \]

\[ \frac{Q}{C} = \frac{1}{1} \]

\[ E = -L \frac{d^2 q}{dt^2} \frac{1}{1} \]

Basic circuit elements

Inductors:

- Will not do RL, RLC circuits
- Will have to calculate inductances, \( L \)
- " " " " \( E \) coming or something using \( L \)
Energy & the Magnetic Field

\[ E = iR + \frac{L}{\text{dt}} \frac{di}{dt} \]

\[ E_i = i^2R + \frac{L}{\text{dt}} \frac{di}{dt} \]

\[ P = iv \]

Power out of EMF

Power dissipated by Resistor

Power in or out of Inductor

Rate at which energy stored in the magnetic field

\[ \text{Power} = \frac{dU_B}{dt} = L \frac{di}{dt} \frac{di}{dt} \]

\[ dU_B = Lidi \]

\[ U_B = \int_0^i Lidi = \frac{1}{2} Li^2 \]

Energy stored in an inductor carrying current \( i \)

(similar to \( U = \frac{1}{2} CV^2 \) for capacitors !)

**Example**

Consider a Solenoid carrying a current \( i \)

\[ i \]

\[ N \text{ turns} / \text{length} \]

What is the Energy density of the Magnetic field inside?

\[ E \text{ in terms of } B \]
B_{solenoid} = \mu_0 n i \quad \text{inside}
= 0 \quad \text{outside}

U_{B3} = \text{energy density} = \frac{U_B}{A l} \quad \text{(Area)(length)}

U_B = \frac{1}{2} L i^2 / A l

Recall for solenoid:
\phi_M \propto i \quad \phi_M = L i \quad \text{amp-turns}

or \quad L = \frac{\phi_M}{i} = \frac{(BA)(n l)}{i}

B_{solenoid} = \mu_0 n i \quad \Rightarrow \quad L = \mu_0 n^2 A l

Sub into \ U_B

U_B = \frac{1}{2} \frac{\mu_0 n^2 A l i^2}{A l} = \frac{1}{2} \frac{\mu_0 i^2 n^2}{A l} = \frac{B^2}{2 \mu_0}

\boxed{U_B = \frac{B^2}{2 \mu_0}} \quad \text{derived \! \! \! \! \! \! \! \! \! \! \! \text{by \! \! \! \! \! \! \! \! \! \! \! solenoid \! \! \! \! \! \! \! \! \! \! \! example}}

\text{True in general!}

Exactly analogous to \ U_e = \frac{1}{2} \epsilon_0 E^2

\underline{Example}

Concentric conducting shells coaxial

(a) Find B everywhere

(b) Find total \ U/2 for system
AC Circuits

Not covered in this course but —

Wire loop

\[ \Phi \propto \cos \theta \]
\[ \theta = \omega t \]
\[ \frac{d\Phi}{dt} \sim \varepsilon \sim \sin \theta \]

Plot \( V_{AB} \) induced

\[ \sinusoidally \]
\[ \text{Varying } \varepsilon \]

Alternating current

\[ \Rightarrow \text{Time dependence in everything!} \]

\[ \Rightarrow \text{More complex than Direct Current} \]

\[ \Rightarrow \text{This is what you get out of the wall} \]

60 Hz frequency

Look for it as a noise source.

Know your limitations!