

The work to carry a charge q_0 from one plate to the other
 is Vq_0 or $q_0 \int \vec{E} \cdot d\vec{s}$

\therefore $\int q_0 \vec{E} \cdot d\vec{s}$

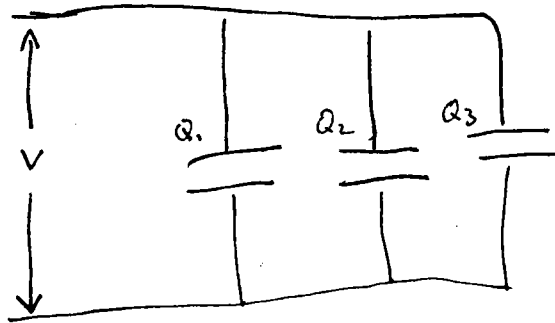
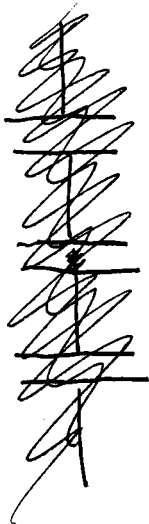
$V = \int \vec{E} \cdot d\vec{s}$

NOW $C = \frac{Q}{V} = \frac{\epsilon_0 \int \vec{E} \cdot d\vec{s} A}{\int \vec{E} \cdot d\vec{s}} = \frac{\epsilon_0 A}{d}$

Depends only on geometry ... As promised!!

Combinations of Capacitors

in Parallel //



TOTAL $Q = Q_1 + Q_2 + Q_3$

but $Q_1 = C_1 V$

$Q_2 = C_2 V$

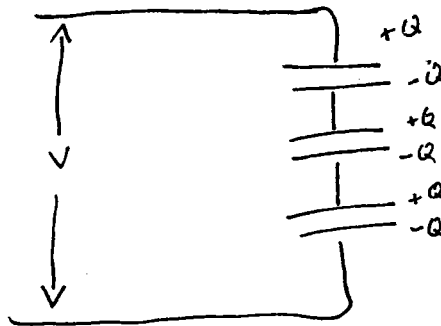
$Q_3 = C_3 V$

$Q_{TOTAL} = V(C_1 + C_2 + C_3)$

\therefore

$C_{cap \text{ in } //} = C_1 + C_2 + C_3$

Capacitors in Series



$$Q = C_1 V_1$$

$$Q = C_2 V_2$$

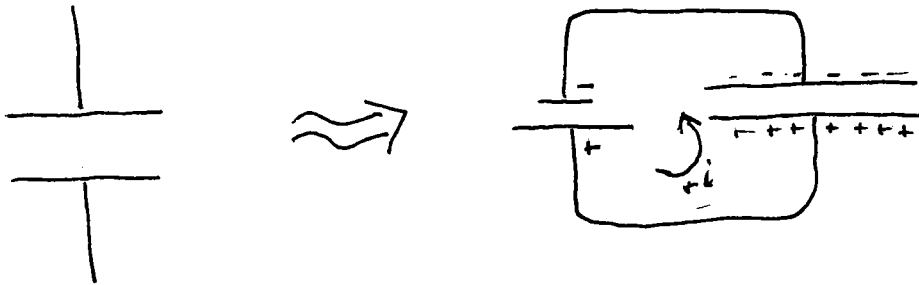
$$Q = C_3 V_3$$

$$V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)$$

$$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

DO EXAMPLE
NEXT PAGE

Energy and the Electric Field



Hook it up ... e^- transferred from + to - side

OR can think

" + current " flows from - to + side

at a given instant V' across plates

$$dW = V' dq' \quad \text{as } q \text{ builds up}$$

↑ work required to transfer dq'

when potential across plates is V'

and q' has been transferred

work required to completely charge up plate as far as possible w/

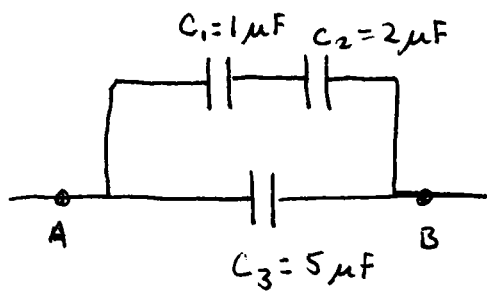
given geometry and total V

allowed by battery

$$W = \int_0^Q \frac{q}{C} dq$$

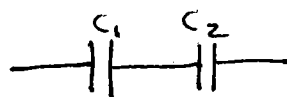
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Example Problems



What is Capacitance between A and B?

1st find $C_{\text{eff.ive top}}$

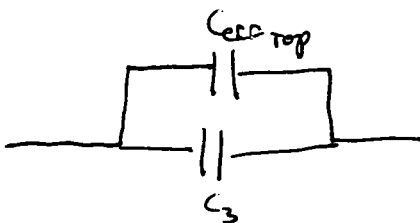


\nearrow
in series

$$\frac{1}{C_{\text{eff top}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

$$C_{\text{eff top}} = \frac{C_1 C_2}{C_1 + C_2}$$

Now



capacitors in //

$$C_{AB} = C_{\text{eff top}} + C_3 = \frac{C_1 C_2}{C_1 + C_2} + C_3$$

$$C_{AB} = \frac{(1\mu\text{F})(2\mu\text{F})}{(3\mu\text{F})} + 5\mu\text{F} = 5\frac{2}{3}\mu\text{F}$$

$$dw = v'dq' = \frac{q'}{C} dq'$$

↗

work to transfer dq'

When ~~plate~~ potential diff bet. plates is v'

due to q' already having been transferred

$$U \equiv W = \int dw = \int_0^Q \frac{q'}{C} dq' = \frac{Q^2}{2C}$$

This is potential energy stored in the capacitor when fully "charged"

$$Q = \frac{V}{C} \Rightarrow U = \frac{CV^2}{2}$$

can view this as PE stored in the electric field between the capacitor plates

Energy density of E $\equiv u = \frac{U}{\text{Volume between plates}}$

$$\frac{(\text{Area})d}{\text{distance bet plates}} \quad u = \frac{CV^2}{2Ad}$$

Earlier found $C = \frac{\epsilon_0 A}{d}$
(Gaussian + work)

$$u = \frac{\epsilon_0 A V^2}{d 2Ad}$$

$$u = \frac{\epsilon_0}{2} \left(\frac{V}{d}\right)^2$$

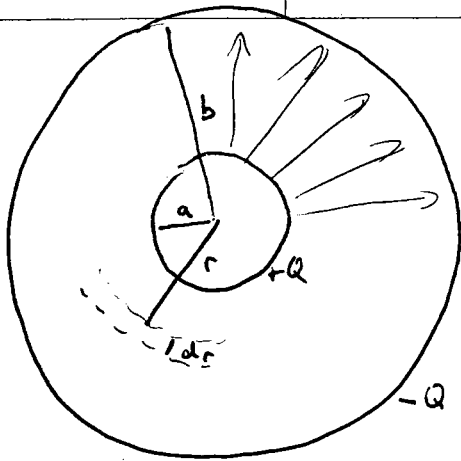
Recall $E = -dV/dx$ but

$$V = Ed \quad \therefore E = \frac{V}{d}$$

Energy density of Electric field

$$u = \frac{\epsilon_0}{2} E^2$$

We used special geometry but this is a general result.



Two concentric conducting spheres

What is energy stored in this system? - $\int E^2$ solve for \vec{E}

This is a great TEST question

$\vec{E} (r < a) = 0$ why?

$$\int E \cdot dA = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{enc} = 0 \quad \therefore E = 0$$

$$\vec{E} (b > r > a) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$

$$\int E \cdot dA = \frac{Q_{enc}}{\epsilon_0} = \frac{+Q}{\epsilon_0}$$

$$E 4\pi r^2 = Q/\epsilon_0$$

$\vec{E} (r > b) = 0$

same as

$$u \equiv \text{energy density} = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \epsilon_0 \left(\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \right)^2$$

$$dU = u dv$$

go from r to $r+dr$ $dv = 4\pi r^2 dr$

$$dU = \frac{1}{2} \epsilon_0 \left(\frac{1}{4\pi\epsilon_0} \right)^2 \frac{Q^2}{r^4} dr 4\pi r^2$$

$$dU = \frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{Q^2}{r^2} dr$$

$$U = \int_a^b \frac{Q^2}{8\pi\epsilon_0} \frac{1}{r^2} dr = \frac{Q^2}{8\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$$

This is not what you learn in calculus. You learn it here in physics.

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