This leads to the Schrödinger eqn

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + U\Psi(x,t) + i\hbar\frac{\partial \Psi(x,t)}{\partial t}$$

Put in Potential and solve for $\Psi(x,t)$

$\Psi\Psi \approx$ probability of finding particle in a certain place

& etc beyond scope of this course

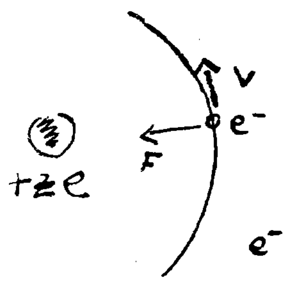
We will NOT be doing formal QM problems

Bohr Model of atom

~~1926~~ 1913 - pre Quantum Mechanics

Extended work of Einstein, Plank

classical view



$$F = \frac{kZe^2}{r^2} = \frac{mv^2}{r}$$ ← velocity

e^- moves in circular orbit

what is potential of Nucleus at radius = r

Potential

$$V(r) = \frac{kZe}{r}$$

recall $V = \frac{W}{q}$ so PE e^- at r $\equiv U = -\frac{Ze^2}{r^2}$ " - " from q of electron

$KE = \frac{1}{2}mv^2$ ← velocity

v^2 (from above) $= \frac{kZe^2}{mr}$

$\therefore KE = \frac{1}{2}\frac{kZe^2}{r}$

$$\text{Total Energy} = PE + KE = -\frac{1}{2}\frac{kZe^2}{r}$$

Such an atom is unstable because

Charge is constantly accelerated

Should radiate energy + spiral inward

Mechanically stable but NOT Electromagnetically stable !

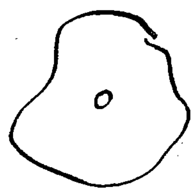
Bohr

⇛ e^- can occupy certain states w/out radiating classically

⇛ e^- can make transitions between these STATES by emitting / Absorbing photons

$$E_\gamma = h\nu = E_i - E_f$$

⇛



Assumed stable orbits happen when Circumference = integral # of λ

$$n\lambda = 2\pi r \qquad n = 1, 2, 3 \ldots$$

$$n\frac{h}{p} = 2\pi r$$

quantized Angular Momentum

$$n\hbar = mvr = L$$

or $v = \frac{n\hbar}{mr}$ — quantized velocity

So

$$F = \frac{kZe^2}{r^2} = \frac{mv^2}{r}$$

$$\frac{kZe^2}{r^2} = \frac{m}{r}\frac{n^2\hbar^2}{m^2r^2}$$

Bohr radius

$$r_n = \frac{n^2\hbar^2}{mkZe^2} = \frac{n^2 a_0}{Z}$$

Assum $Z = 1$, $n = 1$ } orbit of e^- in grnd STATE of H ATOM

$$r = a_0 = \frac{\hbar^2}{mke^2} = 0.0529 \text{ nm}$$

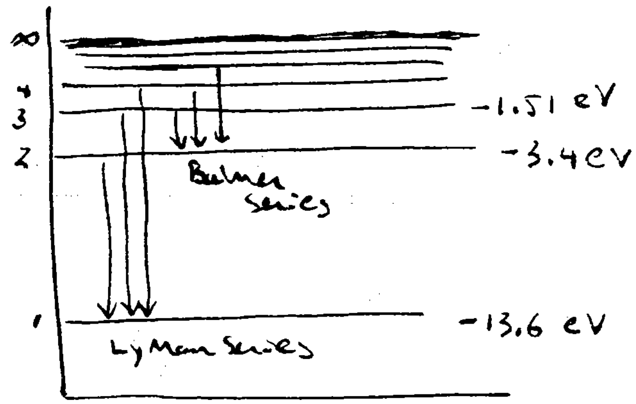
$$E_n = -\frac{1}{2}\frac{kZe^2}{r_n} = -\frac{1}{2}\frac{kZe^2 \, mkZe^2}{n^2\hbar^2}$$

$$E_n = -\frac{mk^2e^4Z^2}{2\hbar^2 n^2} = -\frac{Z^2E_0}{n^2} \qquad n = 1, 2, 3 \cdots$$

↑ Allowed energy levels in the Bohr atom

for H ATOM $Z = 1$

Predicts discrete Spectral emission/Absorption lines



⇒ Agrees w/ experiment!

Big push for Quantum Mechanics!

Examples

(I) What is λ of γ emitted as e^- drops (makes a transition) from the $N=2$ level of hydrogen atom to the $N=1$ level?

$$\Delta E = E_2 - E_1 = -E_0\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = \left(\frac{3}{4}\right)\frac{m_e^2k^2e^4Z^2}{2\hbar^2} = 10.2 \text{ eV}$$

$E = h\nu \qquad 10.2 \text{ eV} = 4.1\times10^{-15} \text{ eV·s } \nu\left(\frac{1}{s}\right) \qquad \nu = 2.49\times10^{15} \frac{1}{s}$ or Hz

$c = \lambda\nu \qquad 3\times10^{8} \text{ m/s} = (\lambda \text{ m})(2.49\times10^{15} \text{ Hz})$

$\lambda = 1.2\times10^{-7} \text{ m}$

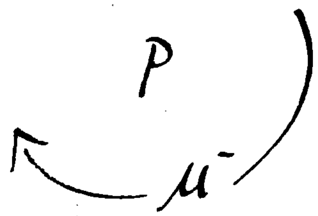
II How Much Energy does it take to remove e^- from H atom
(Minimum Energy)
if it is in $n=2$ energy level

$$\Delta E = E_\infty - E_2 = -E_0\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = E_2 = -3.4\ \text{eV}$$

($\frac{1}{\infty^2} \to 0$)

$$\frac{1}{n_f^2} - \frac{1}{n_i^2}$$

III Suppose one could make "Hydrogen" out of a proton and a Muon (μ^-)



~~[illegible]~~ Muonic Hydrogen

What is the ~~ground state Energy~~ radius of groundstate atom of ~~[illegible]~~ Muonic Hydrogen?

Use Bohr Model – Modify as needed
Bohr's Postulates stay the same

$$n\lambda = 2\pi r$$

$$\hookrightarrow n\hbar = mvr$$

$$v = n\hbar / mr \qquad n = 1, 2, 3 \ldots$$

$$F = \frac{kZe^2}{r^2} = \frac{mv^2}{r} \quad \rightarrow \quad r = \frac{n^2\hbar^2}{m_\mu k Z e^2}$$

e is charge e, μ stays the same!
Z is charge on Nucleus – P, $Z=1$ same

$m_\mu \simeq 2000\, m_e$

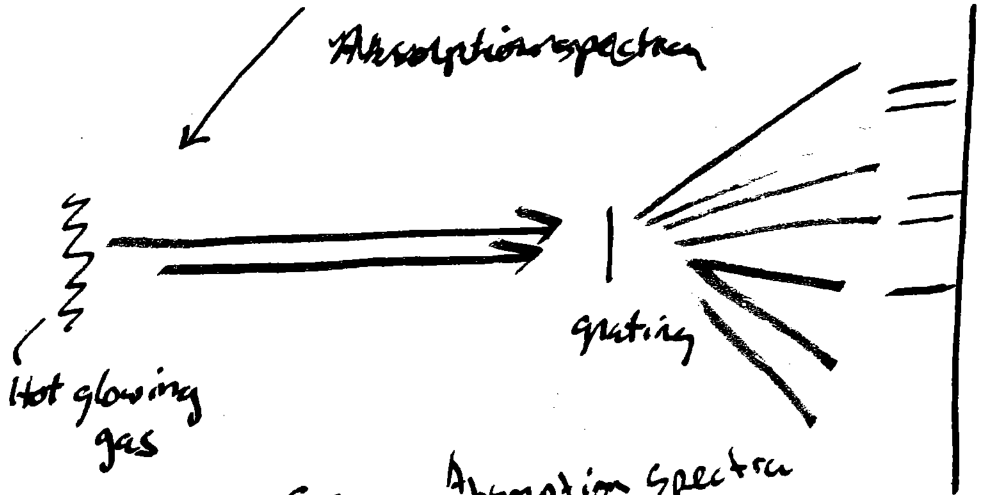
$$r = \frac{n^2\hbar^2}{m_\mu k Z e^2} = \frac{n^2\hbar^2}{2000\, m_e k Z e^2} = \frac{1}{2000} r_0$$

(r_0: Bohr radius, for $n=0$ gnd state)

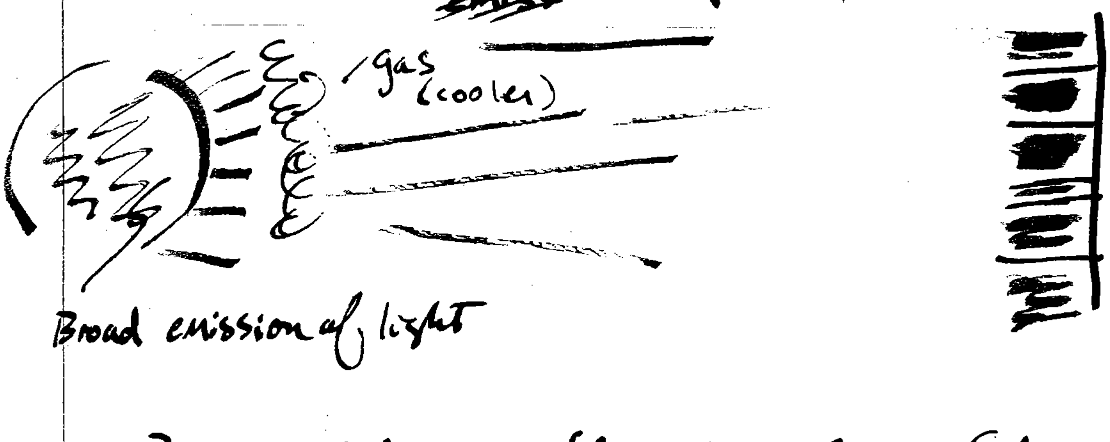
Radius of Muonic hydrogen is $\frac{1}{2000}$ that of Hydrogen

Bohr Model Hugely successful in helping us understand ~~emission~~ emission spectra

Absorption spectra



Absorption Spectra



Bohr Model not sufficient - Solve Schrödinger's Eqn

Can be done Exactly for Single e^- Atom