In one dimensional problems

$$E_x = -\frac{dV(x)}{dx} \quad \text{if} \quad V = V(x)$$

$$E_y = -\frac{dV(y)}{dy} \quad \text{if} \quad V = V(y)$$

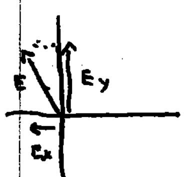
$$E_z = -\frac{dV(z)}{dz} \quad \text{if} \quad V = V(z)$$

$$E_r = -\frac{dV(r)}{dr} \quad \text{if} \quad V = V(r)$$

2 dimensional example -

$$V(x,y) = 3 + 2x - 6y \quad \underbrace{\text{Joules/coulomb}} \text{ or } \equiv \text{ Volts}$$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{dV}{dx}\Big|_{y=\text{const}} = -2 \ \text{NT/coul}$$

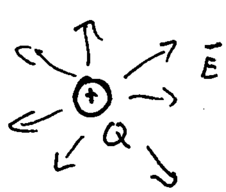


$\Rightarrow$ How do I know this is Joules/coul?

$$E_y = \frac{\partial V}{\partial y} = -\frac{dV}{dy}\Big|_{x=\text{const}} = +6 \ \text{NT/coul}$$

$\Rightarrow$ How do I know this is NT/coul?

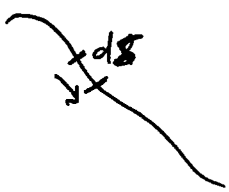
another example - pT charge what is $\vec{E}$ for a pT charge?



Have shown $V(r) = \frac{kQ}{r}$

$$\vec{E}(r) = -\frac{dV(r)}{dr}\hat{r} = +\frac{kQ}{r^2}\hat{r}$$

Equipotential Surface



$\underbrace{dV = 0}$

Surface where V is CONSTANT

Since $dV = -\vec{E}\cdot\vec{ds}$

$\hookrightarrow$ implies $\vec{E} = 0$ or $\underbrace{\vec{E} \perp \text{ to } \vec{ds}}$

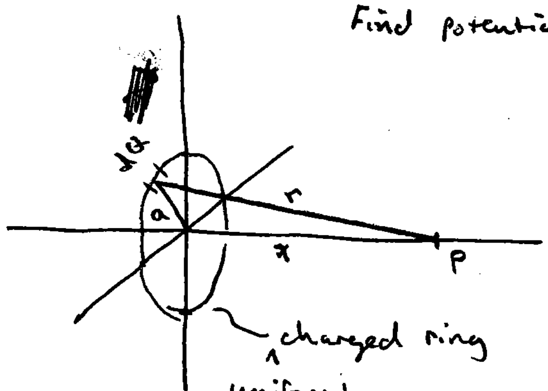
$\llcorner$ More general!

(Harder than $\vec{E}$)

Calculate Potential Directly - often Hard w/out special Techniques beyond scope of course

usually easier than $\vec{E}$ w/ special Techniques.

Find potential at pt P



uniformly charged ring

$$V = \int_{\text{ring}} \frac{k\,dQ}{r}$$

$$V = \int \frac{k\,dQ}{\sqrt{x^2+a^2}}$$

$dQ = \lambda \underbrace{ds}_{\text{arclength}}$

$\frac{1}{\sqrt{x^2+a^2}}$ constant as integrate around ring over ds

$$V = \frac{k\lambda}{\sqrt{x^2+a^2}} \underbrace{\int_0^{2\pi} ds}_{2\pi a} = \frac{k\lambda 2\pi a}{\sqrt{x^2+a^2}} = \frac{kQ}{\sqrt{x^2+a^2}}$$

in limit of x large → get $\frac{kQ}{r}$ potential of pt charge!

Calculate $\vec{E}$ from V

from symmetry E is in $\hat{x}$

$$E_x = -\frac{d\left[\frac{kQ}{(x^2+a^2)^{1/2}}\right]}{dx} = \cancel{kQ\,\frac{1}{2}\,\frac{2x}{\sqrt{x^2+a^2}}} \quad kQ\,\frac{1}{2}\,\frac{2x}{(x^2+a^2)^{3/2}}$$

$$= \frac{kQ\,x}{(x^2+a^2)^{3/2}}$$

⇒ How do I check this?

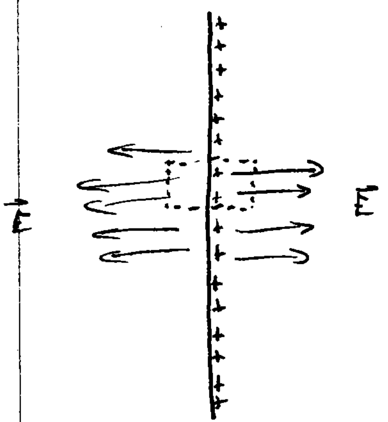
Again in large x limit

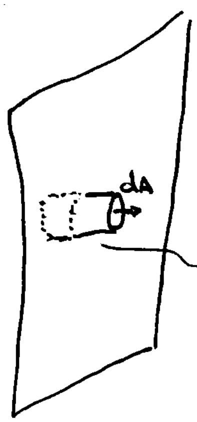
$$E_x = \frac{kQ}{x^2}$$

Field of pt charge w/ net charge Q!

Another example

This time use E to get V





stubby cylinder

$$\int \bar{E}\cdot d\bar{A} = |E|\int dA = \frac{Q_{encl}}{\epsilon_0}$$

$$Q_{encl} = \sigma A$$

2 x ends

$$2|E|A = \frac{\sigma A}{\epsilon_0}$$

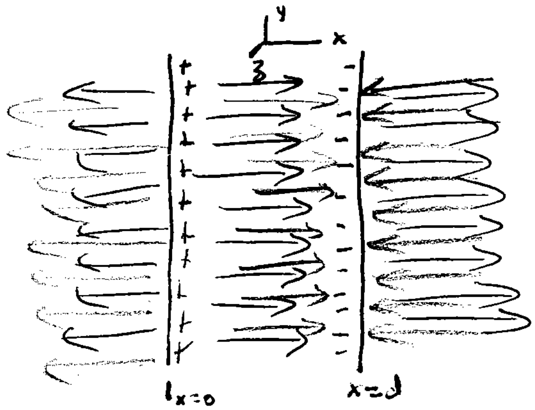
$$|E| = \frac{\sigma}{2\epsilon_0}$$

∞ plane

$dQ = \sigma\, dA$

uniform charge density

Now for problem



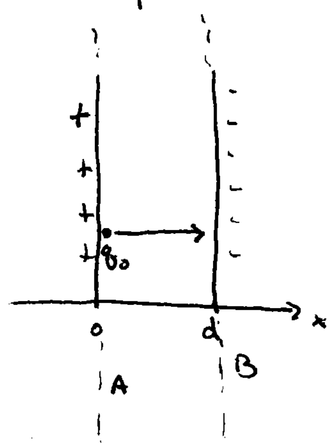
use two transparencies

$$E_{between} = \frac{\sigma}{\epsilon_0}\hat{x}$$

2 ∞ conducting planes // to each other

each w/ σ

Put + plane at $x=0$ - plane at $x = d$



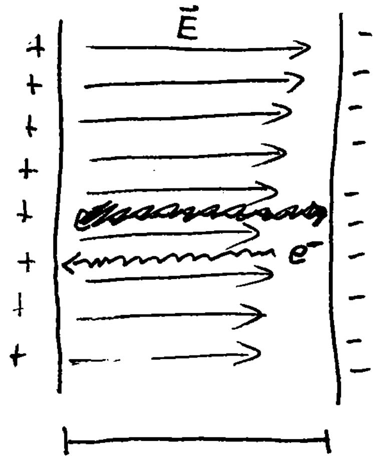
move q_0 from A to B plane

$$dV = -\bar{E}\cdot d\bar{s} = -|E|\,dx$$

$$V_d - V_0 = \int_0^d |E|\,dx = |E|\,d$$

⇒ You ever heard of the electronvolt … chemical ionization energies etc.

The eV ≡ electron-Volt unit of energy



∞ parallel planes

Let e^- go from rest from $\ominus$ side

$\vec{F} = q\vec{E}$ ⇒ is accel. constant?

$\vec{F}$ constant

$\vec{F} = m\vec{a}$

∴ constant acceleration problem

$\Delta V = 1\ \text{Volt} = 1\ \text{Joule}/\text{coulomb}$

All kinematic eqns valid for constant acceleration hold true

for example ....

1d Motion w/ Const. Acceleration

↳
$$v_x = v_{x_0} + a_x t$$
$$x = x_0 + \tfrac{1}{2}(v_{x_0} + v_x)/t$$
remember this?
$$x = x_0 + v_{x_0}t + \tfrac{1}{2}a_x t^2$$
$$v_x^2 = v_{x_0}^2 + 2a_x(x - x_0)$$

We say … "the e^- is accelerated through a potential difference of 1 Volt"

When e^- has reached other side PE changed to KE

$$V = \frac{\Delta W}{q} \times \underbrace{q_{e^-}}_{\text{known as } e} \simeq \Delta W = K.\text{Energy} = 1\ \text{volt} \times |e|$$
$$= 1\ \text{electron-Volt}$$

Better for charged particles in small #'s

$$1\ \text{electron Volt} = (1|e|)(1\ \text{Volt}) = (1.6\times10^{-19}\ \text{Coul})(1\ \text{Volt})$$
$$= 1.6\times10^{-19}\ \text{joule}$$

Calculate $|\vec{E}|$

① use Coulomb's law $\vec{E} = \int \frac{k\,dq}{r^2}\hat{r}$

② use Gauss's law $\int \bar{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$

③ Calculate Potential

one way $\int \frac{k\,dq}{r}\hat{r} = V$

and $\bar{E} = -\text{grad}\,V \rightsquigarrow E_x = -\frac{\partial V}{\partial x}$

⋮ etc.

Calculate potential

① use calculation from charge dist

$\int \frac{k\,dq}{r}\hat{r} = V$

② use $\bar{E}$

$dV = -E \cdot ds$

And integrate

③ other ways we will NOT do