Review of what we have learned in E+M

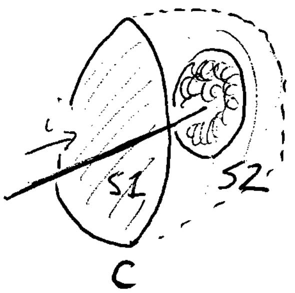
① $\int_s \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$ Gauss' Law

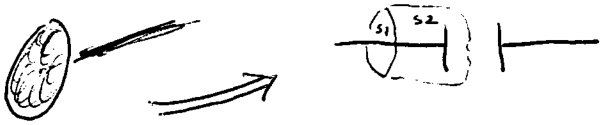
② $\int_s \vec{B}\cdot d\vec{A} = 0$ (Gauss's law for B) No Magnetic Monopoles

③ $\int_c \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\Phi_M = -\frac{d}{dt}\int_s \vec{B}\cdot d\vec{A}$ Faradays law (induced EMF's from changing Magnetic flux)

$\int_c \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$ Ampere's Law

Becomes





Current thru S1 is I

Current thru S2 is 0?

Maxwell fixed this omission by adding a term

④ $$\int_c \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0\epsilon_0 \frac{d\Phi_E}{dt} = \mu_0 I + \mu_0\epsilon_0 \frac{d}{dt}\int_s \vec{E}\cdot d\vec{A}$$

Equations ①-④ are the integral form of Maxwell's Equations

These equations describe classical electromagnetic phenomena

This is most of what you have in your life experience

Note again the slight Asymmetry between $\vec{E}$ and $\vec{B}$

Implications of Maxwell's Equations - radiation, light

Conceptual:

$$\int_c \vec{E}\cdot d\vec{l} \sim \frac{d}{dt}\int_s \vec{B}\cdot d\vec{A} \Longrightarrow \frac{dB}{dt} \rightarrow E \text{ Δ'ing w/ } t$$

$$\int_c \vec{B}\cdot d\vec{l} \sim \frac{d}{dt}\int_s \vec{E}\cdot d\vec{A} \Longrightarrow \frac{dE}{dt} \rightarrow B \text{ Δ'ing w/ } t$$

Changing E induces changing B

Changing B induces changing E

Circular ⇝ Radiation

Mathematical:

Maxwell's Equations

↓ Using Vector Calculus Tricks → Equivalent To

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \frac{\partial^2 E_x}{\partial t^2}\mu_0\epsilon_0$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = \frac{\partial^2 E_y}{\partial t^2}\mu_0\epsilon_0$$

$$\frac{\partial^2 E_z}{\partial x^2} + \dots$$

$$\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} = \frac{\partial^2 B_x}{\partial t^2}\mu_0\epsilon_0$$

$$\frac{\partial^2 B_y}{\partial x^2} + \dots$$

$$\frac{\partial^2 B_z}{\partial x^2} + \dots$$

Six Coupled Partial differential Equations

Looks quite complicated ... And can be complicated

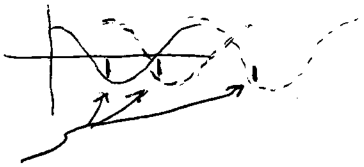
Lets Try A simple situation and see what happens

Physics depends on "Boundary Conditions"

Consider the phase $\omega(t \pm z/c)$ = CONSTANT

$$\frac{\partial(\text{phase})}{\partial t} = \omega$$

$$\frac{\partial(\text{phase})}{\partial z} = \pm\frac{\omega}{c}$$



Points w/ same phase as wave propagates

$$\frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3\times10^{8}\ \text{M/s}$$

$$\frac{\dfrac{\partial(\text{phase})}{\partial t}}{\dfrac{\partial(\text{phase})}{\partial z}} = \frac{\partial z}{\partial t} = \pm c$$

Velocity of propagation of condition of CONSTANT phase

– or – phase velocity of light

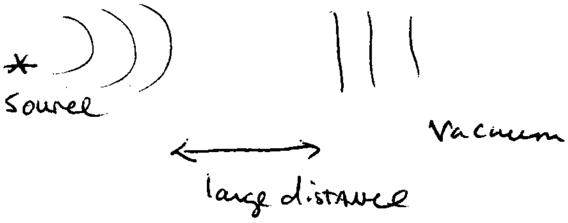
Maxwell's eqns predicts velocity of light in vacuum !

implications of Maxwell's eqns –

With somewhat more Mathematics than we'll want to use one can derive from Maxwell's eqns in a vacuum

- Radiation (light, x rays, infrared, UV ... etc) made up of electric and Magnetic fields;

For case



- Electromagnetic waves TRAVEL w/ $V = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \equiv c$
- $\vec{E}$ + $\vec{B}$ fields ⊥ to eachother and direction of TRAVEL

TRANSVERSE waves

Transparency 39
Figure 32-3, page 1003
Electric and magnetic field vectors in an electromagnetic wave

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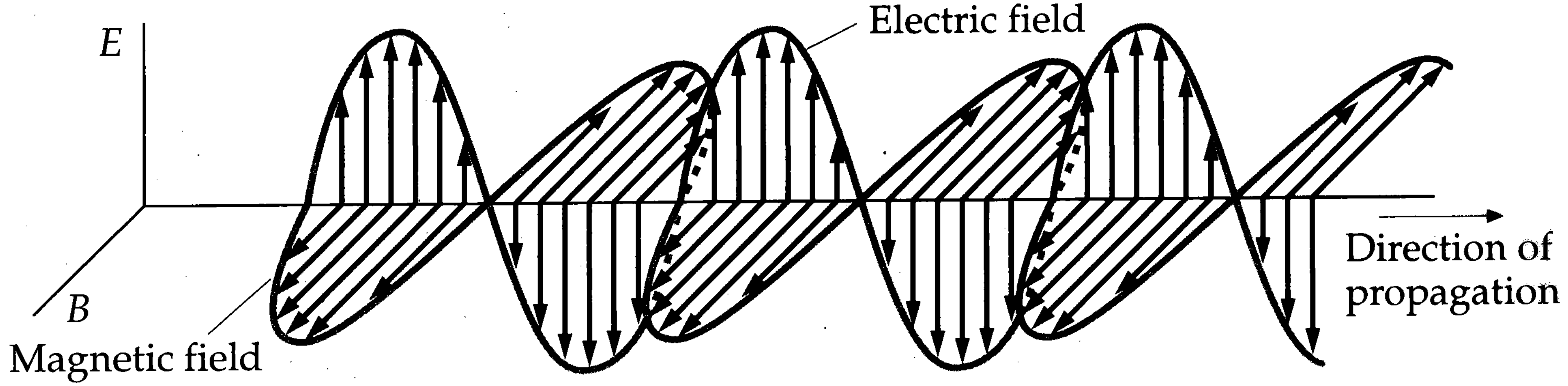
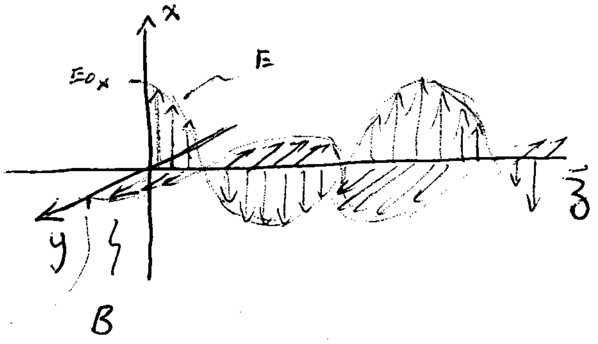


~~Eq.~~ - $\vec{E}$, $\vec{B}$ in phase w/ one another

- $|\vec{E}| = c|\vec{B}|$
- Harmonic in both space + Time
  (waves w/ space + Time dependence)

e.g. $E_x = E_{0x} \cos[\omega_x (t \pm z/c)]$

$B_y = \frac{E_{0x}}{c} \cos[\omega_x (t \pm z/c)]$

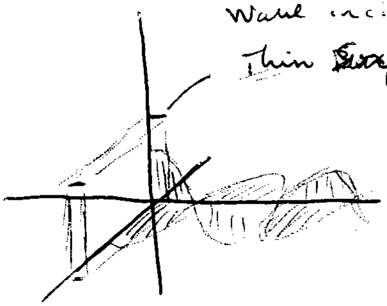


$\frac{E_{0x}}{c} = B_{0x}$

ω is frequency of the electromagnetic radiation

some eqns → $\nu = \frac{1}{T}$ $\omega = \frac{2\pi}{T}$ $k = \frac{2\pi}{\lambda}$ $c = \omega/k$ $c = \lambda\nu$

sometimes see phase written as $\omega t \pm kz$



Wave incident
Thin ~~Sur~~face

- What is $\frac{\text{Power incident}}{\text{Area}}$ = intensity?

$I = u_{ave}\, c$

$u = u_E + u_M = \frac{1}{2}\epsilon_0 E^2 + \frac{B^2}{2\mu_0} = \epsilon_0 E^2 = \frac{EB}{\mu_0 c}$

$c^2 = \frac{1}{\mu_0 \epsilon_0}$ and $E = cB$

Replace $E_{\text{instantaneous}}$, $B_{\text{instantaneous}}$
w/ RMS values ~ Average values

$E_{RMS} = E_0/\sqrt{2}$ $B_{RMS} = B_0/\sqrt{2}$

$$\boxed{I = \frac{1}{2}\frac{E_0 B_0}{\mu_0 c}}$$

define $\vec{S} = \dfrac{\vec{E} \times \vec{B}}{\mu_0}$ Poynting vector

direction gives the direction of propagation of EM wave

$|\vec{S}|$ = intensity

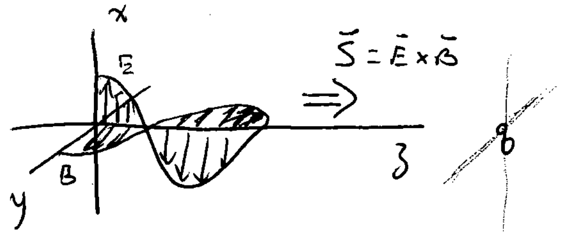
EM waves carry energy

Power incident / unit Area is the intensity $\equiv$ Energy/Time/Area

- radio station energy to run radio (usually amplified where picked up)
- Energy from sun for photosynthesis
- Energy to cause a photochemical reaction w/ film for x rays

etc.

EM waves carry momentum $F = \dfrac{dP}{dt}$



when

F on q due to E is $qE\hat{i}$

q gets $\vec{V} = v\hat{i}$

F from $\vec{B} = (B)\hat{j}$

is

$F_B = q\vec{V} \times \vec{B} = qVB\ \hat{i} \times \hat{j} = qvB\hat{k}$

Force is along direction of wave propagation

relativity $E^2 = (mc^2)^2 + (pc)^2$

$E = pc$

$|P| \equiv$ Momentum carried by EM wave $= \dfrac{\text{Energy}}{c}$

Intensity = Energy/Time/Area   Momentum/Time = Force

$\dfrac{I}{c}$ = Energy/c/Time/Area $= \dfrac{\text{Force}}{\text{Area}}$ ~~Momentum~~ = Pressure radiation exerts pressure on charged particles!

- Sailing Space ships
- radiation Pressure ⟷ gravity

Equilibrium in SUN

Stellar evolution in a nutshell