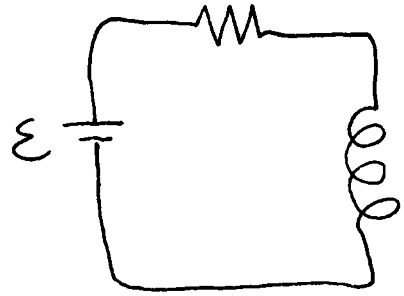
Energy + The Magnetic Field



$$\mathcal{E} = iR + L\frac{di}{dt}$$

$$\mathcal{E}i = i^2R + Li\frac{di}{dt}$$

- $\mathcal{E}i$: $P = iV$ — Power out of EMF
- i^2R: Power dissipated by Resistor
- $Li\frac{di}{dt}$: Power in or out of inductor → rate at which energy stored in the Magnetic field

$$\text{Power} = \frac{dU_B}{dt} = Li\frac{di}{dt}$$

$$dU_B = Li\,di$$

$$U_B = \int_0^I Li\,di = \frac{1}{2}Li^2$$

Energy stored in an inductor carrying current i

(similar to $U = \frac{1}{2}CV^2$ for capacitors!)

Example

Consider a solenoid carrying a current i

n turns/Length

What is the Energy density of the Magnetic field inside?

Find in terms of B.

$$B_{solenoid} = \mu_0 n i \quad \text{inside}$$

(what direction ?)

$$= 0 \quad \text{outside}$$

$$u_B = \text{energy density} = \frac{U_B}{A\ell} \quad \text{(Area)(length)}$$

$$u_B = \tfrac{1}{2} L i^2 / A\ell$$

recall for solenoid

$$\Phi_M \propto i \qquad \Phi_M = L i$$

(# Turns)

$$\text{or} \quad L = \frac{\Phi_M}{i} = \frac{(BA)(n\ell)}{i}$$

$$B_{solenoid} = \mu_0 n i \;\longrightarrow\; L = \mu_0 n^2 A \ell$$

Sub into u_B

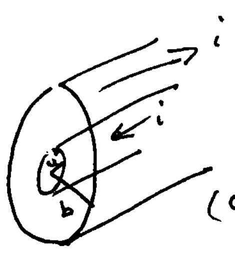
$$u_B = \frac{1}{2}\frac{\mu_0 n^2 A \ell \, i^2}{A\ell} = \frac{1}{2}\mu_0 i^2 n^2 = \frac{B^2}{2\mu_0}$$

$$\boxed{u_B = B^2/2\mu_0}$$

derived w/ Solenoid example
True in general!

Exactly Analogous to $u_E = \frac{1}{2}\epsilon_0 E^2$

<u>Example</u>



Concentric conducting shells
CoAxial

(a) Find B everywhere

(b) Find total U/ℓ for System

Ampere's law

$$\int B \cdot dl = \mu_0 i_{encl}$$

B everywhere

$r < a$ $\quad B\, 2\pi r = 0 \Rightarrow B = 0$

$r > b$ $\quad B\, 2\pi r = \mu_0 i_{enc} = \mu_0(+i - i) = 0$

$\Rightarrow B = 0$

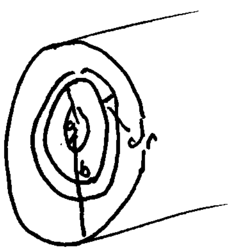
$a < r < b$ $\quad B\, 2\pi r = \mu_0 i$

$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$

Find U_B

$$u_B = \frac{B^2}{2\mu_0} = \frac{\mu_0^2 i^2}{(2\pi)^2 2\mu_0 r^2} = \frac{\mu_0 i^2}{8\pi^2 r}$$

$$dU_B = u_B\, dv = u_B(2\pi r l)\, dr = \frac{\mu_0 i^2}{8\pi^2 r^2} 2\pi r l\, dr$$



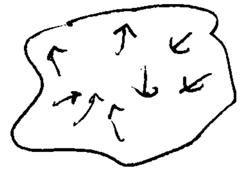
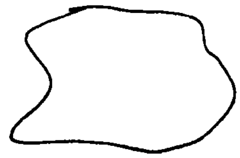
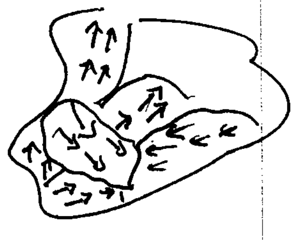
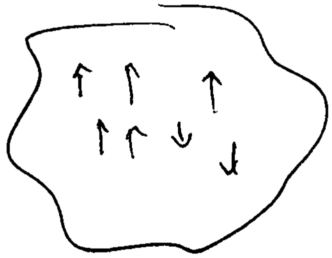
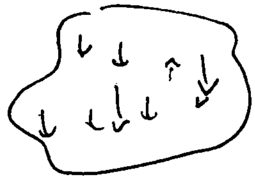
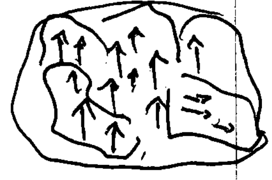
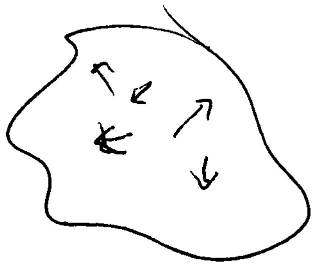
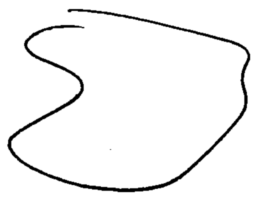
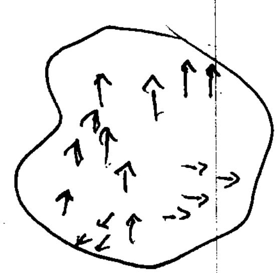
$$dU_B = \frac{\mu_0 i^2 l}{4\pi} \frac{dr}{r}$$

$$U_B = \frac{\mu_0 i^2 l}{4\pi} \int_a^b \frac{dr}{r}$$

$$\boxed{\frac{U_B}{l} = \frac{\mu_0 i^2}{4\pi} \ln\frac{b}{a}}$$

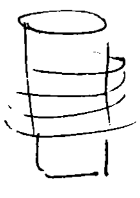
Magnetic Fields and Matter

NOT really covered here but ...

	Para Magnetism	Diamagnetism	Ferro Magnetism
B=0			
	inherent (randomly-oriented) Magnetic Dipole	No inherent Magnetic Dipole	Strong natural Magnetic Dipoles Domains
B large			
	Moments line up w/ field (slight enhancement)	induced Moments opposite to field Weakens field slightly	Domains mostly line up Strong enhancement
B=0			
			Mostly stays lined up

Electromagnets

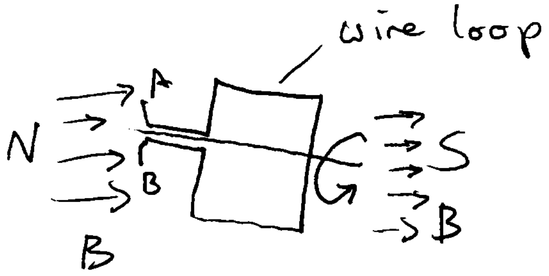
usually have an iron core



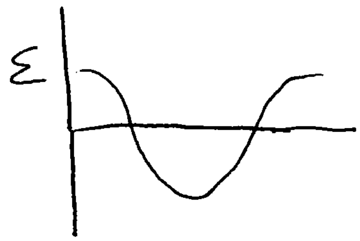
Saturates at 2 Tesla

AC Circuits

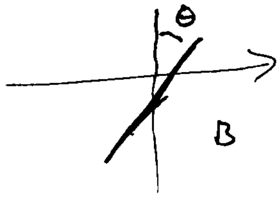
NOT covered in this course but —



Plot V_{AB} induced



Flux depends on Angle



$\phi \propto \cos\theta$

$\theta = \omega t$

$\therefore \frac{d\phi}{dt} \sim \mathcal{E} \sim \sin\theta$

Sinusoidally Varying $\mathcal{E}$

Alternating current

⇒ Time dependence in everything!

⇒ More complex than Direct Current

⇒ This is what you get out of the wall

60 Hz Frequency

Know your limitations!

look for it as a Noise Source.