The work to carry a charge q_0 from one plate to the other is Vq_0 or $q_0|\vec{E}|d$

$\int q_0 \vec{F} \cdot d\vec{s}$

$\therefore$

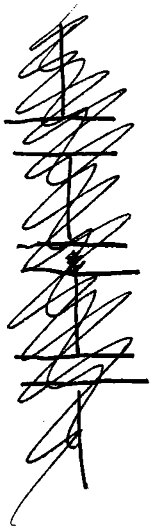
$V = |\vec{E}|d$

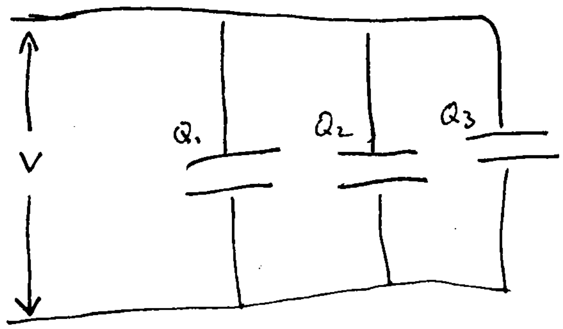
NOW $C = \frac{Q}{V} = \frac{\epsilon_0 |\vec{E}| A}{|\vec{E}| d} = \frac{\epsilon_0 A}{d}$

Depends only on geometry ... As promised.!!

Combinations of Capacitors

in Parallel //





TOTAL $Q = Q_1 + Q_2 + Q_3$

but $Q_1 = C_1 V$

$Q_2 = C_2 V$

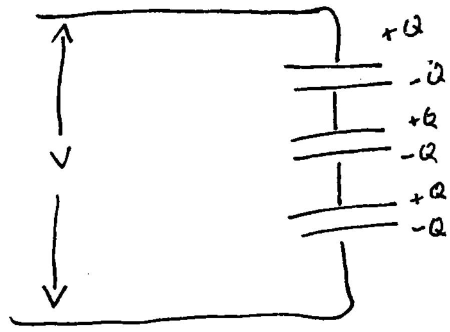
$Q_3 = C_3 V$

$Q_{TOTAL} = V(C_1 + C_2 + C_3)$

$\therefore$

$C_{cap\ in\ //} = C_1 + C_2 + C_3$

Capacitors in Series



$Q = C_1 V_1$

$Q = C_2 V_2$

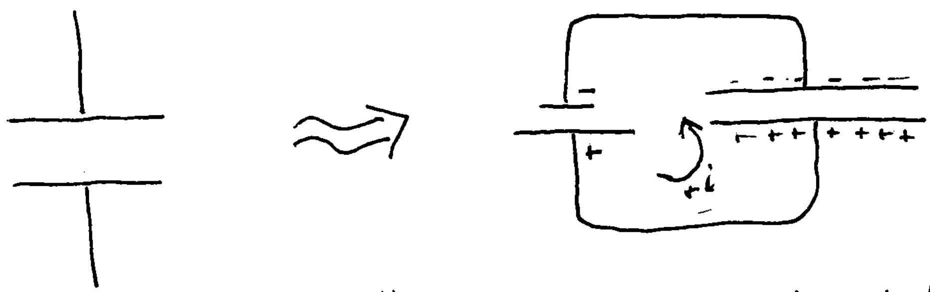
$Q = C_3 V_3$

$$V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

DO EXAMPLE NEXT PAGE

Energy and the Electric field



Hook it up ... e^- transfered from + to - side

or can think

" + current " flows from - to + side

at a given instant V' across plates

$dW = V' dq'$ as q builds up

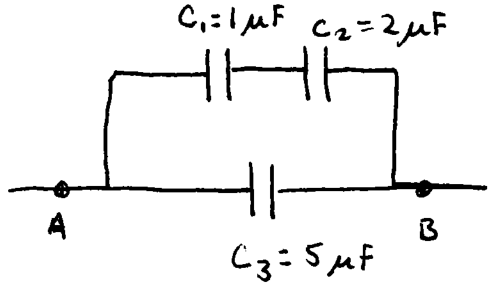
work required to transfer dq' when potential difference across plates is V' and q' has been transfered

Work required to completely charge up plate as far as possible w/ given geometry and total V allowed by battery

~~$W = \int dW = \int_0^Q \frac{q'}{C} dq'$~~

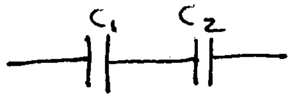
→ Next page

Example Problems



What is Capacitance between A and B?

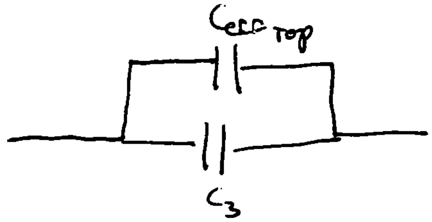
1^{st} find $C_{\text{effective top}}$



$$\frac{1}{C_{\text{eff top}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

$$C_{\text{eff top}} = \frac{C_1 C_2}{C_1 + C_2}$$

Now



capacitors in //

$$C_{AB} = C_{\text{eff top}} + C_3 = \frac{C_1 C_2}{C_1 + C_2} + C_3$$

$$C_{AB} = \frac{(1\,\mu F)(2\,\mu F)}{(3\,\mu F)} + 5\,\mu F = 5\tfrac{2}{3}\,\mu F$$

$$dw = v'\,dq' = \frac{q'}{C}\,dq'$$

↑ work to transfer dq'

When ~~plates~~ potential diff bet. plates is V'

due to q' already having been transferred

$$U \equiv W = \int dw = \int_0^Q \frac{q'}{C}\,dq' = \frac{Q^2}{2C}$$

This is potential energy stored in the capacitor when fully "charged"

$$Q = \frac{V}{C} \Rightarrow U = \frac{CV^2}{2}$$

Can view this as PE stored in the electric field between the capacitor plates

Energy density of E $\equiv u = \dfrac{U}{\text{Volume between plates}}$

(Area) d — distance bet plates

$$u = \frac{CV^2}{2Ad}$$

Earlier found $C = \dfrac{\epsilon_0 A}{d}$ (Gauss's law + work)

$$u = \frac{\epsilon_0 A V^2}{d\,2Ad}$$

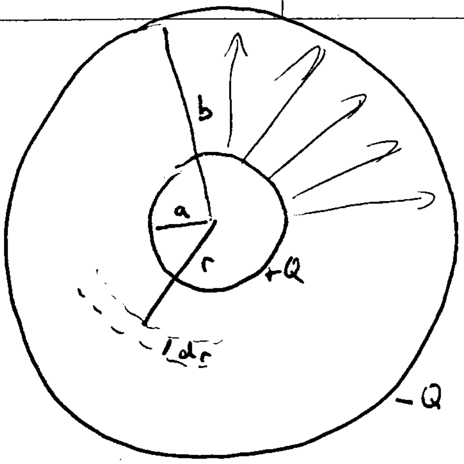
$$u = \frac{\epsilon_0}{2}\left(\frac{V}{d}\right)^2$$

recall $E_x = -dV/dx$ → but $V = Ed \;\therefore\; E = \dfrac{V}{d}$

$$\boxed{u = \frac{\epsilon_0}{2} E^2}$$

Energy density of Electric field

We used special geometry but this is a general result.



Two concentric conducting spheres

What is energy stored in this system? – 1st solve for $\vec{E}$

This is a great test question

$\vec{E} \quad (r<a) = 0 \qquad$ why?

$$\int E \cdot dA = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{encl} = 0 \quad \therefore E = 0$$

$$\vec{E}_{(b > r \geq a)} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$

$$\int E \cdot dA = \frac{Q_{encl}}{\epsilon_0} = \frac{+Q}{\epsilon_0}$$

$$E \, 4\pi r^2 = Q/\epsilon_0$$

$$\vec{E}_{r>b} = 0 \qquad \text{same as}$$

$$u \equiv \text{energy density} = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\right)^2$$

$$dU = u\,dv$$

go from r to $r+dr$ $\quad dv = 4\pi r^2 dr$ – This is NOT what you learn in calculus – You learn it here in physics

$$dU = \frac{1}{2}\epsilon_0\left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{Q^2}{r^4}\,dr\,4\pi r^2$$

$$dU = \frac{1}{2}\frac{1}{4\pi\epsilon_0}\frac{Q^2}{r^2}\,dr$$

$$U = \int_a^b \frac{Q^2}{8\pi\epsilon_0}\frac{1}{r^2}\,dr = \frac{Q^2}{8\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$